

# Measurement of Vibrations of a Plate on Elastic Foundation

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## 1. Introduction

Currently in the European Union social integration of visually disabled persons into society is an important problem. For this integration of blind people devices with Braille elements in their control blocks are designed and produced. Some types of Braille elements experience wear under the influence of small amplitude high frequency vibrations. Also reading of the text represented by Braille elements under the influence of vibrations causes problems and unexperienced readers may not always fully understand the represented text.

In this paper the use of elastic foundation to reduce the vibrations of the elastic structure with Braille elements with the purpose of increasing the durability and decreasing of wear of elements of the control block for various mechanical devices is investigated.

For this purpose, vibrations of a plate on elastic foundation of Winkler type are investigated. A two-dimensional element having six nodal degrees of freedom (three displacements of the lower surface and three displacements of the upper surface) is used. Elastic foundation of Winkler type on the lower surface has been used in the investigation. Eigenmodes are calculated.

Experimental investigations of related problems of analysis of vibrations were presented in [1, 2]. The numerical procedure is based on the material described in [3, 4]. Experimental and numerical investigations of dynamics of similar structures are presented in [5 – 42].

In the previous papers of the authors vibrations of a plate with Braille elements were investigated in detail. As noted there using the results of performed experimental investigations Braille elements do not have substantial effect to the shape of the eigenmodes. Because of this fact numerical investigations of vibrations of the elastic structure without Braille elements is performed. Recommendations for location of Braille elements on a vibrating structure are provided.

The purpose of this investigation is to determine how vibrations influence the dynamics of an elastic structure on the elastic foundation with Braille elements and to determine the places of the elastic structure in which the wear of Braille elements is greatest.

## 2. Numerical model for the analysis of vibrations of a plate on elastic foundation of Winkler type

$x$ ,  $y$  and  $z$  denote the axes of coordinates and  $u$ ,  $v$  and  $w$  denote the corresponding displacements. Displacements in the finite element are represented as:

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \frac{H-z}{H} [\bar{N}] \{\delta\} + \frac{z}{H} [\bar{\bar{N}}] \{\delta\}, \quad (1)$$

where:  $H$  is the thickness of the plate,  $\{\delta\}$  is the vector of nodal displacements and:

$$[\bar{N}] = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & N_1 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & N_1 & 0 & 0 & 0 & \dots \end{bmatrix}, \quad (2)$$

$$[\bar{\bar{N}}] = \begin{bmatrix} 0 & 0 & 0 & N_1 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & N_1 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & N_1 & \dots \end{bmatrix}, \quad (3)$$

where:  $N_1, N_2, \dots, N_9$  are the shape functions of the two-dimensional Lagrange quadratic finite element.

The strains are represented as:

$$\{\varepsilon\} = \frac{1}{H} [B] \{\delta\} + \frac{H-z}{H} [\bar{B}] \{\delta\} + \frac{z}{H} [\bar{\bar{B}}] \{\delta\}, \quad (4)$$

where:

$$[B] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & -N_1 & 0 & 0 & N_1 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & -N_1 & 0 & 0 & N_1 & 0 & \dots \\ -N_1 & 0 & 0 & N_1 & 0 & 0 & \dots \end{bmatrix}, \quad (5)$$

$$[\bar{B}] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \frac{\partial N_1}{\partial y} & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & \frac{\partial N_1}{\partial y} & 0 & 0 & 0 & \dots \\ 0 & 0 & \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & \dots \end{bmatrix}, \quad (6)$$

$$[\bar{\bar{B}}] = \begin{bmatrix} 0 & 0 & 0 & \frac{\partial N_1}{\partial x} & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & \dots \\ 0 & 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial x} & \dots \end{bmatrix}. \quad (7)$$

