ISSN 1392 - 1207. MECHANIKA. 2012 Volume 18(4): 473-478

Development of the method of accuracy measuring of precision spindle

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crossref http://dx.doi.org/10.5755/j01.mech.18.4.2335

1. Introduction

The accuracy of the rotation of a spindle is evaluated by radial, axial and angular errors [1, 2]. During the formation of the circular raster scales and after that – during the calibration of angle standards – circular scales of periodic or coded structure, one of the most influential constituents on its uncertainty are radial errors of the spindle rotation, on which the disc for scale formation or calibration is placed in the plane of the circular scale [3, 4]. An objective point for the determination of accuracy uncertainty of this rotation has a great influence for minimization of these errors or its correction by calculated compensation of the systematic error or during the correction of the production process and rotary tables and angle standards as well.

Here we present an intention to assess disadvantages of the evaluation of these errors according to roundness reference measure (template) and the proposals for elimination of these disadvantages.

2. Subjectivity of assessment of radial rotation errors

Assessment of radial error of rotation in respect of fixed unmovable basis. Applying this method, the reference measure of roundness is fixed to the spindle to be calibrated by means of the centring adjustable device. During this operation, movements of the centre are measured by two pick-ups of small linear displacement. Errors in radial direction are evaluated according to the value of displacement of the centre of the roundness measure in two perpendiculars to each other and to the axis of rotation directions or by assessing of the Lissajouis figures developed by vectorial sum of the displacements mentioned above [5]. During the calibration of precision spindles a glass or of other material made spherical (usually, semi-spherical) or cylindrical body (artefact) is used. The deviations of the cross-section of the artefact from roundness not exceed 0.01 – 0.02 μm. The pick-ups of small displacement are fixed to the unmovable body of the spindle.

During the measurement of the rotation the error in one dimensional direction is assessed by sum of harmonic constituents of all the movements of the centre of a spherical body. During the measurement of rotation error in the plane, the radial error usually is assessed by the displacement of the centre of the sphere in the plane by the width of the ring of trajectory. The result of measurement is evaluated according to trajectory's declinations including constituents of spindle's displacements in the plane, eccentricity of the roundness measurement standard and its cross-section declinations from the roundness in the plane. For accuracy improvement of the calibration, some means are taken to eliminate deviations from roundness of the standard measure although there are no efforts made to eliminate an influence of the eccentricity of roundness standard to the axis of spindle rotation [6]. The influence of this parameter on the results of calibration is demonstrated further.

Displacement of the centre O_2 of the roundness standard in the fixed coordinate system XOY according to the scheme in Fig. 1 is expressed by the equation

$$\left.\begin{aligned} x &= e_1 \cos\left(\beta_0 + \int_0^{t_1} \omega_1 dt\right) + e_2 \cos\left(\tau_0 + \int_0^{t_1} \omega_2 dt\right) \\ y &= e_1 \sin\left(\beta_0 + \int_0^{t_1} \omega_1 dt\right) + e_2 \sin\left(\tau_0 + \int_0^{t_1} \omega_2 dt\right) \end{aligned}\right\} \quad (1)$$

where e_1 is the radius of trajectory of the spindle centre O_1 in the plane of measurement, e_2 is the eccentricity of the roundness standard in the measuring plane according to the spindle axis, β_0 is the angle between positive value of axis X and radius e_1 at the initial position of the spindle, τ_0 is the angle between positive value of axis X_1 and eccentricity e_2, ω_1 and ω_2 are angular velocity of the vector e_1 and of the spindle, t is time.

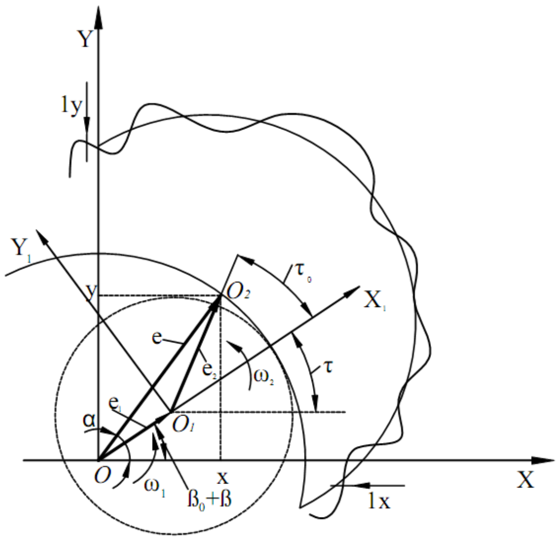

Fig. 1 Calculation scheme of the trajectory of the centre of the roundness standard

