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A nonprobabilistic set model of structural reliability based on satisfaction degree of interval

H.-Z. Huang*, Z. L. Wang, Y. F. Li***, B. Huang****, N. C. Xiao*****, L. P. He******

*University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P.R.China, E-mail: hzhuang@uestc.edu.cn

**University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P.R.China, E-mail: wzhonglai@uestc.edu.cn

***University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P.R.China, E-mail: lyfkjxy@163.com

****University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P.R.China, E-mail: bohuang@uestc.edu.cn

*****University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P.R.China, E-mail: ncxiao@uestc.edu.cn

******University of Electronic Science and Technology of China, Chengdu, Sichuan, 611731, P.R.China, E-mail: hlping0621@163.com

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1. Introduction

Uncertainties in material properties, geometric dimensions, loads and other parameters are always unavoidable in engineering structural problems [1-4]. They have played a more and more important role in the structural reliability analysis. In order to obtain the objective of reliable design, the effects of the various uncertain parameters should be rationally considered and treated. The probabilistic models are widely used to describe the uncertainties and they have been proved very effective in the structural reliability problems [5-7]. However, it is difficult to estimate precise values of parameters to accurately define the probability distributions because of inaccurate and insufficient information. Once the assumption about the probability distributions is not satisfied, the structural reliability analysis seems doubtful and meaningless. Moreover, some researches [8-11] have also indicated that even small deviations from the real probability distributions may cause large errors in the reliability analysis.

The fuzzy set theory provides a useful complement of classic reliability theory, in which the probabilities of the system elements can be not certain. Cai [12] presented different forms of "fuzzy reliability theories". In some recent research, a general fuzzy multistate system model and corresponding reliability evaluation technique - fuzzy universal generating function were proposed in [13] and [14], respectively, for dealing with the fuzziness of engineering systems. Similar with the probabilistic models, the membership functions of the uncertain parameters need to be established before carrying out structural reliability analysis with the fuzzy set theory. The nonprobabilistic reliability method and set model can be another direction for coping with the uncertain parameters. Although obtaining the precise probability distributions or membership functions of the uncertain parameters seems very difficult in many cases, the ranges or bounds of the uncertain parameters can be established relatively easily. Nowadays, there is not a precise method to find the precise intervals or the precise bounds of the uncertain parameters. However, one of the most feasible methods to find the approximate precise intervals or bounds of the uncertain parameters is "expert scoring method". For example, for a system uncertain variable x, several experts can given difference intervals or bounds for the variable. Sometimes, these intervals or bounds are not all the same, the method to handle these intervals or bounds are "average arithmetic". For example, there are n intervals scoring by n experts for an uncertainty variable x such as $x_1^I, x_2^I, x_3^I, \cdots x_n^I$. The result interval of uncertainty variable x expressed as $x^I = \frac{1}{n}\left(x_1^I + x_2^I + x_3^I + \cdots + x_n^I\right)$.

Some researcher such as Ben-Haim [10, 11] proposed that it was more rational to describe the uncertain parameters with the set models instead of the probability models when the statistic information about the uncertain parameters is insufficient. Based on this idea, the concept of nonprobabilistic reliability based on the convex model theory was proposed clearly by Ben-Haim in 1994 [11]. In recent years, the nonprobabilistic reliability theory develops rapidly. Elishakoff [15] discussed the concept of nonprobabilistic reliability and pointed out that the reliability of structures should belong to an interval rather than a certain value. Through interval analysis [16], a nonprobabilistic model of structural reliability was proposed by Guo et al [17] which the reliability was measured as the minimum distance from the coordinate origin to the failure surface. Based on the interval interference model of stress and strength, Wang and Qiu [18] defined the nonprobabilistic reliability index as the ratio of the volume of safe region to the total volume of the region constructed by the basic interval variables. In addition, the nonprobabilistic approaches have already been effectively applied to many practical structure problems in presence of various uncertainties. For example, they were used in the analysis of shells with imperfections in [19, 20], stress concentration at a nearly circular hole with uncertain irregularities in [21] and sandwich plates subject to uncertain loads and initial deflections in [22].

In this paper a new nonprobabilistic set model for reliability assessment of structural system is proposed. Interval variables are used to represent the parameter uncertainty. The nonprobabilistic reliability of structure is defined as the satisfaction degree between the stress-interval and the strength-interval. The interval analysis based on the first-order Taylor series is used to calculate the corresponding reliability. The illustrative example is presented to demonstrate the technique.

