ISSN 1392–1207. MECHANIKA. 2019 Volume 25(5): 341–349

Curvilinear Stress-Strain Relationship for Concrete of EN-2 Regulation in the ZI Method and the Calculation of Beam Strength

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crossref http://dx.doi.org/10.5755/j01.mech.25.5.24453

1. Introduction

The present article may be considered to be another new supplement to the monographs [1, 2]. This supplement deals with the resolution of issues of practical application of the ZI method that was developed by the author and presented in his monographs. The ZI method is a uniform (general) method and is applicable for any stage of load operation when bending moments and /or axial forces are acting. At structural members' cross sections, it is possible to make theoretical calculations of each individual actual value of stress-strain state parameters (crack, compression and tensile zone height, member layer strain and tension at cracks and between cracks). The method directly takes into account of the actual properties of the materials. A very important and complicated problem is resolved, namely that of theoretical estimate of the actual position of the neutral axis. Previously, its position used to be determined either very roughly or through costly experimental equations. The ZI method is suitable for the calculation of variously of reinforced structural members (with not pre-tensioned reinforcement, with tensioned reinforcement, with mixed reinforcement, with not necessarily metallic reinforcement located at any height of the structural member; where there can be any number of rows of reinforcement) of different materials (concrete, reinforced concrete, metal, wood, plastic, etc.), also for members with any kind of cross-section. Stress-strain relationships can be described by various equations, i.e., the stress diagrams may be curvilinear, rectangular, triangular, etc. It should be noted that for the calculations we only need to have stress-strain diagrams. The method enables estimation of the deviation of strain from the flat section. More information on this may be found in monographs [1 and 2] and in articles [3, 4, etc.].

The most complex material is reinforced concrete since members made of reinforced concrete may have cracks in the tension zone. This article focuses on the calculation of strength of reinforced concrete beams based on curvilinear stress-strain relationship $\sigma_c - \varepsilon_c$ for concrete presented in regulations [5–7]. But here still remains one unresolved issue. This relationship is presented for the case where the reliability is 50 %. It is suitable for the analysis of results of reinforced concrete member tests. There is no option for the calculation of the SLS – serviceability limit states – reliability 95 %) or the ULS – ultimate limit states reliability – ~100 %).

To calculate strength of reinforced concrete beams, articles [4, 8] and supplements B and C to the monographs [1, 2] used the eurocode $\sigma_c - \varepsilon_c$ diagram described by the ZI method. The reliability was raised from 50 % to ~100 % not through the increase of the reliability of concrete strength, as it is usually done in the ultimate limit states method, but rather by dividing the beam compression zone strength F_{cm} by factor γ_{Fc}=1.95. The calculation yielded good results – see Table 2.

In order to retain the integrity of the ultimate limit states method, this article offers relationships $\sigma_c - \varepsilon_c$ analogous to the ones used in EN-2 regulations. Their reliability is not 50 %, but 95 % and ~100 %. They are described by polynomials by the ZI method. The latter relationship with reliability of ~100 % is used in the present article to calculate the strength of beam, and the relationship with reliability of 95 % will be used in the next article to examine the serviceability limit states.

One of the aims of this article is to improve the method for calculating the strength of heavily reinforced structural members.

2. Stress-strain diagrams for concrete offered by EN-2 and proposed by the present article

Regulations for reinforced concrete [5–7] present stress-strain relationships as shown in Table 1.

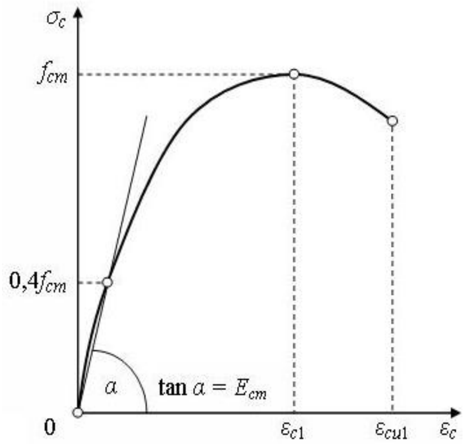

Fig. 1 Stress-strain relationship for concrete as presented in regulation EN-2

The diagram parameters are expressed by the following equations:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1+(k-2)\eta}, \tag{1}$$

$$\eta = \frac{\varepsilon_c}{\varepsilon_{c1}}, \tag{2}$$

$$k = \frac{1.05 E_{cm} \varepsilon_{c1}}{f_{cm}}, \quad (3)$$

here and further ε_{c1} values are considered to be positive. If $E_{cm} = tan\alpha$, $E_c = 1.05 E_{cm}$ and $\nu_{c1} = \frac{f_{cm}}{E_c \varepsilon_{c1}}$, then

$$k = \frac{1}{\nu_{c1}} \text{ and } \sigma_c = \frac{\eta / \nu_{c1} - \eta^2}{1 + (1/\nu_{c1} - 2)\eta} f_{cm}, \quad (4)$$

$$E_{cm} = 22(f_{cm}/10)^{0.3} \text{ (GPa); } (f_{cm}\text{, MPa}), \quad (5)$$

$$\varepsilon_{c1} = 0.7 f_{cm}^{0.31} \le 2.8 \text{ (‰(; } (f_{cm}\text{, MPa}), \quad (6)$$

$$\left.\begin{array}{l} \varepsilon_{cu1} = 3.5 \cdot (‰) \quad for \quad f_{ck} \le 50 \quad \text{MPa} \\ \varepsilon_{cu1} = 2.8 + 27[(98 - f_{cm})/100]^4 (‰) \\ \quad \text{for} \quad f_{ck} \ge 50 \quad \text{MPa} \end{array}\right\}. \quad (7)$$

Relationship $\sigma_c - \varepsilon_c$ for concrete in Fig. 1 in the ZI method can be accurately described by a 5th degree polynomial. The rising part of the diagram and part of the falling part of the diagram and sometimes even all of it can be described quite accurately also by a much simpler 3rd degree polynomial. The latter option has been chosen in this article. A3-degree graph is shown in Fig. 2.

Equations of Figs. 1 and 2 and Eqs. (1)–(7) mostly use mean parameter values, their index is *cm*, and their reliability is 50 %. The method is suitable for the analysis of test data. In this case in Fig. 2 $E_{cm} = tan\alpha$, , $E_c = 1.05 E_{cm}$, $\sigma_{c1} = f_{cm}$, $\varepsilon_{c1} = 0.7 f_{cm}^{0.31} \le 2.8$.