XOY is fixed coordinate system, the initial point O of which coincides with the hole of the spindle axis rotation; $X_1O_1Y_1$ is movable coordinate system an initial point O_1 of which coincides with the axis of spindle; β - angle of rotation of radius e_1; τ is angle of declination of the movable coordinate system $X_1O_1Y_1$, angle of rotation of the spindle; $1x$ and $1y$ are directions of the measurement of radial displacement of the roundness standard; O_2 is centre of the roundness standard in the measuring plane.

The radius e of the trajectory of centre O_2 of roundness measure and polar angle α in polar coordinate system are described by the equations

$$\left.\begin{aligned} e &= \sqrt{e_1^2 + e_2^2 + 2e_1e_2\cos\left[\psi_0 + \int_0^{t_1} \omega_2 - \omega_1 \; dt\right]} \\ tg\alpha &= \frac{y}{x} \end{aligned}\right\} \tag{2}$$

where $\psi_0=\tau_0-\beta_0$ is angle between the radii e_1 and e_2 at the initial position of the spindle.

Aerostatic and hydrodynamic bearings as well as roll bearings of special construction are mainly used in precision measuring and technological equipment [7, 8]. The trajectory of the axis of such bearings in the measuring plane can be described by Fourier series with adequate accuracy

$$e_1 = e_{10} + \sum_{i=1}^{n} A_i \sin \; i\omega_2 t + \zeta_{0i} \;, \tag{3}$$

where e_{10} is mean value of e_1, A_i is the amplitude of vector e_1 by change of frequency $i\omega_2$, ζ_{0i} is initial phase value of i-th harmonic of e_1, i is harmonic's number.

It is the feature of aerostatic spindles an irregular trajectory of the axis rotation. By assumption hat the trajectory has an elliptic pattern, it can be strictly described by the Eq. (3) in case of $i = 2$ and $\omega_1 = \omega_2$. Then the radius of rotation standard centre O_2 and polar angle α in polar coordinate system in the plane of measurement will be described by the Eq. (4).

Analysis of these equations shows that the centre trajectory of the rotation standard in the measuring plane depends on eccentricity e_2 and angle ψ_0. During their variation, essential changes of its form occur as well as orientation and according to its ring‘s width an radial rotation error can be determined.

This can be illustrated by the graphs of radial error of aerostatic spindles in the measuring plane in case when eccentricity of the roundness measure according to the rotation error of the spindle differs (Fig. 2).

Radial error assessed by the ring width of the trajectory at different centring in the plane differs by 0.24 μm to 0.44 μm.

$$\left.\begin{aligned} e &= \sqrt{\left[e_{10} + A_2\sin\left(2\omega_2 t + \zeta_{02}\right)\right]^2 + e_2^2 + 2\left[e_{10} + A_2\sin\left(2\omega_2 t + \zeta_{02}\right)\right]e_2\cos\psi_0} \\ a &= arctg\,\frac{\left[e_{10} + A_2\sin\left(2\omega_2 t + \zeta_{02}\right)\right]\times\sin\left(\beta_0 + \omega_2 t\right) + e_2\sin\left(\tau_0 + \omega_2 t\right)}{\left[e_{10} + A_2\sin\left(2\omega_2 t + \zeta_{02}\right)\right]\times\cos\left(\beta_0 + \omega_2 t\right) + e_2\cos\left(\tau_0 + \omega_2 t\right)} \end{aligned}\right\} \tag{4}$$