2. Interval variable and its operations

Before further discussion on the nonprobabilistic set model of structural reliability, a brief view of the definitions of the interval variable and its operations is provided. Assume that x denotes an uncertain parameter in the structural reliability problem, and it varies within a closed interval $x^I=\left[\underline{x},\overline{x}\right]$, then

$$x\in x^I=\left[\underline{x},\overline{x}\right] \tag{1}$$

is defined as an interval variable; $\underline{x}$ and $\overline{x}$ is the lower bound and upper bound of the interval x^I, respectively. Similar with the random variable, interval variable has its own center x^c and radius x^r, which can be defined as follows

$$x^c=\frac{\underline{x}+\overline{x}}{2}\ ,\ x^r=\frac{\overline{x}-\underline{x}}{2} \tag{2}$$

According to Eq. (2), interval x^I and interval variable x can be denoted in the following standardized form

$$x^I=x^c+x^r\Delta^I\ , x=x^c+x^r\delta \tag{3}$$

where $\Delta^I=\left[-1,1\right]$ is the standardized interval, $\delta\in\Delta^I$ is the standardized interval variable.

Let $x\in x^I=\left[\underline{x},\overline{x}\right]$ and $y\in y^I=\left[\underline{y},\overline{y}\right]$ be two interval variables, then the operations for x^I+y^I, x^I-y^I, $x^I\cdot y^I$ and x^I/y^I are obtained as [23]

$$x^I+y^I=[\underline{x},\overline{x}]+[\underline{y},\overline{y}]=[\underline{x}+\underline{y},\overline{x}+\overline{y}] \tag{4}$$

$$x^I-y^I=[\underline{x},\overline{x}]-[\underline{y},\overline{y}]=[\underline{x}-\overline{y},\overline{x}-\underline{y}] \tag{5}$$

$$\begin{aligned}x^Iy^I&=[\underline{x},\overline{x}][\underline{y},\overline{y}]=\\&=\left[min\left\{\underline{x}\underline{y},\overline{x}\underline{y},\underline{x}\overline{y},\overline{x}\overline{y}\right\},max\left\{\underline{x}\underline{y},\overline{x}\underline{y},\underline{x}\overline{y},\overline{x}\overline{y}\right\}\right]\end{aligned} \tag{6}$$

$$x^I/y^I=[\underline{x},\overline{x}]/[\underline{y},\overline{y}]=[\underline{x},\overline{x}]\cdot[1/\overline{y},1/\underline{y}] \tag{7}$$

Supposed that $I\left(R\right)$ denotes the sets of all closed real intervals. $x_i^I\in I\left(R\right)$, $x_i\in x_i^I(1,2,\cdots,n)$ are arbitrary interval variables which are independent with each other. The linear combination of these interval variables can be formed as follows

$$y=\sum_{i=1}^{n}a_ix_i\ ,\ i=1,2,\cdots,n \tag{8}$$

where $a_i\in R$ are arbitrary real numbers. Because y is the linear combination of interval x_i, y is also an interval variable. If the center and radius of interval variables x_i are denoted with x_i^c and x_i^r, then the center and radius of interval variable y are

$$y^c=\sum_{i=1}^{n}a_ix_i^c\ ,\ y^r=\sum_{i=1}^{n}\left|a_i\right|x_i^r\ ,\ i=1,2,\cdots,n\ . \tag{9}$$

3. Satisfaction degree of the relation $x^I\le y^I$

Different with the size relation of two real numbers, the size relation of two intervals is a kind of partial-order relation [24] which is usually denoted with the satisfaction degree of the two intervals. Here the concept of satisfaction degree of the relation $x^I\le y^I$ is actually a fuzzy set definition which represents the possibility that one interval is larger or smaller than the other. It is often used to compare intervals. Assumed that there are two intervals $x^I=[\underline{x},\overline{x}]$ and $y^I=[\underline{y},\overline{y}]$, consider the related rectangle in the $\left(x,y\right)$ - plane having the sides given by the two intervals. There are five case between $x^I\le y^I$ which is expressed in Fig. 1. The area value of the set $\left\{\left(x,y\right):\underline{x}\le x\le\overline{x},\underline{y}\le y\le\overline{y}\right\}$ can be computed as $\omega\left(x^I\right)\bullet\omega\left(y^I\right)$. The area value of shadow part can express as

$$area\left(\bullet\right)=\begin{cases}\omega\left(x^I\right)\bullet\omega\left(y^I\right) & case\,1\\ \omega\left(x^I\right)\bullet\omega\left(y^I\right)-\int_{\underline{y}}^{\overline{x}}dx\int_{\underline{y}}^{x}dy & case\,2\\ \dfrac{1}{2}\omega\left(x^I\right)\bullet\omega\left(y^I\right) & case\,3\\ \int_{\underline{x}}^{\overline{y}}dx\int_{x}^{\overline{y}}dy & case\,4\\ 0 & case\,5\end{cases} \tag{10}$$

where $area\left(\bullet\right)$ denotes the area value of shadow part.