The following integrals are used in the expressions of element matrixes:

$$\int_0^H \frac{H-z}{H} \frac{z}{H} dz = \frac{H}{6}, \quad (8)$$

$$\int_0^H \left(\frac{H-z}{H}\right)^2 dz = \int_0^H \left(\frac{z}{H}\right)^2 dz = \frac{H}{3}, \quad (9)$$

$$\int_0^H \frac{1}{H} \frac{H-z}{H} dz = \int_0^H \frac{1}{H} \frac{z}{H} dz = \frac{1}{2}, \quad (10)$$

$$\int_0^H \left(\frac{1}{H}\right)^2 dz = \frac{1}{H}. \quad (11)$$

The mass matrix has the form:

$$[M] = \int \left( [\bar{N}]^T \rho \frac{H}{6} [\bar{N}] + [\bar{\bar{N}}]^T \rho \frac{H}{6} [\bar{\bar{N}}] + [\bar{N}]^T \rho \frac{H}{3} [\bar{N}] + [\bar{\bar{N}}]^T \rho \frac{H}{3} [\bar{\bar{N}}] \right) dx dy, \quad (12)$$

where:  $\rho$  is the density of material of the investigated elastic structure.

The stiffness matrix has the form:

$$[K] = \int \left( [B]^T [D] \frac{1}{2} [\bar{B}] + [\bar{B}]^T [D] \frac{1}{2} [B] + [B]^T [D] \frac{1}{2} [\bar{\bar{B}}] + [\bar{\bar{B}}]^T [D] \frac{1}{2} [B] + [\bar{B}]^T [D] \frac{H}{6} [\bar{\bar{B}}] + [\bar{\bar{B}}]^T [D] \frac{H}{6} [\bar{B}] + [\bar{B}]^T [D] \frac{H}{3} [\bar{B}] + [\bar{\bar{B}}]^T [D] \frac{H}{3} [\bar{\bar{B}}] + [B]^T [D] \frac{1}{H} [B] + [\bar{N}]^T \begin{bmatrix} \mu & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu_z \end{bmatrix} [\bar{N}] \right) dx dy, \quad (13)$$

where:  $\mu$  is the stiffness of the elastic foundation of Winkler type for plane motion and  $\mu_z$  is the stiffness of the elastic foundation of Winkler type for motion in the direction of the  $z$  axis and:

$$[D] = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K + \frac{4}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K - \frac{2}{3}G & K + \frac{4}{3}G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{G}{1.2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{G}{1.2} \end{bmatrix}, \quad (14)$$

where:  $K = \frac{E}{3(1-2\nu)}$ ,  $G = \frac{E}{2(1+\nu)}$ , where:  $E$  is modulus of elasticity and  $\nu$  is Poisson's ratio.

### 3. Eigenmodes of vibrations of the plate with elastic foundation of Winkler type

The structure is a circle with internal radius 0.02 m and external radius 0.04 m. Thickness of the structure  $H = 0.004$  m. The following parameters are assumed:

modulus of elasticity  $E = 6 \cdot 10^8$  Pa, Poisson's ratio  $\nu = 0.3$ , density of the material  $\rho = 785 \frac{\text{kg}}{\text{m}^3}$ . Elastic foundation is assumed on the lower surface of the structure for the elements located in the region from the internal radius of the structure up to the average radius of the structure with the parameters  $\mu = 10000 \frac{\text{N}}{\text{m}^3}$ ,  $\mu_z = 400000 \frac{\text{N}}{\text{m}^3}$ .

The first eigenmode is presented in Fig. 1, the second eigenmode is presented in Fig. 2, ..., the eighth eigenmode is presented in Fig. 8. When the motion of the lower surface and the motion of the upper surface are very similar, then only the motion of the upper surface is presented. For the investigated eigenmodes out of plane motions of the upper and lower surfaces are very similar, while plane motions of the upper and lower surfaces are very similar only for the first eigenmode.