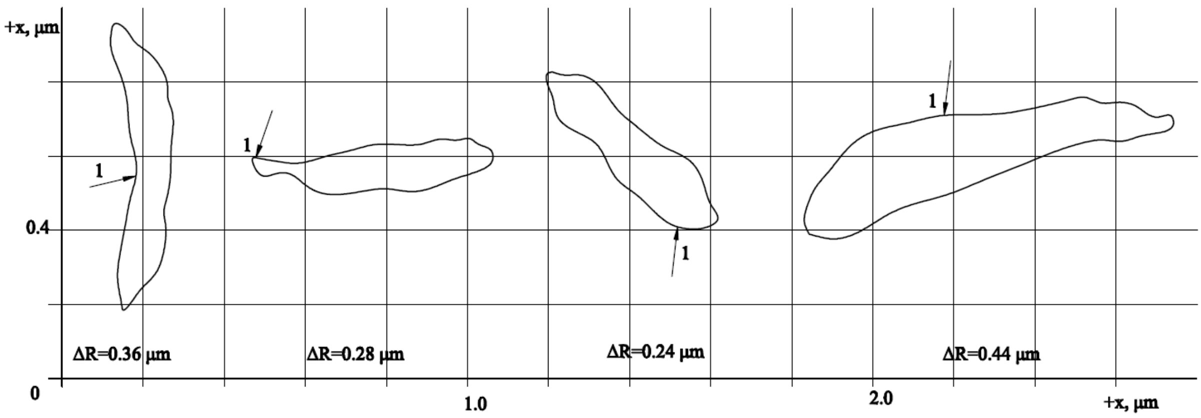

Fig. 2 Diagrams of measurement of the radial error of the aerostatic at different centring

Characteristic feature of hydrodynamic bearings is trajectory of the spindle approximately near to circular, its angular velocity being twice smaller than velocity of the spindle itself. It can be expressed as $e_1 = e_{10}$, $\omega_1 = 0.5\omega_2$.

In this case the form and orientation of the centre trajectory of the roundness standard depends essentially on the value of eccentricity e_2 and its orientation (angle ψ_0). Also, the diagram of trajectory repeats every two revolutions of the spindle. If $\omega_1 \neq 0{,}5\omega_2$, or is approximately equal, then the trajectory will rotate around the centre of the spindle and will not repeat at every two revolutions.

Despite of small errors due to not coinciding of the measurement direction with the required and of the influence of the roundness standard's displacements in the directions of axis movement to the displacements into the other directions, the readings D_x and D_y in the direction of ordinate axis X and Y are expressed by the equations

$$\left.\begin{aligned} D_x &= k_x\left(x + \delta_x\right) \\ D_y &= k_y\left(y + \delta_y\right) \end{aligned}\right\} \tag{5}$$

where k_x, k_y are coefficients of amplification of measuring signals for x and y respectively, δx, δ_y are roundness deviations of the cross-section of the roundness standard in the measuring plane in X and Y axis respectively.

By inserting the values x and y from the Eq. (1) into the Eq. (5) a member consisting from two harmonic oscillations appears. Frequency of the one is (i-1), of the other (i+1). Amplitudes of both oscillations are equal to $A_i/2$. This value of method‘s error except of harmonic constituent equal to the frequency of revolution depends on β_0 and ζ_{0i}.

For example, in case of aerostatic bearing, when $i = 2$ and $\omega_1 = \omega_2 = \text{const}$, frequency of the one oscillation is equal to the frequency of spindle rotation ω_2, and of the other is three times more. Amplitudes of both frequencies are equal to $A_2/2$. The frequency ω_2 is eliminated from the results of measurement as coinciding with the eccentricity. The next frequency will be assessed as radial error of the spindle rotation having the frequency ω_3 that does not exist as it is an error of the method.

Method, when a pick-up of small displacements is fixed to the spindle to be calibrated. By using this method the tip of the pick-up contacts the surface of the roundness standard that is mounted to the unmovable body via centring device. Relative radial displacements of the surface of the roundness standard and the spindle axis trajectory are measured in the measuring plane. The radial errors of rotation are assessed according to the readings of these measurements.