The satisfaction degree of the relation $x^I\le y^I$ or reliability can be defined as

$$P\left(x^I\le y^I\right)=\frac{area\left(\bullet\right)}{\omega\left(x^I\right)\bullet\omega\left(y^I\right)} \tag{11}$$

Then

$$P\left(x^I\le y^I\right)=\begin{cases}1 & case\,1\\ 1-\dfrac{\int_{\underline{y}}^{\overline{x}}dx\int_{\underline{y}}^{x}dy}{\omega\left(x^I\right)\bullet\omega\left(y^I\right)} & case\,2\\ \dfrac{1}{2} & case\,3\\ \dfrac{\int_{\underline{x}}^{\overline{y}}dx\int_{x}^{\overline{y}}dy}{\omega\left(x^I\right)\bullet\omega\left(y^I\right)} & case\,4\\ 0 & case\,5\end{cases} \tag{12}$$

where “P” means possibility, $\omega\left(x^I\right)$ and $\omega\left(y^I\right)$ denotes the width of interval x^I and y^I, respectively. That is [25]

$$\omega\left(x^I\right)=\overline{x}-\underline{x}\,,\ \omega\left(y^I\right)=\overline{y}-\underline{y} \tag{13}$$

It can be found according to Eq. (12) and Fig. 1 that $P\left(x^I \le y^I\right)$ is equal to 1 for case 1 as interval x^I is always smaller than interval y^I. For case 5, $P\left(x^I \le y^I\right)$ is equal to 0 as interval x^I is always larger than interval y^I. For case 2 to 4, the value of $P\left(x^I \le y^I\right)$ is between [0,1] as interval x^I interferes with interval y^I.

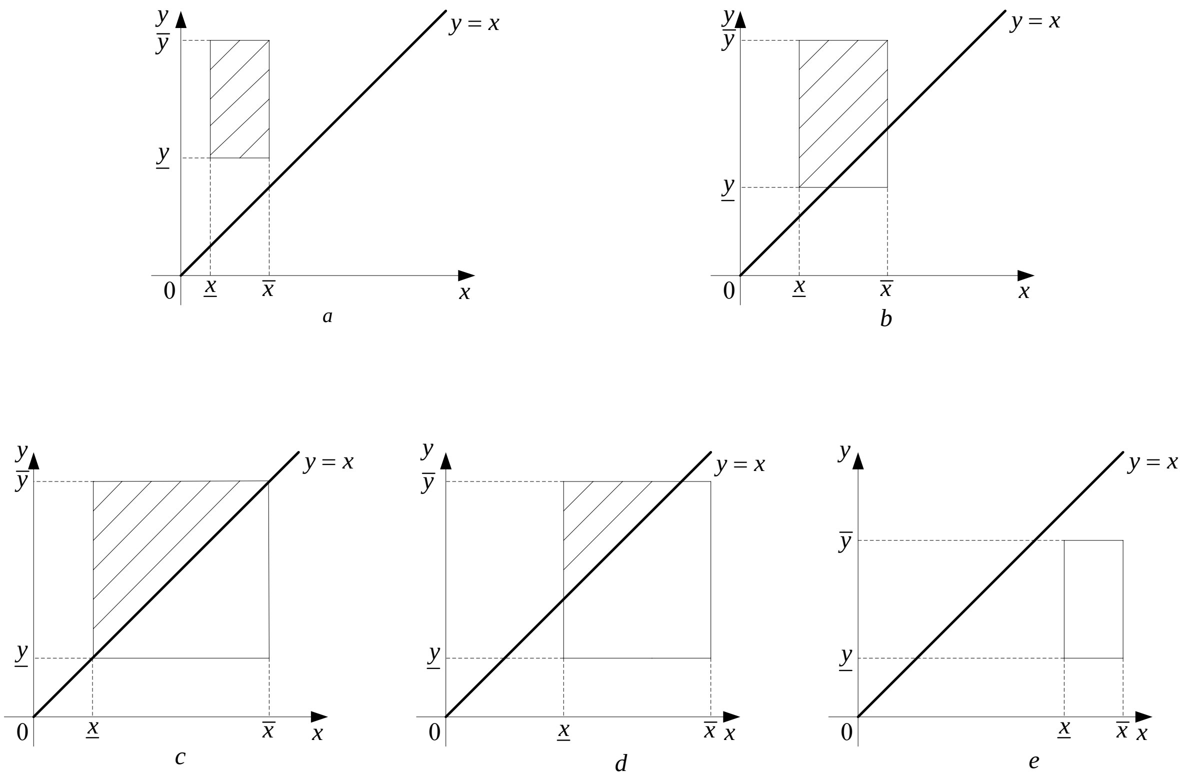

Fig. 1 Five cases for the relation $x^I \le y^I$: a - case 1; b - case 2; c - case 3; d - case 4; e - case 5