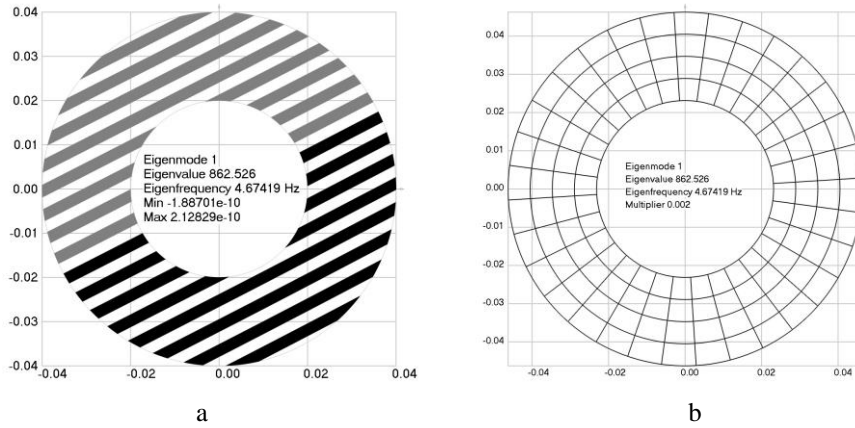


Fig. 1 The first eigenmode: a - out of plane motion of the upper surface, b - plane motion of the upper surface

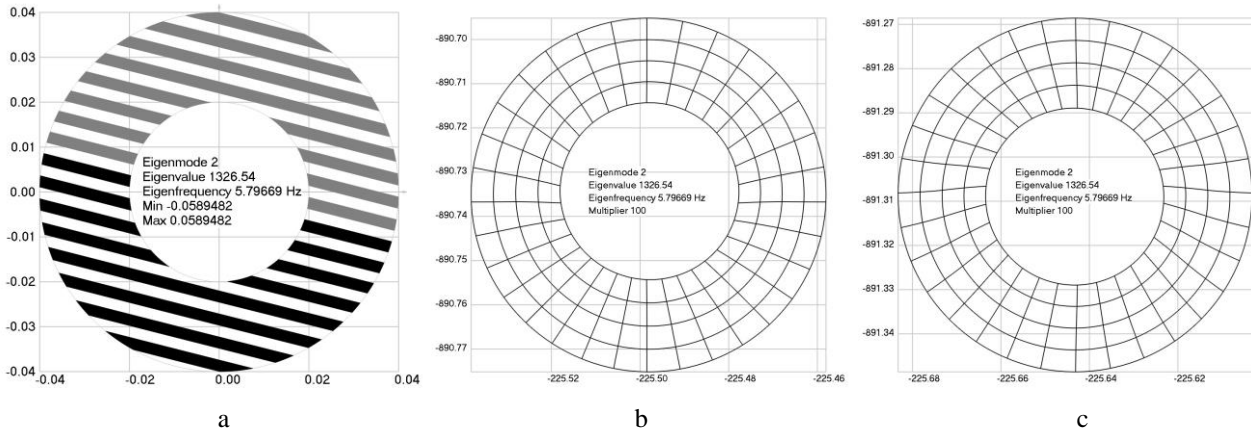


Fig. 2 The second eigenmode: a - out of plane motion of the upper surface, b - plane motion of the lower surface, c - plane motion of the upper surface