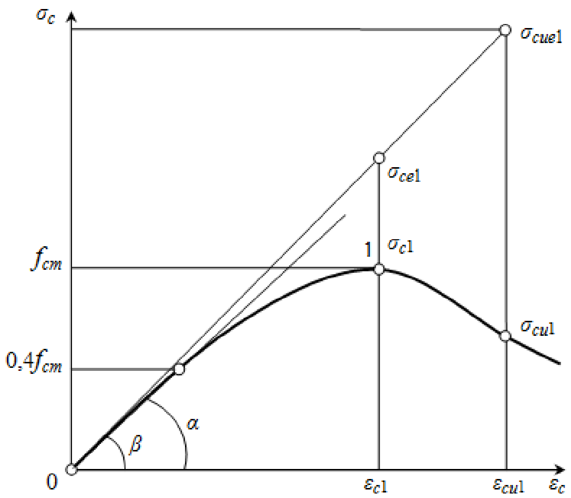

Fig. 2 The ZI methodology takes into account 3rd grade stress-strain relationship for concrete

In order to have reliability of 95 %, which is used for the calculation of the serviceability limit states (SLS states), the author of the present article suggests in Eqs. (5 and 6) (rather than using mean values of parameters) assuming characteristic values – index *k*. Then in Fig. 2 $E_{ck} = tan\beta$, $\sigma_{c1} = f_{ck}$, $\varepsilon_{ck1} = 0.7 f_{ck}^{0.31} \le 2.8$. This option is planned to be further analysed in the author's next article.

When reliability of ~100% is required, the author of the present article proposes when calculating the ultimate limit states (ULS states) in Eqs. (5 and 6) (rather than assuming mean values of parameters) to assume the calculated values index *d*. Then, in Fig. 2 $E_{cd} = tan\beta$, $\sigma_{c1} = f_{cd}$, $\varepsilon_{cd1} = 0.7 f_{cd}^{0.31} \le 2.8$. This option is analysed in greater detail in this article. We obtain good results– Tables 1 and 2.

3. The main equations of the ZI method when a 3rd degree polynomial is used

When the function of Fig. 2 is expressed by the ZI method's 3rd degree polynomial, the following simple equations are used:

$$\sigma_c = E_c \varepsilon_c \left(1 + c_1 \eta + c_2 \eta^2\right) = \nu_c E_c \varepsilon_c = \nu_c \sigma_{ce}, \quad (8)$$

$$\nu_c = 1 + c_1 \eta + c_2 \eta^2 = 1 + (3\nu_{c1} - 2)\eta + (1 - 2\nu_{c1})\eta^2, \quad (9)$$

$$\nu_{c1} = \frac{\sigma_{c1}}{\sigma_{ce1}} = \frac{f_{cm}}{E_{cm} \varepsilon_{c1}}, \quad (10)$$

$$\eta = \varepsilon_c / \varepsilon_{c1}, \quad (11)$$

$$F_c = \int_0^{x_w} \sigma_c b (\delta x_c) = \omega_{nc} \varepsilon_w E_c b x_w, \quad (12)$$

$$M_c = \int_0^{x_w} \sigma_c b x_c (\delta x_c) = \omega_{mc} \varepsilon_w E_c b x_w^2, \quad (13)$$

$$e_c = \frac{M_c}{F_c} = \frac{\omega_{mc}}{\omega_{nc}} x_w, \quad (14)$$

$$\left.\begin{array}{l} \omega_{nc} = \frac{1}{2} + \frac{c_1}{3}\eta + \frac{c_2}{4}\eta^2 \\ \omega_{mc} = \frac{1}{3} + \frac{c_1}{4}\eta + \frac{c_2}{5}\eta^2 \end{array}\right\}. \quad (15)$$

ε_w is maximum (edge) strain of the compression zone of the beam.

For more information, see the examples.

4. Assumptions and calculations made in the present article

4.1. Assumptions

1. For the concrete of beam compression zone, a curvilinear EN-2 stress diagram $\sigma_c - \varepsilon_c$ is assumed that is shown in Fig. 1, which is described by the ZI method's 3rd degree polynomial as shown in Fig. 2.
2. For the reinforcement of tensile zone of beams analysed in the article, the diagram $\sigma_s - \varepsilon_s$ as shown in Fig. 3 is used.
3. The strength of a beam is considered to be the state when the stress of the concrete compression zone's stresses

$\sigma_c = \sigma_{c1}$ and strain $\varepsilon_c = \varepsilon_{c1}$ or when the strain of the reinforcement in tension $\varepsilon_s = \varepsilon_{su}$.

4. Hypothesis of plane sections (Bernoulli) is applied.
5. The impact of the tensioned concrete over the crack is disregarded.

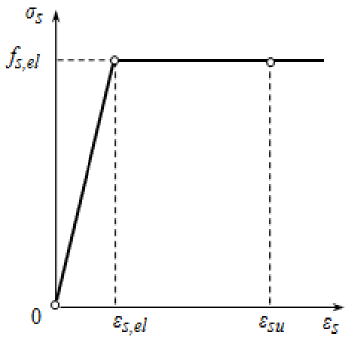

Fig. 3 Reinforcement stress-strain diagram is assumed in the article

4.2. Calculations made in the article

Scope of the research. Four reinforced concrete beam's tensile zone's reinforcement variants – little (reinforcement factor $\rho_l \approx 0.44$ %), average ($\rho_l \approx 1.07$ % and $\rho_l \approx 1.60$ %) and large ($\rho_l \approx 2.13$ %). All the [5–7] strength classes of regular concrete as presented in the regulations: f_{ck} from 08 to 90 MPa. Beam cross-section parameters: $b = 0.20$ m, $h = 0.50$ m, $d = 0.46$ m (Fig. 4).

4.3. Research results

The results of the calculations are presented in Table 2.

Further in the text we supply examples of the calculation and explanations of the results presented in the tables.

Examples of calculations of $M_{Rd,EN-2}$ by EN-2 equations and calculation of $M_{Rd,ZI}$ by the ZI method when $\gamma_{Fc} = 1.95$ coefficient is used are presented in example to monographs [1, 2]. Results of the calculation are presented in Table 2.

The article presents examples of variants of calculation of $M_{Rdu,ZI}$ by the ZI method, when the calculated expression is used of the diagram $\sigma_c - \varepsilon_c$ presented in the EN-2 regulation described by the ZI method. There are three possible cases: Case 1, where $\sigma_c = f_{cd}$ and $\varepsilon_{s,el} \le \varepsilon_s \le \varepsilon_{su}$ – these are regularly (economically) reinforced beams; Case 2, where $\sigma_c = f_{cd}$ and $\varepsilon_s < \varepsilon_{s,el}$, – these are abundantly reinforced beams and Case 3, where $\sigma_c = f_{cd}$ and $\varepsilon_s \ge \varepsilon_{su}$ economical reinforcement – when the calculation is made on the assumption that $\sigma_c = f_{cd}$ and $\varepsilon_s = \varepsilon_{su}$. In all cases, the calculated version of the curvilinear EN-2 diagram described by the ZI method is used, i. e. partial material strength factors are used that are accepted in the limit state method. In the examples provided for the simplification of the calculations, the impact of tensile concrete over the crack is disregarded, as in most cases it is insignificant. If necessary, it is possible to factor in also the impact of the tensioned concrete over the crack, the impact of the axial force, and the impact of the flanges.