To sum up, the satisfaction degree of interval $P\left(x^I \le y^I\right)$ has the following properties

(1) $0 \le P\left(x^I \le y^I\right) \le 1$

(2) $P\left(x^I \le y^I\right)+P\left(x^I \ge y^I\right)=1$

(3) if $P\left(x^I \le y^I\right)=P\left(x^I \ge y^I\right)$, then $P\left(x^I \le y^I\right)=P\left(x^I \ge y^I\right)=0.5$, and $x^I = y^I$

(4) if $x^I \le y^I$, then $P\left(x^I \le y^I\right)=1$

(5) if $x^I \ge y^I$, then $P\left(x^I \le y^I\right)=0$.

4. Nonprobabilistic set model of structural reliability

As described in the introduction, structural reliability is subjected to many uncertain parameters. Therefore, the stress S and strength R of the structure can be denoted as the functions of these uncertain parameters

$$S=S\left(X_S\right)=S\left(x_{S1},x_{S2},\cdots,x_{Sl}\right) \tag{14}$$

$$R=R\left(X_R\right)=R\left(x_{R1},x_{R2},\cdots,x_{Rm}\right) \tag{15}$$

where $X_S=\left\{x_{Si}\right\}\left(i=1,2,\cdots,l\right)$ is the parameter set impacting on the stress S, such as concentration forces, distribution forces, bending moments and so on. $X_R=\left\{x_{Ri}\right\}\left(i=1,2,\cdots,m\right)$ is the parameter set impacting on the strength R, such as material properties, geometric dimensions, surface cracks and so on. According to the basic idea of nonprobabilistic reliability presented by Ben-Haim, all the uncertain parameters are described with interval variables in this paper, which are

$$x_{Si}\in x_{Si}^I=[\underline{x}_{Si},\overline{x}_{Si}],\left(i=1,2,\cdots,l\right) \tag{16}$$

$$x_{Ri}\in x_{Ri}^I=[\underline{x}_{Ri},\overline{x}_{Ri}],\left(i=1,2,\cdots,m\right) \tag{17}$$

Based on Eqs. (2) and (3), the interval variables x_{Si} and x_{Ri} can be transformed into their standardized forms. That is

$$x_{Si}=x_{Si}^c+x_{Si}^r\delta\,,\left(i=1,2,\cdots,l\right) \tag{18}$$

$$x_{Ri}=x_{Ri}^c+x_{Ri}^r\delta\,,\left(i=1,2,\cdots,m\right) \tag{19}$$

where x_{Si}^c and x_{Si}^r are the center and radius of the interval variables x_{Si}^I; x_{Ri}^c and x_{Ri}^r are the center and radius of the interval variables x_{Ri}; $\delta_i \in \Delta^I=[-1,1]$ are the standardized interval variables.

Because the stress S and strength R are functions of these interval variables respectively, they will vary within some closed intervals S^I and R^I. In order to obtain the upper bounds and the lower bounds of the intervals S^I and R^I, Eqs. (14) and (15) can be respectively expanded at the center x_{Si}^c and x_{Ri}^c of the uncertain interval variables x_{Si} and x_{Ri} by using the first-order Taylor series

$$\begin{aligned} S &= S(X_S) = S(x_{S1}, x_{S2}, \cdots, x_{Sl}) \approx \\ &\approx S(x_{S1}^c, x_{S2}^c, \cdots, x_{Sl}^c) + \frac{\partial S}{\partial x_{S1}}(x_{S1} - x_{S1}^c) + \\ &+ \frac{\partial S}{\partial x_{S2}}(x_{S2} - x_{S2}^c) + \cdots + \frac{\partial S}{\partial x_{Sl}}(x_{Sl} - x_{Sl}^c) \end{aligned} \tag{20}$$

$$\begin{aligned} R &= R(X_R) = R(x_{R1}, x_{R2}, \cdots, x_{Rm}) \approx \\ &\approx R(x_{R1}^c, x_{R2}^c, \cdots, x_{Rm}^c) + \frac{\partial R}{\partial x_{R1}}(x_{R1} - x_{R1}^c) + \\ &+ \frac{\partial R}{\partial x_{R2}}(x_{R2} - x_{R2}^c) + \cdots + \frac{\partial R}{\partial x_{Rm}}(x_{Rm} - x_{Rm}^c) \end{aligned} \tag{21}$$