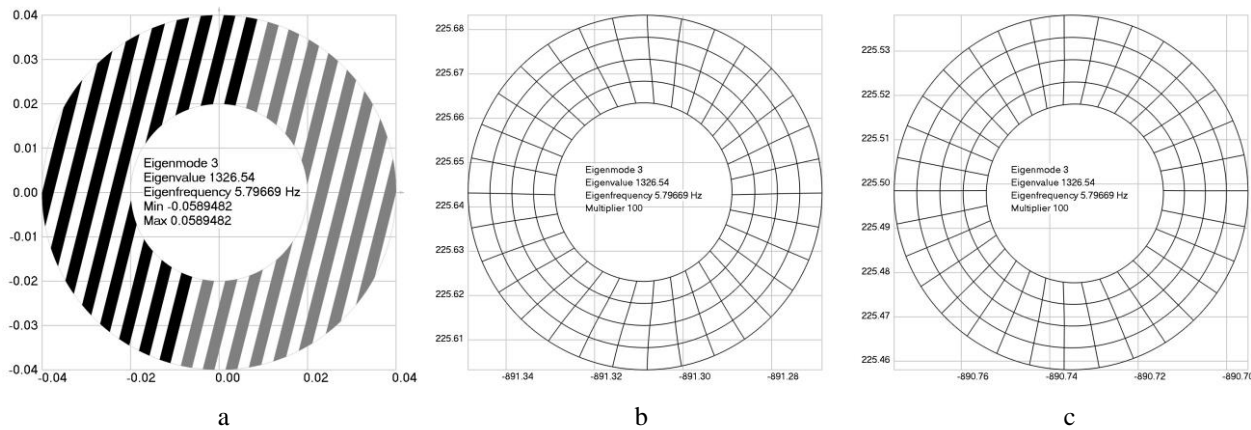


Fig. 3 The third eigenmode: a - out of plane motion of the upper surface, b - plane motion of the lower surface, c - plane motion of the upper surface

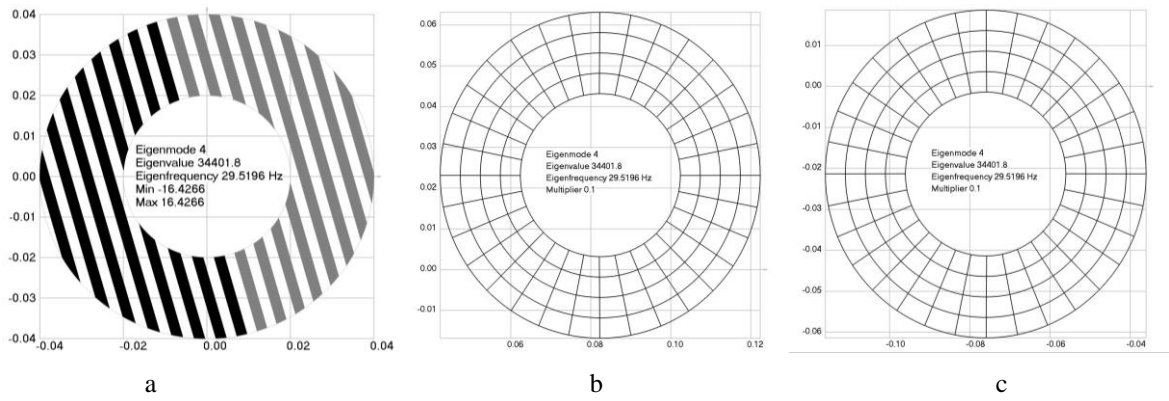


Fig. 4 The fourth eigenmode: a - out of plane motion of the upper surface, b - plane motion of the lower surface, c - plane motion of the upper surface

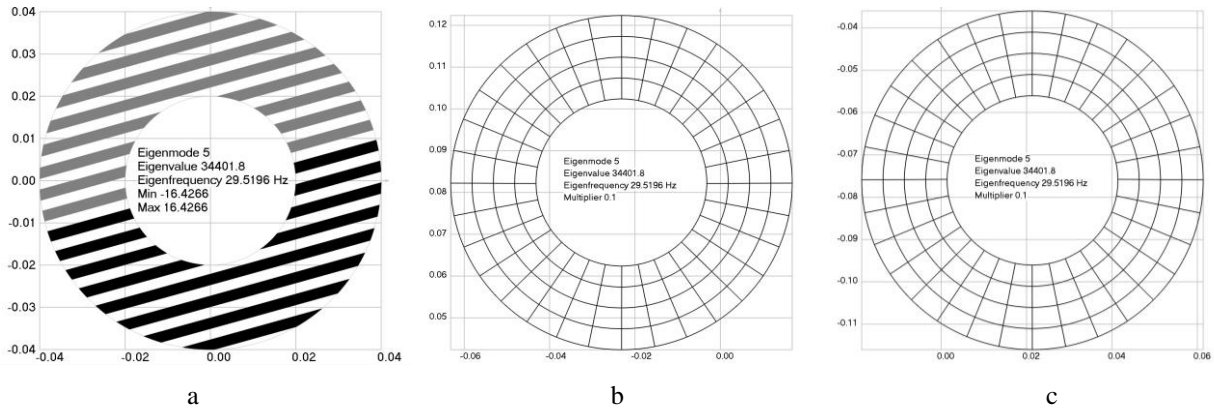


Fig. 5 The fifth eigenmode: a - out of plane motion of the upper surface, b - plane motion of the lower surface, c - plane motion of the upper surface