In examples 1, 2 and 3, $M_{Rdu,ZI}$ is calculated by the ZI method by applying to the compression zone concrete partial factor $\gamma_c = 1.5$.

Example 1. Regularly (economically) reinforced beam

When $\sigma_c = f_{cd}$ and $\varepsilon_{s,el} \le \varepsilon_s \le \varepsilon_{su}$, strength of regularly (economically) reinforced concrete beams with rectangular cross-section is calculated by the method of ultimate limit states of both zones (tension zone reinforcement and compression zone concrete). $f_{ck} = 25$ MPa.

$$f_{cd} = f_{ck}/1.5 = 25/1.5 = 1.6667 \quad \text{MPa.}$$

$$2\varphi16 \ \text{mm} \rightarrow A_s = 4.02 \ \text{cm}^2.$$

$$f_{sd} = \frac{f_{sk}}{\gamma_s} = \frac{400}{1.1} = 363.636 \cong 364 \quad \text{MPa,}$$

$$F_{sd} = f_{sd}A_s = 363.6 \cdot 10^6 \cdot 4.02 \cdot 10^{-4} = 146.167 \quad \text{kN.}$$

$$x_{w1} = \frac{f_{sd}A_s + P}{\omega_{nc1}\varepsilon_{cd1}E_{cd}b} = \frac{146.167 \cdot 10^3 + 0}{0.27743 \cdot 42.9375 \cdot 10^6 \cdot 0.20} =$$

$$= 0.061352 \quad \text{m.}$$

$$\varepsilon_s = \frac{\varepsilon_{cd1}}{x_{w1}}(d - x_{w1}) = \frac{1.6744}{6.1352}(46 - 6.1352) = 10.8798 \quad ‰ >$$

$$> \varepsilon_{sd,el} = \frac{f_{sd}}{E_s} = \frac{363.6 \cdot 10^6}{200 \cdot 10^9} = 1.818 \cdot 10^{-3} = 1.818 \quad ‰.$$

For instance, if $\varepsilon_{su} = 35$ ‰, then $\varepsilon_s = \varepsilon_{su}$. Control:

$$F_{cd} = \omega_{nd1}\varepsilon_{cd1}E_{cd}bx_{w1} =$$

$$= 0.27743 \cdot 1.6744 \cdot 10^{-3} \cdot 25.6435 \cdot 10^9 \cdot 0.20 \cdot 0.061352 =$$

$$= 0.146167 \quad \text{MN} = 146.167 \quad \text{KN} = F_{sd}.$$

Distance between F_c and F_s:

$$z_c = z_{cd1} = z_{sd1} = d - (1 - \omega_{mc}/\omega_{nc})x_{w,c1} =$$

$$= 0.46 - (1 - 0.16920/0.27743) \cdot 0.061352 = 0.43507 \quad \text{m.}$$

$$M_{Ru} = F_{sd}z_{sd1} = 146.167 \cdot 0.43607 = 63.739 \quad \text{kNm.}$$

If we factor in the impact of tensioned concrete above the crack, then we get M_{Ru} = 63.747 kN·m, i.e. almost the same result.

If we factor in not only the tension zone reinforcements force F_{st} and compression zone concrete force F_{cd}, but also tension zone over the crack concrete force and the forces, values of which do not depend on x_{w1} (axial and tension forces N and P, compression zone reinforcement $F_{sc} = f_{scd}A_{sc}$ and flanges $F_{fd} = \eta f_{cd}(b_f - b)h_f$ forces (Fig. 5), then $x_{w1} = \dfrac{N_{const}}{(\omega_{nc1}\varepsilon_{cd1} - \eta_{tu\varepsilon}\omega_{nt}\varepsilon_{ctu})E_{cd}b}$,

$$N_{const} = N + P + F_{cf} - F_{sc} - F_{cf} .$$

Example 2. $\varepsilon_s < \varepsilon_{s,el}$, i.e. case of abundant reinforcement

$f_{sk} = 20$ MPa, $4\varphi25$ mm $\rightarrow A_s = 19.63$ cm^2,

$$\rho_l = 19.63 \cdot 100/(20 \cdot 46) = 2.1337 \approx 2.134 \quad \%,$$

$$F_{sd} = f_{sd} A_s = 363.6 \cdot 19.63 \cdot 10^{-4} = 713.747 \quad \text{kN},$$

$$\alpha_e = \frac{E_s}{E_{cd}} = \frac{200 \cdot 10^9}{23.9830 \cdot 10^9} = 8.3392403 \ .$$

From monographs [1, 2] equations relative (not actual) values are calculated:

$$s = \frac{\alpha_{est}\rho_l}{\omega_{nc1}} = \frac{8.3392403 \cdot 2.1337 \cdot 10^{-2}}{0.26123} = 0.681140643,$$

$$\xi_{w1} = -\frac{s}{2} + \sqrt{\left(\frac{s}{2}\right)^2 + s} = -\frac{0.68114064}{2} +$$

$$+\sqrt{\left(\frac{0.68114064}{2}\right)^2 + 0.68114064} = 0.55225037 \ .$$

$$x_{w1} = \xi_{w1} d = 0.55225037 \cdot 0.46 = 0.25403517 \quad \text{m}.$$

$$\varepsilon_s = \frac{\varepsilon_{cd1}}{x_{w1}}\left(d - x_{w1}\right) =$$

$$= \frac{1.5625}{0.25403517}\left(0.46 - 0.25403517\right) = 1.2668 \quad ‰ <$$

$$< \varepsilon_{y,el} = \frac{f_{cd}}{E_s} = \frac{363.6 \cdot 10^6}{200 \cdot 10^9} = 1.818 \cdot 10^{-3} = 1.818 \quad ‰.$$

Actual values:

$$F_{sd} = \varepsilon_s E_s A_s = 1.2668 \cdot 10^{-3} \cdot 200 \cdot 10^9 \cdot 19.63 \cdot 10^{-4} =$$

$$= 4973.585 \cdot 10^2 \quad \text{N} = 497.3585 \quad \text{kN}.$$

$$F_{cd} = \omega_{nc1}\varepsilon_{cd1}E_{cd} b x_{w1} =$$

$$= 0.26123 \cdot 1.5625 \cdot 10^{-3} \cdot 23.9830 \cdot 10^9 \cdot 0.20 \cdot 0.254035 =$$

$$= 0.4973611 \quad \text{MN} = 497.3611 \quad \text{kN}.$$

$$z_c = z_{cd1} = z_{sd1} = d - \left(1 - \omega_{mc1}/\omega_{nc1}\right) x_{w1} =$$

$$= 0.46 - \left(1 - 0.15786 / 0.26123\right) \cdot 0.25403517 = 0.359477 \quad \text{m}.$$

$$M_{Ru} = F_c z_c = F_s z_s = F_{sd} z_{sd1} = 497.360 \cdot 0.359477 =$$

$$= 178.78976 \quad \text{kNm}.$$

Example 3. $\varepsilon_s < \varepsilon_{su}$**=35‰. In this case, the procedure of the calculation is the same as that of the calculation of example of the monographs [1, 2].**

The same beam as in example 1, but here not $f_{ck} = 25$ MPa, but $f_{ck} = 70$ MPa.