where $\frac{\partial S}{\partial x_{Sl}}$, $(i = 1, 2, \cdots, l)$ is the first-order partial derivative of the stress S at the center x_{Si}^c; $\frac{\partial R}{\partial x_{Rm}}$, $(i = 1, 2, \cdots, m)$ is the first-order partial derivative of the strength R at the center x_{Ri}^c. Substituting Eqs. (18) and (19) into Eqs. (20) and (21) respectively, Eqs. (20) and Eq. (21) can be rewritten as follows

$$\begin{aligned} S &= S(X_S) = S(x_{S1}, x_{S2}, \cdots, x_{Sl}) \approx \\ &\approx S(x_{S1}^c, x_{S2}^c, \cdots, x_{Sl}^c) + \sum_{i=1}^{l} \frac{\partial S}{\partial x_{Si}} x_{Si}^r \delta \end{aligned} \tag{22}$$

$$\begin{aligned} R &= R(X_R) = R(x_{R1}, x_{R2}, \cdots, x_{Rm}) \approx \\ &\approx R(x_{R1}^c, x_{R2}^c, \cdots, x_{Rm}^c) + \sum_{i=1}^{m} \frac{\partial R}{\partial x_{Ri}} x_{Ri}^r \delta \end{aligned} \tag{23}$$

According to Eqs. (8), (9) and (22), the center S^c and radius S^r of the interval S^I can be determined as follows

$$S^c = S(x_{S1}^c, x_{S2}^c, \cdots, x_{Sl}^c), \; S^r = \sum_{i=1}^{l} \left| \frac{\partial S}{\partial x_{Si}} \right| x_{Si}^r \tag{24}$$

Therefore, stress-interval S^I the of structure is

$$S^I \approx \left[S^c - S^r, S^c + S^r \right] \tag{25}$$

According to Eqs. (8), (9) and (23), the center R^c and radius R^r of the interval R^I can be determined as follows

$$R^c = R(x_{R1}^c, x_{R2}^c, \cdots, x_{Rm}^c), \; R^r = \sum_{i=1}^{m} \left| \frac{\partial R}{\partial x_{Ri}} \right| x_{Ri}^r \tag{26}$$

Therefore, strength-interval R^I of the structure is

$$R^I \approx \left[R^c - R^r, R^c + R^r \right] \tag{27}$$

According to the stress-strength interference model, the reliability criterion of structure design is that the stress of the structure is less than or equal to the strength of the structure. Therefore, based on the principle of satisfaction degree of interval, a nonprobabilistic reliability of the structure can be defined as the satisfaction degree between the stress-interval S^I and the strength-interval R^I. For the definition of the satisfaction degree of the relation $x^I \le y^I$ in Eq. (11), there are also five cases between $S^I \le R^I$ as same as the $x^I \le y^I$ which shown in Fig. 2. The satisfaction degree of the relation $S^I \le R^I$ or reliability becomes

$$P(S^I \le R^I) = \begin{cases} 1 & case\,1 \\ 1 - \dfrac{\int_{\underline{R}}^{\overline{S}} dS \int_{\underline{R}}^{S} dR}{\omega(S^I) \cdot \omega(R^I)} & case\,2 \\ \dfrac{1}{2} & case\,3 \\ \dfrac{\int_{\underline{S}}^{\overline{R}} dS \int_{S}^{\overline{R}} dR}{\omega(S^I) \cdot \omega(R^I)} & case\,4 \\ 0 & case\,5 \end{cases} \tag{28}$$

By the definition of the satisfaction degree of the relation $S^I \le R^I$, the value of $P(\bullet)$ varies from 0 to 1. When $P(\bullet)$ is equal to 1, it means that the stress-interval S^I is absolutely smaller than the strength-interval R^I and the structure is in the state of safety which is denoted by case 1 in Fig. 2. When $P(\bullet)$ is equal to 0, it means that the stress-interval S^I is absolutely larger than the strength-interval R^I and the structure is in the state of failure which is denoted by case 3 in Fig. 2. When $P(\bullet)$ is equal to some value between 0 and 1, it means that the stress-interval S^I interfered with the strength-interval R^I and the structure may be safety or may be failure.

5. Illustrative example

Gears are widely used in many practical engineering systems. The gear transmission system plays an important role in modern industry. However, in the process of gear meshing, contact stress will be produced which causes pitting. Systems including gears meshing shocks with the increase of the pitting, which will lead to the decrease of the transmission efficiency and accuracy. Therefore, contact fatigue analysis is necessary and important for increasing the reliability of gear transmission. In this section, the nonprobabilistic reliability of the contact fatigue of a pair of spur gear meshing of a reducer is calculated. Main parameters of the gear pairs used in the example are described as: modulus $m = 4$ mm; tooth number of two gear are $z_1 = 14$, $z_2 = 47$; torques are $T_1 = 353$ Nm,

$T_2 = 1180$ Nm; rotation speed are $n_1 = 76.5$ r/min, $n_2 = 22.8$ r/min; pitch diameters are $d_1 = 56.57$ mm, $d_2 = 189.89$ mm respectively; width of the tooth $b = 46$ mm; material of the pinion: 20MnTiB, HRC = 56~62; material of the gear: 40Cr, HRC = 50~56; life of the reducer: 1000 h.