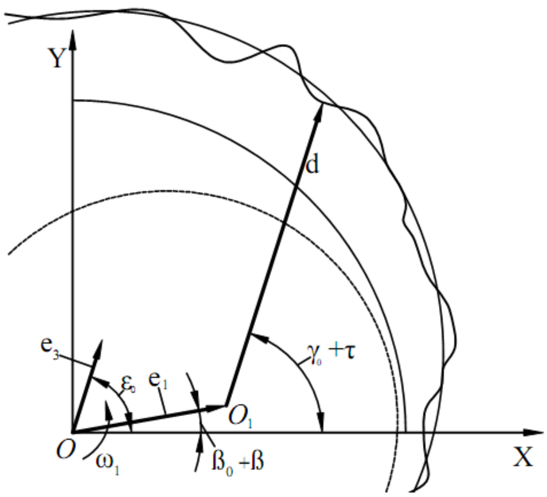

Fig. 3 Diagram of radial errors of rotation in the fixed direction to the spindle

As in the case of using the first method, this time also is impossible an ideal centring and always remains an eccentricity between the spindle axis of rotation and the roundness standard of unknown value and direction in the measuring plane.

According to the measuring diagram according this method shown in Fig. 3, applying several simplifications and mathematical rearrangement, displacement of the measuring tip of the pick-up in the perpendicular direction to the axis of the spindle will be expressed by the equation

$$d = e_1 \cos\left[\left(\gamma_0 - \beta_0\right) + \int_0^{t_1}\left(\omega_2 - \omega_1\right)dt\right] - e_3\left(\cos\varepsilon_0 - \gamma_0 - \int_0^{t_1}\omega_2 dt\right) \quad (6)$$

where γ_0 is angle between the positive coordinate axis X and the direction of measurement at initial position of the spindle; ε_0 is angle between the positive coordinate axis X and the eccentricity e_3 of the roundness standard at initial position of the spindle. Other designations are the same as in Fig. 1.

In this case of measurement of radial error, its phase and amplitude of the graph completed by the readings of the measurement depend on the angle between the radius e_1 and measuring direction at initial position of the spindle.

Recording the results of measurement in polar coordinate system occur additional errors of eccentricity of the graph that depend on mean radius of the graph, eccentricity of the roundness standard, angles β_0, γ_0, ε_0.

3. Assessment of radial error of rotation by centrodes

Objective assessment of radial errors requires elimination of the uncontrollable parameters that make influence on the results of measurement. Centrodes can be used for this purpose as they unambiguously determine a movement of flat figure.

The displacement of the centre of cross-section of the roundness standard in the measuring plane perpendicular to the axis rotation X_{O_1} of the spindle, without evaluation of the influence of small axial displacements of the spindle, can be determined by rotation of mobile centrode along the fixed centrode and rigidly connected to the spindle.

Coordinates of the moment centre P of velocities in nonmobile plane are described by the equations

$$\left.\begin{aligned} x_P &= x_{O_1} - \frac{\dot{y}_{O_1}}{\omega_2} \\ y_P &= y_{O_1} - \frac{\dot{x}_{O_1}}{\omega_2} \end{aligned}\right\} \quad (7)$$

where x_{O1}, y_{O1} are coordinates of centre O_1, ω_2 are angular velocity of rotation of flat figure.

XOY is not mobile coordinate system the beginning of which O coincides with the centre of the body of the spindle; X`O¸Y` is mobile coordinate system the beginning of which O_1 coincides with the centre of the first roundness standard. Coordinate axis X_1 is directed across the centre of the second roundness standard O_2; P is moment velocity centre; φ_0 is angle between the not mobile X and mobile X`, coordinate axis at the initial time; 1x, 1y and 2x are the directions of measurement of radial displacement of the roundness standard.

As angular velocity of the figure is undetermined, there are three unknown variables x_p, y_p and ω_2 in two Eq. (7). For solving the third equation with the same variables is needed.

The third equation can be formed by measuring the movement of one more point O_2 connected with the spindle (Fig. 4)

$$x_P = x_{O_2} - \frac{\dot{y}_{O_2}}{\omega_2} \quad (8)$$

where x_{O2}, y_{O2} are coordinates of centre O_2 in the system of not mobile coordinate system.