Calculation by method of Example 1 resulted in the following $\varepsilon_s = 41.25936‰ > \varepsilon_{su} = 35‰$. This is a rather rare case, so because of the limited scope of the article we only provide guidelines for further calculations.

$$E_{su} = \frac{f_{sd}}{\varepsilon_{yu}} = \frac{363.6 \cdot 10^6}{35 \cdot 10^{-3}} = 10.3885714 \quad \text{GPa}.$$

$$\nu_s = \frac{E_{su}}{E_s} = \frac{10.3885714 \cdot 10^9}{200 \cdot 10^9} = 0.05194286 \ .$$

$$\alpha_e = \frac{E_s}{E_{cd}} = \frac{200 \cdot 10^9}{34.9241 \cdot 10^9} = 5.72670448 \ .$$

$$a_\varepsilon = a_{si} = 0.46 \quad \text{m}.$$

$$Z_{\kappa n/b\sigma} = \frac{\Sigma(P_i \nu_{Si}/\nu_{pi}) + \Sigma N_i}{bE\ \varepsilon_\varepsilon / k_\varepsilon} = 0 \ .$$

From [1, 2]:

$$\left.\begin{aligned} S_{n0} &= a_\varepsilon^2\left(\Sigma Z_{si/b} a_{si} + a_\varepsilon Z_{\kappa n/b\sigma}\right) = 58.196941 \cdot 10^{-6} \\ S_{n1} &= a_\varepsilon\left[\Sigma Z_{si/b}\left(a_\varepsilon + 2a_{si}\right) + 3a_\varepsilon Z_{\kappa n/b\sigma}\right] = \\ &= 379.545268 \cdot 10^{-6} \end{aligned}\right\}$$

$$\left.\begin{aligned} S_{n2} &= \Sigma Z_{si/b}\left(2a_\varepsilon + a_{si}\right) + 3a_\varepsilon Z_{\kappa n/b\sigma} = 825.098408 \\ S_{n3} &= \Sigma Z_{si/b} + Z_{\kappa n/b\sigma} = 597.897397 \cdot 10^{-6} \end{aligned}\right\}$$

$$\left.\begin{aligned} B_{n0} &= 12\left(W_{n0} + S_{n0}\right) = 698.363293 \cdot 10^{-6} \\ B_{n1} &= 12\left(W_{n1} + S_{n1}\right) = 4554.543216 \cdot 10^{-6} \\ B_{n2} &= 12\left(W_{n2} + S_{n2}\right) = 9901.180896 \cdot 10^{-6} \\ B_{n3} &= 12\left(W_{n3} + S_{n3}\right) = 7174.768769 \cdot 10^{-6} \end{aligned}\right\}$$

From [1 and 2]:

$$\eta_{0\varepsilon c} = \frac{\varepsilon_{0\varepsilon}}{-\varepsilon_{cd1}} = \frac{35}{-2.3040} = -15.1909722 \ .$$

$$u_{1c} = c_{d1}\eta_{0\varepsilon c} = -0.2601 \cdot (-15.1909722\ ) = 3.951172 \ ,$$

$$u_{2c} = c_{d2}\eta_{0\varepsilon c}^2 = -0.1599 \cdot (-15.1909722\ )^2 = -36.899425 \ ,$$

$$n_{0c} = 6a_\varepsilon^2 = 6 \cdot 0.46^2 = 1.2696 \quad \text{m}^2,$$

$$n_{1c} = 4 \cdot \left(3 + u_{1c}\right) a_\varepsilon = 4 \cdot \left(3 + 3.951172\ \right) \cdot 0.46 = 12.790157 \quad \text{m}$$

$$n_{2c} = 6 + 4u_{1c} + 3u_{2c} = 6 + 4 \cdot 3.951172 + 3 \cdot \left(-36.899425\right) =$$

$$= -88.893588.$$

$$\left.\begin{aligned} C_{n2} &= k_c n_{0c} = 1 \cdot 1.2696 = 1269600 \cdot 10^{-6} \\ C_{n3} &= k_c n_{1c} = 1 \cdot 12.7901565 = 12790156.5 \cdot 10^{-6} \\ C_{n4} &= k_c n_{2c} = -88893588 \cdot 10^{-6} \end{aligned}\right\}.$$

From [1, 2]:

$$\left.\begin{aligned} a_{n0} &= B_{n0} + T_{n0} = B_{n0} = 698.363293 \cdot 10^{-6} \\ a_{n1} &= B_{n1} + T_{n1} = B_{n1} = 4554.54321\ 6 \cdot 10^{-6} \\ a_{n2} &= B_{n2} + T_{n2} - C_{n2} = B_{n2} - C_{n2} = -1259698.82 \cdot 10^{-6} \\ a_{n3} &= B_{n3} + T_{n3} - C_{n3} = B_{n3} - C_{n3} = -12782981.7\ 3 \cdot 10^{-6} \\ a_{n4} &= T_{n4} - C_{n4} = -C_{n4} = 88893588 \cdot 10^{-6} \end{aligned}\right\}$$

From [1, 2]:

$$698.363293 + 4554.543216 x_w - 1259698.82 x_w^2 -$$

$$-12782981.73 x_w^3 + 88893588 x_w^4 = 0.$$

$$x_{w1} = -0.02578063 \cong -0.0257806 \quad \text{m}.$$

$$x_{c1} = -x_w = 0.02578063 \cong -0.0257806 \quad \text{m}.$$

$$\varepsilon_w = \frac{\varepsilon_s}{d - x_{w1}} x_{w1} = \frac{35 \cdot 2.578063}{46 - 2.578063} = 2.0780327 \quad ‰ <$$

$$< \varepsilon_{cd1} = 2.3040 \quad ‰.$$

$$\eta_w = \frac{x_w}{x_{cm}} = \frac{\varepsilon_w}{\varepsilon_{cm}} = \frac{2.0780327}{2.3040} = 0.9019239.$$

$$\omega_{nc} = \frac{1}{2} + \frac{c_1}{3}\eta_w + \frac{c_2}{4}\eta_w^2 = \frac{1}{2} + \frac{-0.2601148}{3} \cdot 0.9019239 +$$

$$+ \frac{-0.1599236}{4} \cdot 0.9019239^{\ 2} \cong 0.38927562 \ .$$

$$F_c = \phi E_c b \omega_{nc} x_c^2 = \frac{\varepsilon_w}{x_c} E_c b \omega_{nc} x_c^2 = \omega_{nc} E_c \varepsilon_w b x_c =$$

$$= 0.3892756 \cdot 34.9241 \cdot 10^9 \cdot 2.0780327 \cdot 10^{-3} \cdot 0.20 \cdot$$

$$\cdot 0.0257806 = 0.14566604445 \cdot 10^6\, N \cong 145.6660 \quad \text{kN}.$$

$$F_s = \sigma_{yd} A_s = f_{sd} A_s = 363.6 \cdot 10^6 \cdot 4.02 \cdot 10^{-4} =$$
$$= 1461.672 \cdot 10^2 = 146.1672 \quad \text{kN}.$$
$$e_{c0} = \frac{\omega_{mc}}{\omega_{nc}} x_c = \frac{0.248664}{0.389276} \cdot 0.0257806 = 0.0164683 \quad \text{m}.$$