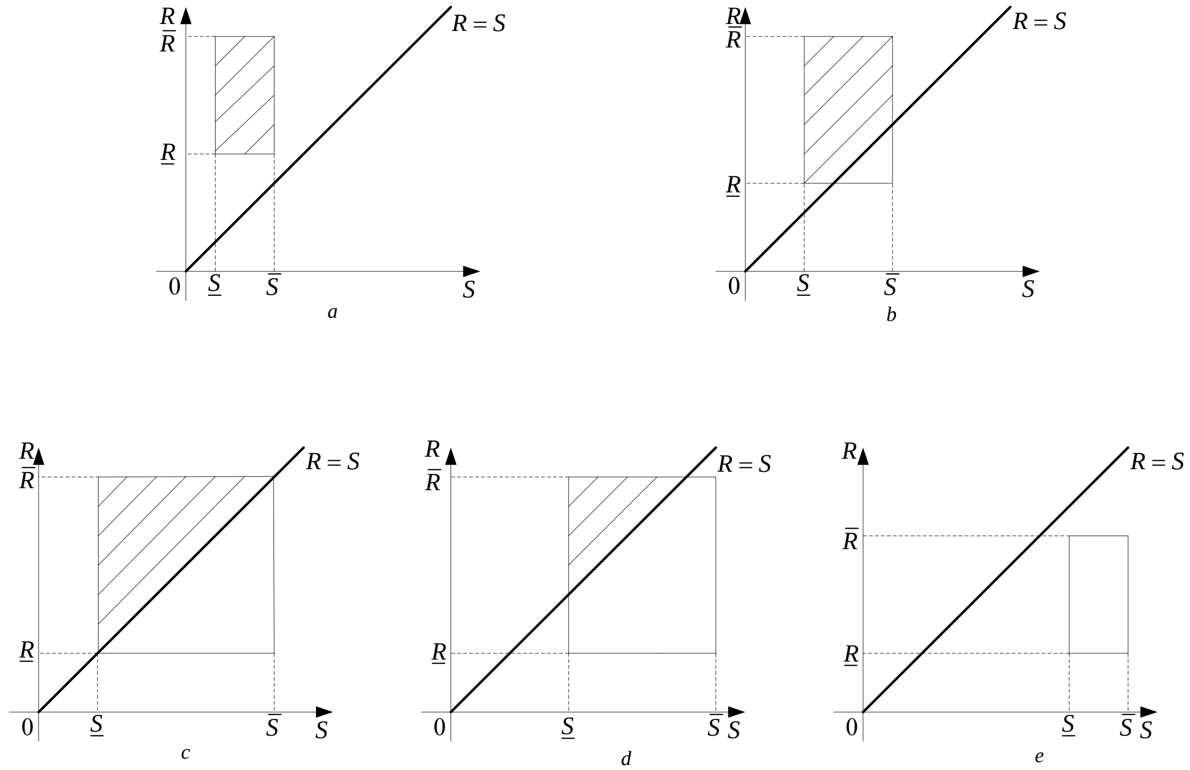

Fig. 2 Five cases for the relation $S^I \le R^I$: a - case 1; b - case 2; c - case 3; d - case 4; e - case 5

According to reference [26], the calculated contact stress σ_H is denoted by the formula

$$\sigma_H = Z_E \sqrt{F_t K_O K_V K_S \frac{K_H}{bd_1} \frac{Z_R}{Z_I}} \tag{29}$$

where Z_E is an elastic coefficient; F_t is the transmitted tangential load; K_O is the overload factor; K_V is the dynamic factor; K_S is the size factor; K_H is the load-distribution factor; b is the width of the tooth; d_1 is the pitch diameter of the pinion; Z_R is the surface condition factor; Z_I is the geometry factor.

According to the nonprobabilistic reliability model presented in this paper, all the parameters in Eq. (29) are described with interval variables.