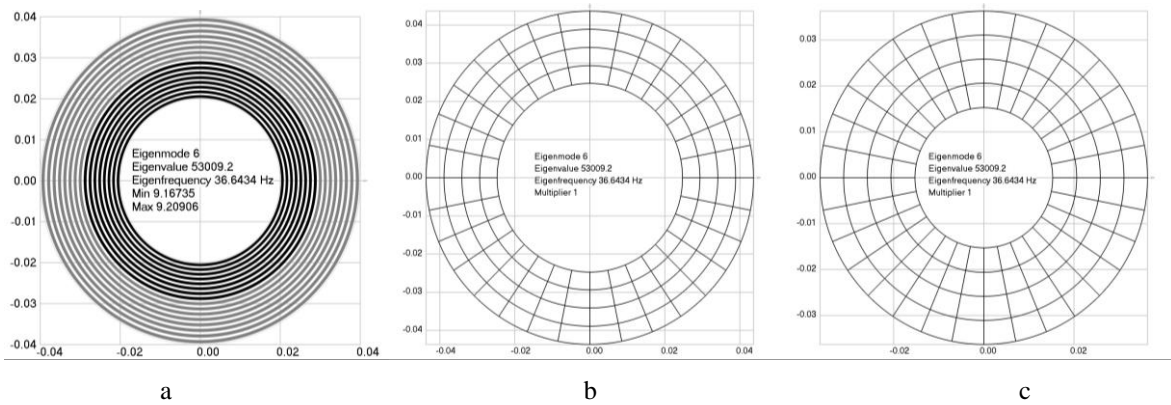


Fig. 6 The sixth eigenmode: a - out of plane motion of the upper surface, b - plane motion of the lower surface, c - plane motion of the upper surface

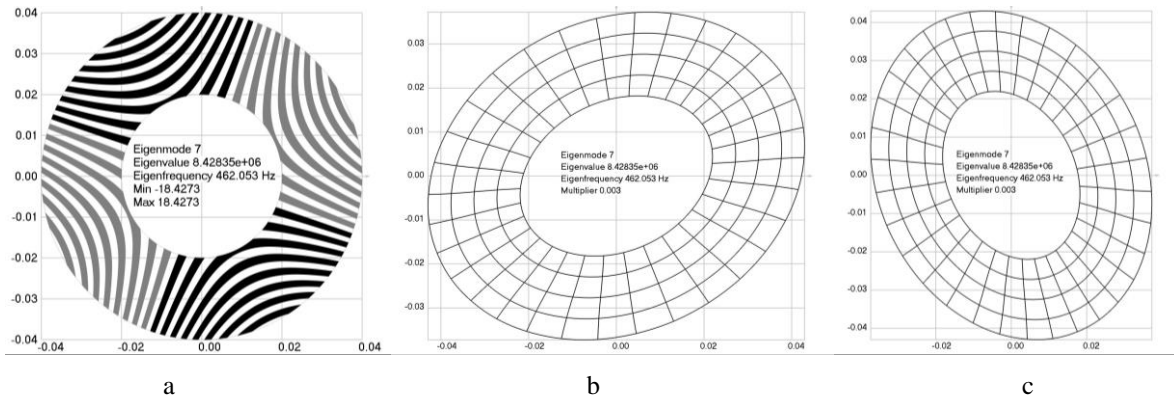


Fig. 7 The seventh eigenmode: a - out of plane motion of the upper surface, b - plane motion of the lower surface, c - plane motion of the upper surface