Distance between Z_c and Z_s:

$$z_c = z_{cd1} = z_{sd1} = d - \left(1 - \omega_{mc1} / \omega_{nc1}\right) x_{w1} =$$
$$= 0.46 - \left(1 - \frac{0.248664}{0.389276}\right) \cdot 0.0257806 = 0.45068768 \quad \text{m}.$$
$$M_{Ru} = M_{cs} = N_c \left(d - x_c + e_{c0}\right) =$$
$$= 145.666 \cdot \left(0.46 - 0.0257806 + 0.0164683\right) =$$
$$= 145.666 \cdot 0.4506877 = 65.6498745 \cong 65.650 \quad \text{kNm}.$$

If in Example 3 instead of $\varepsilon_s = \varepsilon_{su} = 35‰$ we take $\varepsilon_s = \varepsilon_{yu} = 41.25936‰$, then we get the same result as the result of the calculation according to both zones ULS method of Example 1, i.e.

$x_{w1} = 2.4329$ cm, $\varepsilon_{w1} = 2.3040$ ‰,

$\varepsilon_s = \varepsilon_{yu} = 41.25936$ ‰ and $M_{Ru} = 65.932$ kNm.

This confirms the correctness of the method of Example 3.

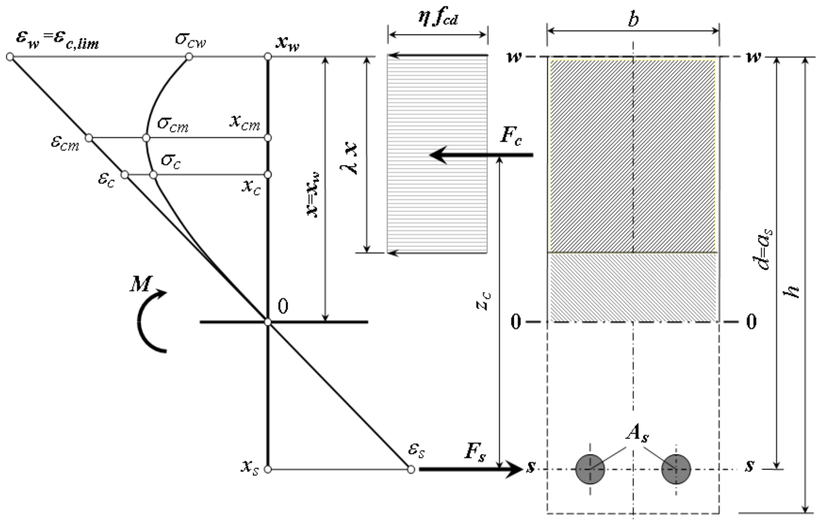

Fig. 4 Cross-section of a member with rectangular cross-section and stress-strain diagrams