By means of Eq. (23), the center and radius of the calculated contact stress σ_H are

$$\sigma_H^c = 1350.04 \text{ MPa}, \sigma_H^r = 118.05 \text{ MPa} \tag{30}$$

According to reference [26], the contact fatigue strength σ_{HS} is denoted by the formula

$$\sigma_{HS} = \frac{\sigma_{HP}}{S_H} \frac{Z_N Z_W}{Y_\theta} \tag{31}$$

where σ_{HP} is the surface fatigue strength; S_H is the AGMA factor of safety; Z_N is the stress cycle life factor; Z_W is the hardness ratio factor; Y_θ is the temperature factor. Similarly, all the parameters in Eq. (31) are described with interval variables.

The center and radius of interval variables in Eqs. (29) and (31) are expressed in Table [27].

By means of Eq. (26), the center and radius of the contact fatigue strength σ_{HS} are

$$\sigma_{HS}^c = 1661.33 \text{ MPa}, \sigma_{HS}^r = 207.67 \text{ MPa} \tag{32}$$

Thus, from the relation $\sigma_H^I \le \sigma_{HS}^I$ shown in Fig. 3, the satisfaction degree of the relation $\sigma_H^I \le \sigma_{HS}^I$ or the reliability of the contact fatigue is

$$P\left(\sigma_H^I \le \sigma_{HS}^I\right) = 1 - \frac{\int_{\underline{\sigma}_{HS}}^{\overline{\sigma}_H} d\sigma_H \int_{\underline{\sigma}_{HS}}^{\sigma_H} d\sigma_{HS}}{\omega\left(\sigma_H{}^I\right) \bullet \omega\left(\sigma_{HS}{}^I\right)} = 0.9989 \tag{33}$$

Table

Center and Radius of uncertain parameters

Uncertain parameters	Center	Radius
$Z_E\left(\sqrt{\text{MPa}}\right)$	189.8	17.1
F_t (N)	12480	1248
K_O	1	0.01
K_V	1.04	0.04
K_S	1.00	0.01
K_H	1.496	0.40
b (mm)	46	0.01
d_1 (mm)	56.57	0.01
Z_R	1.02	0.02
Z_I	1.07	0.01
$\sigma_{HP}\left(\sqrt{\text{MPa}}\right)$	1495.2	164.9
S_H	1.35	0.03
Z_N	1.5	0.04
Z_W	1.00	0.02
Y_θ	1.00	0.01

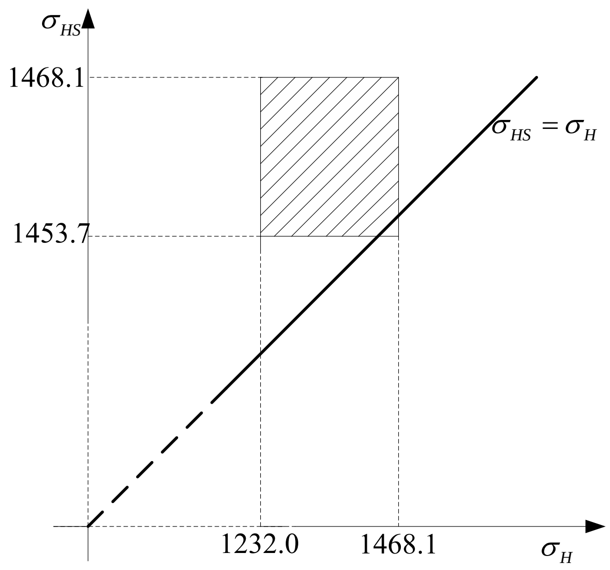

Fig. 3 The relation of $\sigma_H^I \le \sigma_{HS}^I$

From Eq. (33), the satisfaction degree of the relation $\sigma_H^I \le \sigma_{HS}^I$ or the reliability is very close to 1. It indicates that the gear transmission of the reducer is very reliable. If all the parameters in the example are of uniform distribution, for example, σ_{HP} follows the uniform distribution $[1330.3, 1660.1]$, from the Monte Carlo simulation, the reliability $R \approx 1$, Obviously, the nonprobabilistic reliability is a little smaller than the probabilistic reliability and it means that if the calculated result by nonprobabilistic approach is thought to be reliable, the calculated result by probabilistic approach is absolutely reliable. From the result there is a conclusion that the method proposed in the paper is not as same as the probabilistic reliability method which assumes that all the variables are of uniform distribution. The nonprobabilistic method is more conservative than probabilistic method because there is no human assumption for system parameters distribution.

6. Conclusions

1. For the structural reliability analysis, the stress and strength are the function of several interval variables. The approximations $S \approx S\left(x_{S1}^c, x_{S2}^c, \cdots, x_{Sl}^c\right) + \sum_{i=1}^{l} \frac{\partial S}{\partial x_{Si}} x_{Si}^r \delta$ and $R \approx R\left(x_{R1}^c, x_{R2}^c, \cdots, x_{Rm}^c\right) + \sum_{i=1}^{m} \frac{\partial R}{\partial x_{Ri}} x_{Ri}^r \delta$ for the stress and strength are implemented with the first order Taylor series to guarantee the computational efficiency and accuracy of the reliability analysis.