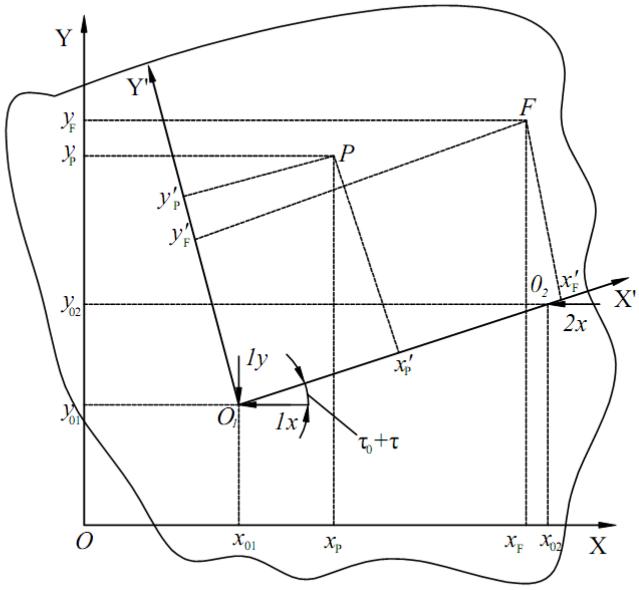

Fig. 4 Diagram of evaluation of radial error of rotation using centrodes

During flat displacement of the figure, velocities of its two points O_1 and O_2 are connected by the dependence

$$\vec{V}_{O_2} = \vec{V}_{O_1} + \vec{\omega}_2\, \vec{e}_4 \tag{9}$$

where $e_4 = O_1O_2$.

Then projection of the velocities into the axis Y can be determined

$$V_{O_2 y} = V_{O_1 y} + \omega_2 e_4 \cos\left(\varphi_0 + \tau\right) \tag{10}$$

where $\tau = \int_0^{t_1} \omega_2 dt$ is angle of rotation of the spindle, mobile coordinate system's angle of rotation; φ_0 is angle between the non mobile X and mobile X', coordinate axis at initial moment.

$$V_{O_1 y} = \dot{y}_{O_1}\,,\; V_{O_2 y} = \dot{y}_{O_2}$$

By assessing Eq. (10) from (7) and (8) Eqs., the system of three equation can be formed with three variables

$$\left.\begin{aligned} x_P &= x_{O_1} - \frac{\dot{y}_{O_1}}{\omega_2} \\ y_P &= y_{O_1} - \frac{\dot{x}_{O_1}}{\omega_2} \\ x_P &= x_{O_2} - e_4 \cos\left(\varphi_0 + \int_0^{t_1} \omega_2 dt\right) - \frac{\dot{y}_{O_1}}{\omega_2} \end{aligned}\right\} \tag{11}$$

By solving this equation the coordinates of non-mobile centrode are determined in the non-mobile coordinate system and variable angular velocity of the spindle

$$\left.\begin{aligned} x_P &= x_{O_1} - \frac{\dot{y}\sqrt{e_4^2 - \left(x_{O_2} - x_{O_1}\right)^2}}{\dot{x}_{O_1} - \dot{x}_{O_2}} \\ y_P &= y_{O_1} - \frac{\dot{x}_{O_1}\sqrt{e_4^2 - \left(x_{O_2} - x_{O_1}\right)^2}}{\dot{x}_{O_1} - \dot{x}_{O_2}} \\ \omega_2 &= \frac{\dot{x}_{O_1} - \dot{x}_{O_2}}{\sqrt{e_4^2 - \left(x_{O_2} - x_{O_1}\right)^2}} \end{aligned}\right\} \tag{12}$$

Coordinates of instantaneous velocities centre P and nonmobile centrode graph are not dependant on the selection of points O_1 and O_2 and also from their position according to the axis of the spindle. They unambiguously determine the displacement of flat figure in respect of non-mobile elements of the spindle.