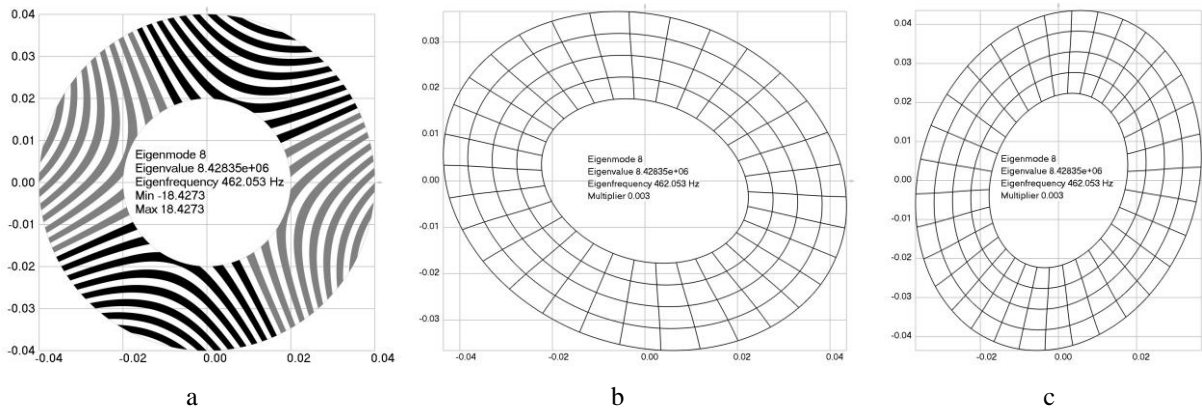


Fig. 8 The eighth eigenmode: a - out of plane motion of the upper surface, b - plane motion of the lower surface, c - plane motion of the upper surface

From the presented results the out of plane and plane motions of the structure are clearly seen. In many of the investigated eigenmodes motions of the lower and upper surfaces are very similar, but there are eigenmodes where the motions of the surfaces differ.

Using the presented results places for recommended locations of Braille elements are determined. It is recommended to locate them at the places where the amplitude of vibrations is smallest.

#### 4. Results of experimental study of vibrations of the plate with elastic foundation

Experimental investigation of vibrations of the investigated elastic structure was performed by using the vibrometer Polytec PSV – 500. Structural scheme of experimental setup is presented in Fig. 9.

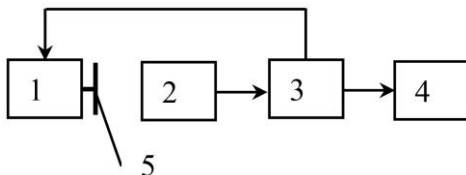
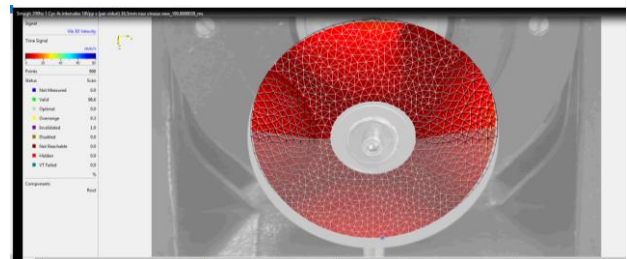


Fig. 9 Structural scheme of experimental setup: 1 – exciter of vibrations, 2 – scanning heads, 3 – vibrometer Polytec PSV-500, 4 – personal computer, 5 – elastic structure with Braille elements