2. Comparison of results between the proposed nonprobabilistic method and the probabilistic method has shown that the reliability by using the proposed nonprobabilistic method ($R = 0.9989$) is a little smaller than using the probabilistic method ($R \approx 1$). Hence it is reliable with the proposed nonprobabilistic method.

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H.-Z. Huang, Z.L. Wang, Y.F. Li, B. Huang, N.C. Xiao, L.P. He

KONSTRUKCINIS NETIKIMYBINIS AIBĖS PATIKIMUMO MODELIS PAGRĮSTAS PATIKIMU INTERVALO DYDŽIU

R e z i u m ė

Inžinerinėse konstrukcijose daugiausia susiduriama su dviejų tipų neapibrėžtumu. Pažintinis neapibrėžtumas atsiranda dėl informacijos neišsamumo arba jos ignoravimo, o rizikingas neapibrėžtumas – dėl paveldėto nepastovumo. Priklausomai nuo daugelio neapibrėžtumų ir neaiškumų įtakos gautai informacijai, visos atsitiktinių dydžių tikimybės arba tikimybių pasiskirstymas yra arba tiksliai žinomi, arba tiksliai jų nustatyti negalima. Sprendžiant daugelį konstrukcinio patikimumo problemų, trūksta informacijos apie neapibrėžtus parametrus. Intervalo kintamojo parinkimas yra patogus ir efektyvus būdas apibūdinant neapibrėžtumą. Remiantis šiuo metodu, straipsnyje siūlomas naujas netikimybinis konstrukcinio patikimumo aibės modelis, paremtas patikimu intervalu ir jo analize. Konstrukcijos netikimybinis patikimumas yra nustatytas kaip leistinas dydis tarp įtempių ir jėgos intervalų. Šiame darbe aprašytas netikimybinio patikimumo modelis yra panaudotas praktinei inžinerinei krumpliaratinės pavaros kontaktinio nuovargio patikimumo analizei. Gauti patikimi ir svarbūs rezultatai.

H.-Z. Huang, Z. L. Wang, Y. F. Li, B. Huang, N. C. Xiao, L. P. He

A NONPROBABILISTIC SET MODEL OF STRUCTURAL RELIABILITY BASED ON SATISFACTION DEGREE OF INTERVAL

S u m m a r y

In engineering structural systems, two types of uncertainty exist in systems widely. Epistemic uncertainty comes from incomplete information or ignorance while aleatory uncertainty derives from inherent variations. Due to the influence of many uncertainties and vagueness in the available information, all probabilities or probability distributions of random variables are precise known or perfect determination is impossible. For many structural reliability problems lacking information of the uncertain parameters, interval variable is a convenient and effective selection for the uncertainty description. According to this method, this paper suggests a new nonprobabilistic set model of structural reliability based on interval analysis and the satisfaction degree of the interval. The nonprobabilistic reliability of a structure is defined as the satisfaction degree between the stress-interval and the strength-interval. With the nonprobabilistic reliability model presented in this paper, a practical engineering example of the contact fatigue reliability analysis for the gear transmission is calculated and the result is reasonable and reliable.

Х.-З. Хуанг, З.Л. Ванг, И.Ф. Ли, Б. Хуанг, Н.Ц. Хиао, Л.П. Хе

КОНСТРУКЦИОННАЯ НЕВЕРОЯТНОСТНАЯ МОДЕЛЬ НАДЕЖНОСТИ МНОЖЕСТВА, ОСНОВАННАЯ НА НАДЕЖНОЙ ВЕЛИЧИНЕ ИНТЕРВАЛА

Р е з ю м е

В инженерных конструкциях в основном встречаются два типа неопределенности. Различительная неопределенность познания появляется из-за неполной информации или ее игнорирования, а рискованная неопределенность – из-за наследственного непостоянства. В зависимости от влияния многих неопределенностей и неясностей на полученную информацию все вероятности случайных величин или распределения вероятностей являются точно известными или точное их определение невозможно. Для многих проблем конструкционной надежности неполная информация из-за неопределенности параметров. Подбор переменной интервала является подходящим и эффективным выбором для описания неопределенности. Применяя этот метод, авторы предлагают новую невероятностную модель конструктивной надежности множества, основанную на анализе и надежности интервала. Невероятностная надежность конструкции определяется как допустимая величина между интервалами напряжения и силы. Модель невероятностной надежности, предложенная в данной работе, применялась для практического инженерного анализа контактной усталости зубчатой передачи. Получены достоверные результаты.

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