The position of the point under measurement in accordance to the spindle is determined by using mobile centrode that determines geometric position of instantaneous centres of velocity in the moving body. Its coordinates in the mobile system of coordinates $X'O_1Y'$ can be written

$$\left.\begin{aligned} x'_P &= \frac{\left(\dot{x}_{O_1} \sin\varphi - y_{O_1} \cos\varphi\right)\sqrt{e_4^2 - \left(x_{O_2} - x_{O_1}\right)^2}}{\dot{x}_{O_1} - \dot{x}_{O_2}} \\ y'_P &= \frac{\left(\dot{x}_{O_1} \cos\varphi + y_{O_1} \sin\varphi\right)\sqrt{e_4^2 - \left(x_{O_2} - x_{O_1}\right)^2}}{\dot{x}_{O_1} - \dot{x}_{O_2}} \\ \varphi &= \varphi_0 + \int_0^{t_1} \frac{\dot{x}_{O_1} - \dot{x}_{O_2}}{\sqrt{e_4^2 - \left(x_{O_2} - x_{O_1}\right)^2}}\, dt \end{aligned}\right\} \tag{13}$$

The graph of mobile centrode also depends only on the displacement of flat figure and does not depend on chosen centres O_1 and O_2. If exact distance between the centres O_1 and O_2, angle φ_0 between the axis X and line O_1O_2, by a single measurement the data is received for the calculation of nonmobile coordinates x_P, y_P of centrode and mobile coordinates of centrode x'_P, y'_P and for angular velocity ω_2 of the spindle. When this is known, unambiguous determination of the coordinates of every point P in the non-mobile coordinate system

$$\left.\begin{aligned} x_F &= x_P + \begin{bmatrix} x'_F - x'_P - \\ -\left(y'_F - y'_P\right)\sin\left(\varphi_0 + \tau\right) \end{bmatrix} \cos\left(\varphi_0 + \tau\right) \\ y_F &= y_P + \begin{bmatrix} y'_F - y'_P + \\ -\left(x'_F - x'_P\right)\sin\left(\varphi_0 + \tau\right) \end{bmatrix} \cos\left(\varphi_0 + \tau\right) \end{aligned}\right\} \tag{14}$$

The method proposed can be accomplished by using the roundness standard with three reference cylindrical surfaces.

Two cylindrical reference surfaces are concentric and the surface in the middle is concentric to those (Fig. 5). The value of eccentricities O_1 O_2 is determined by measurements. Such standard of roundness is fixed to the spindle to be calibrated via the centring devise. Displacements in the coordinate directions X and Y of the middle cylindrical surface are measured by two measuring systems mounted on sturdy pedestal on two perpendicular directions 1*x* ir 1*y*. Using the third measuring system 2*x* the displacement of other two cylindrical surfaces are measured in the direction of X axis. Capacitance transducers with the cylindrical surface are used for measurement of the displacement of reference cylindrical surfaces. Such transducers eliminate the influence of form errors of the roundness standard on the measurement result. Three surfaces are used for the purpose of elimination of calibration errors occurring by angular biases of the spindle during its rotation.