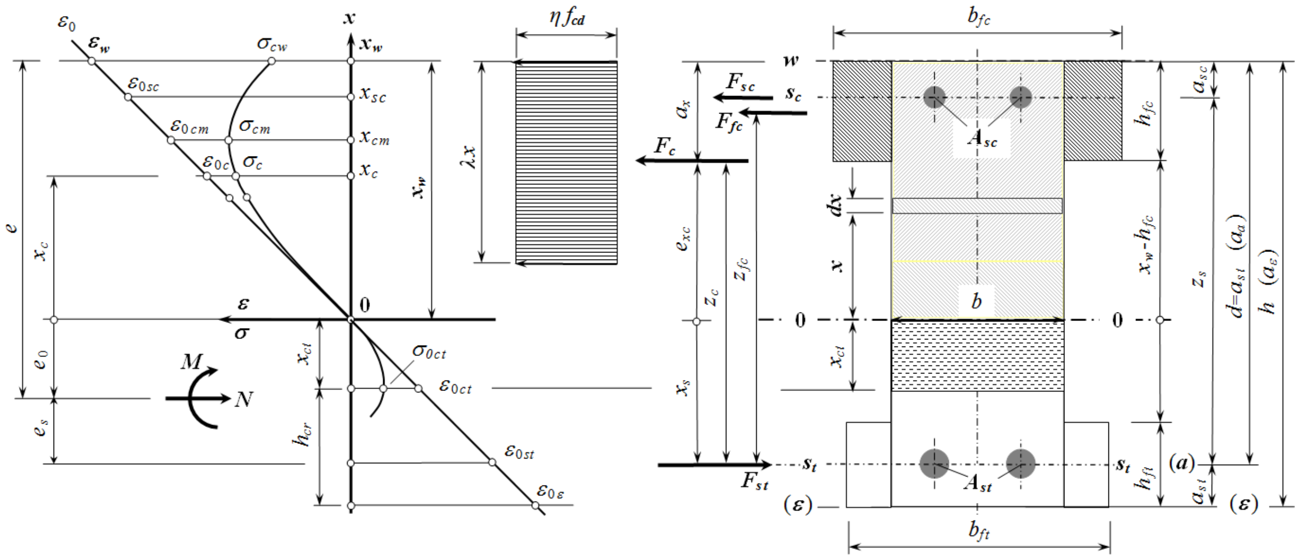

Fig. 5 Cross-section of a member with flanges and stress-strain diagrams

Table 1

Strength and deformation characteristics for concrete

Parameters	Strength classes for concrete														
f_{ck}, **MPa**	**8**	**12**	**16**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**70**	**80**	**90**
Mean values – **used for the analysis of data results – reliability 50%**															
$f_{cm} = f_{ck} + 8$ (MPa)	16	20	24	28	33	38	43	48	53	58	63	68	78	88	98
$E_{cm} = 22(f_{cm}/10)^{0.3}$ (E_{cm}, GPa, f_{cm}, MPa)	25.3314	27.0852	28.6079	29.9620	31.4758	32.8366	34.0771	35.2205	36.2832	37.2779	38.2142	39.0999	40.7428	42.2442	43.6305
$E_{cm0} = 1.05 E_{cm}$ (GPa)	26.5979	28.4394	30.0383	31.4600	33.0496	34.4784	35.7810	36.9815	38.0973	39.1418	40.1249	41.0549	42.7800	44.3565	45.8121
$\varepsilon_{cm1} = 0.7 f_{cm}^{0.31} \le 2.8$ (‰); (f_{cm}, MPa)	1.6534	1.7718	1.8748	1.9666	2.0694	2.1619	2.2463	2.3242	2.3968	2.4647	2.5287	2.5893	2.7018	2.8	2.8
$\varepsilon_{cmu1} = 3.5$ ‰, when $f_{ck} < 50$ MPa; $\varepsilon_{cu1} = 2.8 + 27\left[(98 - f_{cm})/100\right]^4$ (‰), when $f_{ck} \ge 50$ MPa															
ε_{cmu1}, (‰)	3.5									3.4912	3.2052	3.0187	2.8432	2.8027	2.8
$\sigma_{cme1} = \varepsilon_{cm1} E_{cm0}$, (MPa)	43.9770	50.3889	56.3158	61.8692	68.3928	74.5389	80.3749	85.9524	91.3116	96.4728	101.464	106.303	115.583	124.198	128.274
$\nu_{cm1} = f_{cm}/(1.05 E_{cm} \varepsilon_{c1})$ (f_{cm}, MPa; E_{cm}, GPa; ε_{c1}, ‰)	0.3638	0.3969	0.4262	0.4526	0.4825	0.5098	0.5350	0.5584	0.5804	0.6012	0.6209	0.6397	0.6748	0.7086	0.7640
$c_{m1} = 3\nu_{cm1} - 2$,	-0.9086	-0.8093	-0.7214	-0.6422	-0.5525	-0.4706	-0.3950	-0.3245	-0.2588	-0.1964	-0.1373	-0.0809	0.0244	0.1258	0.2920
$c_{m2} = 1 - 2\nu_{cm1}$	0.2724	0.2062	0.1476	0.0948	0.0350	-0.0196	-0.0700	-0.1170	-0.1608	-0.2024	-0.2418	-0.2794	-0.3496	-0.4172	-0.5280
Design values **for the calculation of the ultimate limit states (ULS) – reliability ~ 100%**															
$f_{cd} = f_{ck}/1.5$, (MPa)	5.3333	8	10.6667	13.3333	16.6667	20	23.3333	26.6667	30	33.3333	36.6667	40	46.6667	53.3333	60
$E_{cd} = 22(f_{cd}/10)^{0.3}$, (GPa)	18.2189	20.5755	22.4301	23.9830	25.6435	27.0852	28.3672	29.5266	30.5886	31.5709	32.4866	33.3458	34.9241	36.3515	37.6589
$\varepsilon_{cd1} = 0.7 f_{cd}^{0.31} < 2.8$, (‰)	1.1762	1.3337	1.4581	1.5625	1.6744	1.7718	1.8585	1.9371	2.0091	2.0758	2.1381	2.1965	2.3040	2.4014	2.4907
$\varepsilon_{cdu1} = 3.5$ ‰, when $f_{ck} < 50$ MPa; $\varepsilon_{cdu1} = 2.6 + 35\left[(90 - f_{ck})/100\right]^4$ (‰), when $f_{ck} \ge 50$ MPa															
ε_{cdu1}, (‰)	3.5									3.4960	3.1252	2.8835	2.6560	2.6035	2.6000
$\sigma_{cde1} = \varepsilon_{cd1} E_{cd}$, (MPa)	21.4291	27.4415	32.7053	37.4734	42.9375	47.9896	52.7204	57.960	61.4556	65.5349	69.4596	73.2440	80.4651	87.2945	93.7970
$\nu_{cd1} = f_{cd}/\varepsilon_{cd1} E_{cd}$ (f_{cd}, MPa; E_{cd}, GPa; ε_{cd1}, ‰)	0.2489	0.2915	0.3261	0.3558	0.3882	0.4168	0.4426	0.46624	0.4882	0.5086	0.5279	0.5461	0.5800	0.6110	0.6397
$c_{d1} = 3\nu_{cd1} - 2$	-1.2533	-1.1254	-1.0216	-0.9326	-0.8355	-0.7497	-0.6723	-0.6013	-0.5355	-0.4741	-0.4163	-0.3617	-0.2601	-0.1671	-0.0810
$c_{d1} = 1 - 2\nu_{cd1}$	0.5022	0.4169	0.3477	0.2884	0.2237	0.1665	0.1148	0.0675	0.0237	-0.0173	-0.0558	-0.0922	-0.1599	-0.2219	-0.2794
When $\varepsilon_w = \varepsilon_{cd1}$; $\eta_{d1} = 1$															
$\omega_{nd1} = \frac{1}{2} + \frac{c_1}{3}\eta_{d1} + \frac{c_2}{4}\eta_{d1}^2$	0.20778	0.22909	0.24639	0.26123	0.27743	0.29173	0.30460	0.31644	0.32743	0.33764	0.34728	0.35638	0.37333	0.38883	0.40315
$\omega_{md1} = \frac{1}{3} + \frac{c_{d1}}{4}\eta_{d1} + \frac{c_{d1}}{5}\eta_{d1}^2$,	0.12045	0.13536	0.14747	0.15786	0.16920	0.17921	0.18822	0.19651	0.20420	0.21135	0.21810	0.22447	0.23633	0.24718	0.25720
$\omega_{md1}/\omega_{nd1}$	0.57970	0.59086	0.59852	0.60430	0.60988	0.61430	0.61793	0.62100	0.62364	0.62596	0.62802	0.62986	0.63303	0.63570	0.63798
$F_{cd}/x_{w1} = \omega_{nd1} \varepsilon_{cd1} E_{cd} b$	0.89051	1.25731	1.61165	1.95784	2.38243	2.80000	3.21173	3.66817	4.02448	4.42544	4.82439	5.22054	6.00801	6.78854	7.56285
$x_{lim1} = \varepsilon_{cd1} d/(\varepsilon_{cd1} + \varepsilon_{sd,el})$	18.0700	19.4657	20.4733	21.2616	22.0543	22.7040	23.2534	23.7295	24.1485	24.5228	24.8610	25.1685	25.718	26.180	26.5909