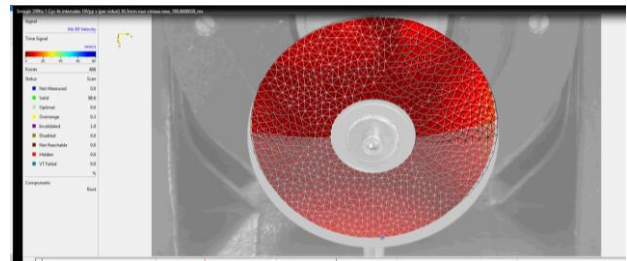
Some of the obtained results of quantitative investigations of vibrations of the elastic structure on elastic foundation with Braille elements for frequency of vibrations equal to 20 Hz and for various moments of time are presented in Fig. 10. In the numerical results presented earlier the eigenfrequencies are indicated in the figures of the eigenmodes. As seen from the presented results the frequency which is near to 20 Hz is observed in the fourth eigenmode (see Fig. 4). It is not possible to expect very precise correspondence of experimental and numerical results because of the simplified model of elastic foundation of Winkler type used in the numerical calculations. In order to obtain more precise correspondence of experimental and numerical results more complicated model of elastic foundation must be used in the numerical model.

From the obtained experimental results places for recommended locations of Braille elements as well as stiffness parameters of the elastic foundation are deter-

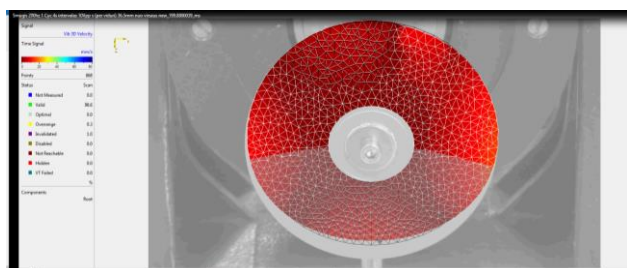
mined. The stiffness parameters of the elastic foundation must be chosen in such a way that enables to shift the eigenfrequencies of the elastic structure from the excitation frequencies of the investigated mechanical device. It is recommended to locate Braille elements at the places where the amplitude of vibrations is smallest.



a



b



c

Fig. 10 Results of experimental investigations of vibrations of elastic structure on elastic foundation with Braille elements by using the vibrometer: a - 20 Hz, 279.7 ms, b - 20 Hz, 280.9 ms, c - 20 Hz, 297.3 ms

#### 5. Conclusions

Vibrations of a plate on elastic foundation of Winkler type are investigated. A two-dimensional element

having six nodal degrees of freedom (three displacements of the lower surface and three displacements of the upper surface) is used. Elastic foundation of Winkler type on the lower surface is used in the numerical model. Eigenmodes are calculated.

From the presented results the out of plane and plane motions of the structure are clearly seen. In many of the eigenmodes motions of the lower and upper surfaces are very similar, but there are eigenmodes where the motions of the surfaces differ.

Experimental investigations of vibrations of the investigated elastic structure have been performed. Using the presented results places for recommended locations of Braille elements are determined. It is recommended to locate them at the places where the amplitude of vibrations is smallest.

Results of the performed investigations enabled to determine the regions for location of Braille elements where the intensity of vibrations has smallest influence into the effect of wear of Braille elements.

The stiffness parameters of the elastic foundation must be chosen in such a way that enables to shift the eigenfrequencies of the elastic structure from the excitation frequencies of the investigated mechanical device.

The obtained results are used in the process of design of elastic vibrating structures in devices with Braille elements.

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#### MEASUREMENT OF VIBRATIONS OF A PLATE ON ELASTIC FOUNDATION

#### S u m m a r y

Vibrations of a plate on elastic foundation of Winkler type are investigated. A two-dimensional element having six nodal degrees of freedom (three displacements of the lower surface and three displacements of the upper surface) is used. Elastic foundation of Winkler type on the lower surface is used in the numerical model. Eigenmodes are calculated. Experimental investigations of vibrations were performed using a special experimental setup and typical experimental results are presented. Using the presented results places for recommended locations of Braille elements are determined. It is recommended to locate them at the places where the amplitude of vibrations is smallest. The stiffness parameters of the elastic foundation must be chosen in such a way that enables to shift the eigenfrequencies of the elastic structure from the excitation frequencies of the investigated mechanical device.

**Keywords:** plate, elastic foundation, Winkler, vibrations, eigenmodes, experimental setup, experimental results, Braille elements.

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