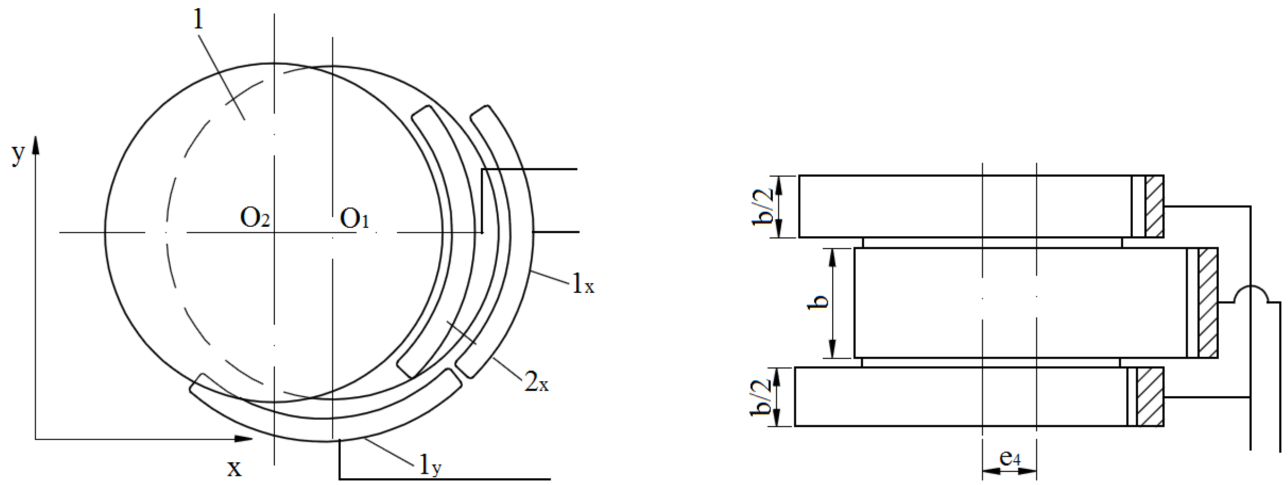

Fig. 5 Diagram of reference roundness measure for the determination of radial rotation error according to centrodes

4. Conclusions

1. Various methods of measurement of radial errors of the precision spindles using the roundness standard give different results of measurement of the same measurement due to uncontrollable values.

2. The trajectory of the centre of roundness standard used for the measurement of radial errors of the spindle depends on radial error of the spindle's axis trajectory in the measuring plane and on its value and direction of eccentricity in respect of the spindle axis.

3. Function and value of radial error measurement depend on the uncontrollable errors of centring and on the direction of measurement during the calibration.

4. The trajectory of the spindle is evaluated unambiguously according to the centrodes.

5. The centrodes can be calculated in the measuring plane according to the displacement of two eccentrically situated roundness standards in the same plane.

6. The trajectory of every point connected with the spindle can be determined unambiguously and the metrological or technological errors according to the analysis of measuring results can be determined using centrodes for the calculation.

Acknowledgments

Publication is prepared based on information received by implementing project "Creation of the absolute, high resolution, precision angular encoder with capabilities of self diagnostics" which is supported by EU structural funds and Republic of Lithuania. (project Nr. VP2-1.3-ŪM-02-K-03-025).

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A. Kasparaitis, V. Giniotis

PATOBULINTAS METODAS PRECIZINIO SŪKLIO SUKIMOSI PAKLAIDOMS ĮVERTINTI

R e z i u m ė

Analitiškai parodyta precizinių sūklių radialiųjų paklaidų matavimo pavyzdiniu apskritumo matu rezultatų priklausomybė nuo proceso metu nekontroliuojamų pavyzdinio apskritumo mato centravimo parametrų – ekscentriciteto dydžio ir jo krypties kalibruojamo sūklio atžvilgiu. Išnagrinėti dažniausi naudojami radialiosios sukimosi paklaidos matavimo fiksuotomis nepaslankios bazės atžvilgiu ir kryptimis bei fiksuota sūklio atžvilgiu kryptimi variantai. Daroma išvada apie tokio metodo subjektyvumą, nes dėl šių faktorių kitimo realiose ribose keičiasi užrašoma sūklio sukimosi radialios paklaidos forma ir iki kelių kartų matavimo rezultato skaitinė reikšmė. Analitinių tyrimų rezultatai ir išvados patvirtinti eksperimentiniais rezultatais. Pasiūlytas radialiųjų sukimosi paklaidų įvertinimas pagal centroides metodas, leidžiantis ojektyviai, nepriklausomai nuo apskritumo mato centravimo parametrų, nustatyti bet kurio su sūkliu susieto taško trajektoriją užduotoje matavimo plokštumoje. Tai leidžia įvertinti paklaidas, atsirandančias formuojant ir matuojant kampinius matus bei kitais precizinės technologijos ir metrologijos atvejais.

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DEVELOPMENT OF THE METHOD OF ACCURACY MEASURING OF PRECISION SPINDLE

S u m m a r y

The paper deals with methods of measurement of radial errors of the precision spindles using the roundness standard and the results of measurement of the same measurement due to uncontrollable values. The trajectory of the centre of roundness standard used for the measurement of radial errors of the spindle depends on radial error of the spindle‘s axis trajectory in the measuring plane and on its value and direction of eccentricity in respect of the spindle axis. Function and value of radial error measurement depend on the uncontrollable errors of centring and on the direction of measurement during the calibration. The trajectory of the spindle is evaluated unambiguously according to the centrodes using the method proposed and described in the paper.

Keywords: accuracy measuring, precision spindle.

Received April 05, 2011
Accepted June 29, 2012