Continuation of Table 1

Strength and deformation characteristics for concrete

Design values **for the calculation of the ultimate limit states (ULS) – reliability ~ 100%**															
f_{ck}, **MPa**	**8**	**12**	**16**	**20**	**25**	**30**	**35**	**40**	**45**	**50**	**55**	**60**	**70**	**80**	**90**
Below for all types classes of concrete when $2\phi 16mm \rightarrow A_s = 4.02cm^2$, $\rho_l = 4.02 \cdot 100 / (20 \cdot 46) = 0.4370$ **%**, $N_{sd} = f_{sd} A_s = 363.6 \cdot 4.02 \cdot 10^{-4} = 1461.672 \cdot 10^{-4} MN = 146.167$ **kN**															
x_{w1}, cm	16.4139	11.6254	9.0694	7.4657	6.1352	5.2203	4.5510	3.9847	3.6445	3.3029	3.0298	2.7998	2.4329	2.1531	1.9327
$\varepsilon_s = \varepsilon_{cd1}(d - x_{w1})/x_{w1}$, ‰	2.1201	3.9436	5.9374	8.0648	10.8798	13.8410	16.9264	20.4249	23.3496	26.8344	30.3242	33.8909	41.2594	48.9024	56.7903
$z_c = z_{cd1} = z_{sd1} =$ $= d - (1 - \omega_{md1}/\omega_{nd1}) x_{w1}$, cm	39.101	41.244	42.359	0.4305	43.607	43.987	44.261	44.490	44.628	44.7656	44.8730	44.9637	45..107	45.216	45.300
$M_{Ru} = F_{cd} z_s = F_{sd} z_s$, kN·m	57.153	60.285	61.915	62.919	63.739	64.294	64.695	65.029	65.232	65.431	65.590	65.722	65.932	66.090	66.214
Below for all types classes of concrete when $2\phi 25mm \rightarrow A_s = 9.82cm^2$, $\rho_l = 9.82 \cdot 100 / (20 \cdot 46) = 1.0674 \approx 1.067$ **%** $N_{sd} = f_{sd} A_s = 363.6 \cdot 9.82 \cdot 10^{-4} = 3570.552 \cdot 10^{-4} MN = 357.055$ **kN**															
x_{w1}, cm	**23.9283**	**22.2459**	**18.2897**	18.2372	14.9870	12.7520	11.1172	9.73387	8.87208	8.06824	7.40104	6.83943	5.94298	5.25967	4.72117
$\varepsilon_s = \varepsilon_{cd1}(d - x_{w1})/x_{w1}$, ‰	**1.08494**	**1.42417**	**1.72766**	2.37862	3.46488	4.61957	5.83148	7.21718	8.40769	9.75910	11.1509	12.5765	15.5295	18.6008	21.7771
$z_c = z_{cd1} = z_{sd1} =$ $= d - (1 - \omega_{md1}/\omega_{nd1}) x_{w1}$, cm	**35.943**	**36.898**	**37.547**	38.784	40.153	41.082	41.752	42.311	42.661	42.982	43.247	43.468	43.819	44.084	44.291
$M_{Ru} = F_{cd} z_s = F_{sd} z_s$, kN·m	**76.588**	**103.204**	**127.403**	138.480	143.368	146.685	149.078	151.074	142.323	153.469	154.415	155.205	156.458	157.404	158.143
Below for all types classes of concrete when $\approx 3\phi 25mm \rightarrow A_s = 14.68cm^2$ $\rho_l = 14.68 \cdot 100 / (20 \cdot 46) = 1.5957 \approx 1.596$ **%**, $N_{sd} = f_{sd} A_s = 363.6 \cdot 14.68 \cdot 10^{-4} = 5337.648 \cdot 10^{-4} MN = 533.8$ **kN**															
x_{w1}, cm	**27.0843**	**25.3563**	**24.1124**	**23.1429**	**22.1735**	19.0630	16.6139	14.5513	13.2630	12.0613	11.0639	10.2243	8.8842	7.8883	7.0577
$\varepsilon_s = \varepsilon_{cd1}(d - x_{w1})/x_{w1}$, ‰	**0.82146**	**1.08583**	**1.32356**	**1.54321**	**1.79922**	2.50364	3.28725	4.18654	4.95908	5.84100	6.75142	6.88168	9.62547	11.6022	13.7429
$z_c = z_{cd1} = z_{sd1} =$ $= d - (1 - \omega_{md1}/\omega_{nd1}) x_{w1}$, cm	**34.616**	**35.626**	**36.319**	**36.842**	**37.350**	38.647	39.653	40.485	41.008	41.489	41.884	42.216	42.740	43.126	43.445
$M_{Ru} = F_{cd} z_s = F_{sd} z_s$, kN·m	**83.489**	**113.575**	**141.137**	**166.930**	**197.303**	206.286	211.655	216.095	218.888	221.452	223.564	225.332	228.130	230.193	231.894
Below for all types classes of concrete when $4\phi 25mm \rightarrow A_s = 19.63cm^2$ $\rho_l = 19.63 \cdot 100 / (20 \cdot 46) = 2.1337 \approx 2.134$ %; $N_{sd} = f_{sd} A_s = 363.6 \cdot 19.63 \cdot 10^{-4} = 7137.468 \cdot 10^{-4} MN = 713.747$ **kN**															
x_{w1}, cm	**29.3678**	**27.6466**	**26.3912**	**25.4035**	**24.4083**	**23.5935**	22.2231	19.4579	17.7354	16.1283	14.7946	13.6719	11.7992	10.5140	9.4375
$\varepsilon_s = \varepsilon_{cd1}(d - x_{w1})/x_{w1}$, ‰	**0.66613**	**0.88539**	**1.08338**	**1.26683**	**1.48118**	**1.68267**	1.98844	2.64237	3.20188	3.84466	4.50979	5.19377	6.61727	8.10501	9.64935
$z_c = z_{cd1} = z_{sd1} =$ $= d - (1 - \omega_{md1}/\omega_{nd1}) x_{w1}$, cm	**33.657**	**34.689**	**35.404**	**35.948**	**36.478**	**36.900**	37.509	38.625	**39.325**	39.967	40.497	40.939	41.640	42.170	42.583
$M_{Ru} = F_{cd} z_s = F_{sd} z_s$, kN·m	**88.020**	**120.579**	**150.587**	**178.790**	**212.122**	**243.767**	267.720	275.688	280.682	285.266	289.044	292.204	297.207	300.985	303.938

Table 2

Comparison of the calculations of strength of *beams* M_{Rd} by ZI method and by EN-2 method **(in case of abundant reinforcement, the figures in the table are highlighted)**

		f_{ck} (MPa)	08	12	16	20	25	30	35	40	45	50	55	60	70	80	90
Reinforcement factor for tensile zone of reinforced concrete beams	ρ_l = 0.437 %	$M^*_{Rd,ZI}$		61.547	62.458	63.103	63.688	64.163	64.500	64.767	65.000	65.176	65.348	65.478	65.695	65.869	66.007
		$M_{Rd,EN-2}$	56.106	59.819	61.675	62.786	63.678	64.270	64.694	65.008	65.256	65.454	65.578	65.673	65.826	65.929	66.002
		$M^*_{Rd,ZI}/M_{Rd,EN-2}$		1.0289	1.0127	1.0050	1.0002	0.9983	0.9970	0.9963	0.9961	0.9958	0.9965	0.9970	0.9980	0.9991	1.0001
		$M_{Rdu,ZI}$	57.153	60.285	61.915	62.919	63.739	64.294	64.695	65.029	65.232	65.431	65.590	65.722	65.932	66.090	66.214
		$M_{Rdu,ZI}/M_{Rd,EN-2}$	1.0187	1.0078	1.0039	1.0021	1.0010	1.0004	1.0000	1.0003	0.9996	0.9996	1.0002	1.0022	1.0016	1.0024	1.0032
	ρ_l = 1.067 %	$M^*_{Rd,ZI}$		**107.506**	135.728	139.578	143.069	145.901	147.916	149.505	150.897	151.927	152.974	153.748	154.377	156.082	156.907
		$M_{Rd,EN-2}$	**78.798**	**118.197**	131.039	137.680	143.001	146.535	149.070	150.963	152.445	153.623	154.337	154.926	155.819	156.426	156.872
		$M^*_{Rd,ZI}/M_{Rd,EN-2}$		**0.9095**	**1.0358**	1.0138	1.0005	0.9957	0.9923	0.9903	0.9898	0.9890	0.9912	0.9929	0.9907	0.9978	1.0002
		$M_{Rdu,ZI}$	**76.588**	**103.204**	**127.403**	138.480	143.368	146.685	149.078	151.074	152.323	153.469	154.415	155.205	156.458	157.404	158.143
		$M_{Rdu,ZI}/M_{Rd,EN-2}$	**0.9720**	**0.8732**	**0.9723**	1.0058	1.0056	1.0010	1.0000	1.0007	0.9991	0.9990	1.0005	1.0018	1.0041	1.0063	1.0081
	ρ_l = 1.596 %	$M^*_{Rd,ZI}$		**107.506**	**129.953**	**151.588**	198.21	204.544	209.052	212.604	215.714	218.017	220.356	222.091	224.990	227.302	229.147
		$M_{Rd,EN-2}$	**78.798**	**118.197**	**157.597**	186.189	198.07	205.97	211.63	215.87	219.15	221.79	223.40	224.73	226.70	228.09	229.05
		$M^*_{Rd,ZI}/M_{Rd,EN-2}$		**0.9095**	**0.8245**	**0.8150**	1.0007	0.9931	0.9878	0.9849	0.9843	0.9830	0.9864	0.9883	0.9925	0.9965	1.0004
		$M_{Rdu,ZI}$	**83.489**	**113.575**	**141.137**	**166.930**	**197.303**	206.286	211.655	216.095	218.888	221.452	223.564	225.332	228.130	230.193	231.894
		$M_{Rdu,ZI}/M_{Rd,EN-2}$	**1.0595**	**0.9609**	**0.8956**	**0.89656**	**0.9961**	1.0015	1.0001	1.0010	0.9988	0.9985	1.0007	1.0027	1.0063	1.0092	1.01242
	ρ_l = 2.134 %	$M^*_{Rd,ZI}$		**107.506**	**129.953**	**151.588**	**178.620**	**207.434**	263.074	269.424	274.985	279.101	283.284	286.386	291.570	295.703	299.001
		$M_{Rd,EN-2}$	**78.798**	**118.197**	**157.597**	**196.996**	243.423	257.556	267.691	275.257	281.145	285.856	288.746	291.102	294.635	297.097	298.846
		$M^*_{Rd,ZI}/M_{Rd,EN-2}$		**0.9095**	**0.8246**	**0.7695**	**0.7338**	**0.8054**	0.9828	0.9788	0.9781	0.9764	0.9811	0.9838	0.9896	0.9953	1.0005
		$M_{Rdu,ZI}$	**88.020**	**120.579**	**150.587**	**178.790**	212.122	243.767	267.720	275.688	280.682	285.266	289.044	292.204	297.207	300.985	303.938
		$M_{Rdu,ZI}/M_{Rd,EN-2}$	**1.1170**	**1.0202**	**0.9555**	**0.9076**	**0.8714**	**0.9465**	**1.0001**	**1.0016**	0.9984	0.9979	1.0010	1.0038	1.0087	1.0131	1.0486

The moments $M^*_{Rd,ZI}$ marked by asterisk were calculated using not the concrete f_{ck} strength partial factor $\gamma_c = 1.5$, but rather the partial factor of the beam concrete compression zone $\gamma_{Fc} = 1.95$.

$M^*_{Rd,ZI}/M_{Rd,EN-2}$ – here, for the calculation of the value of $M^*_{Rd,ZI}$, in equation (18) we assumed $\gamma_{Fc} = 1.95$.

$M_{Rdu,ZI}$ was calculated by the method proposed by the article assuming partial factor $\gamma_c = 1.5$ – reliability ~100 %.

$M_{Rd,ZI}/M_{Rd,EN-2}$ – here, for the calculation of the value of $M_{Rdu,ZI}$ we assumed $\gamma_{Fc} = 1.5$.

Conclusions

1. Description of non-linear diagrams $\sigma_c - \varepsilon_c$ for concrete as presented in regulations EN-2 with reliability of 50%, by ZI method provides a possibility to have a reliability not only of 50 %, but also of 95 % and ~100 %. The calculations made in the article confirm that the proposal is realistic.
2. When calculating the strength of reinforced concrete beams using the ZI method, there is no need to have a limit value for the thickness of the concrete layer of the compression zone, which at present is traditionally calculated either from empirical equations or theoretically very roughly.
3. Calculation of the strength of reinforced concrete beams using the ZI method presented in the article is logical and gives actual values of normal and abundantly reinforced beam stress-strain state at the crack. No empirical equations are required for these calculations. This is especially important for the calculation of abundantly reinforced beams, as their calculation that is used at present is either complex or, alternatively, the simplified calculation that is made is imprecise.
4. The data presented in Table 1 makes it possible to simplify the calculations. The data in Table 1 show that the proposals are realistic. In addition to that data in Table 2 shows that the ULS required reliability can be achieved in two ways: (1) through the use of diagram $\sigma_c - \varepsilon_c$ with reliability of 50 % and reinforced concrete compression zone concrete force factor $\gamma_{Fc} = 1.95$ or (2) through the use of a diagram $\sigma_c - \varepsilon_c$ with reliability of ~100 %; reinforced concrete compression zone *concrete (not force)* factor $\gamma_c = 1.5$ is used. The second option is in line with the limit states methods that are currently used.

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CURVILINEAR STRESS-STRAIN RELATIONSHIP FOR CONCRETE OF EN-2 REGULATION IN THE ZI METHOD AND THE CALCULATION OF BEAM STRENGTH

S u m m a r y

The article illustrates the possibilities of the practical application of the ZI method [1 and 2] when calculating the strength of reinforced concrete beams. The article presents variants of description of the EN-2 regulation curvilinear diagram for concrete $\sigma_c - \varepsilon_c$ with reliability of 50 % by the ZI method with reliability of 50 %, 95 % and ~100 %. The article demonstrates how, when calculating the strength of normally and abundantly reinforced concrete beams by the ZI method, it is possible to do without the calculation of the limit value of the thickness of the concrete layer of the beam compression zone. This is important in the case of the calculation of the strength of abundantly reinforced beams. The method for calculating the strength of abundantly reinforced beams has been improved. When calculating strength, we also obtain actual values of stress-strain parameters at the crack. The tables provide data supporting the proposed innovations and facilitating calculations.

Keywords: ZI method**,** reliability of curvilinear diagram**,** reinforced concrete beam strength**,** abundantly reinforced beams**,** limit value compression zone**.**

Received June 12, 2019
Accepted October 14, 2019