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Alternative method for the calculation of stress-strain state parameters in normal sections of structural members

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1. Introduction

Analysis of stress-strain state in cross-sections of reinforced concrete members is often required for the design and investigation of structures. It is performed for different loading stages of reinforced concrete members: before cracking, in cracking stage, in the service stage of reinforced concrete members with cracks, for the failure stage of the member. For the calculation of parameters of these states, simple but very conventional and inaccurate stress diagrams are used: triangular, rectangular, rectangular-curved line (the curved line diagrams are not clearly defined).

Since there is no general method of the analysis and quite distorted calculation diagrams are employed the calculation has many defects [1].

Many of the problems may be solved using more realistic nonlinear stress-strain diagrams for materials [2-4]. Nonlinear diagrams are proposed in the regulations [5, 6]. In the latter years much attention is being payed to the generation and application of nonlinear diagrams [7-16]. The existing general methods for the calculation of stress-strain state parameters, with all their great possibilities, cannot replace simpler but more convenient models for the solution of specific problems. Unfortunately, there is no such general but convenient method for direct use of nonlinear stress-strain relationships without replacing them, for instance, by broken lines comprised of segments of a straight line [10] and that would take into account the possible deviations of the strains of the materials from the hypothesis (Bernoulli's) of the plane sections [11].

This paper develops the method published in [1], created for the calculation of stress-strain state parameters in normal sections. The new method presented in this paper is more general in comparison with the aforementioned one since it gives us a possibility to analyze layered structures. The method can be applied for analysis of concrete, reinforced concrete, timber, metal and other structural members in various loading stages. At present, it is very important to have a method enabling us to use curvilinear strain-stress relationships. In the case of concrete and reinforcement, the relationships of the Eurocode [5] are adopted in the regulation [6].

2. Definitions and symbols used in the paper

Structural member (hereafter - member) is flexural, eccentrically compressed or eccentrically tensioned bar made of concrete, reinforced concrete, metal, wood or other materials;

the main material of the member is material constituting the biggest part of the member into which materials of other parts are reduced for the analysis: in case of concrete and reinforced concrete – concrete, in case of timber members – timber, etc.;

effective cross-section is the part of cross-sectional area of the member that is allowed for in the analysis and at the time considered is subject to the normal stress;

Z_c *and* Z_t are the zones of effective cross-sectional area of the member in compression and tension respectively; for the zone Z_c characteristics for materials in compression are applied, while for the zone Z_t – characteristics for these in tension;

member strengthening is strengthened parts of the members; e.g. stronger members imbedded in to concrete members or fixed to the timber member;

member weakening is ducts in the member, e.g. for the prestressed reinforcement, etc.;

cross-section layer is rectangular layer in the effective cross-section of the member, with dimensions and characteristics of materials typical to the layer;

first edge of the cross-section layer (marked by Index 1*)* is the edge with the lowest arithmetical value of acting stress*;

second edge of the cross-section layer (marked by Index 2*)* is the edge with the highest arithmetical value of acting stress*;

values of the parameters x_i, v_i, ω_i, ϖ_i of the first edge are marked by Index 1; while these of the second edge are marked by Index 2;

symbols i, j, k, n are used to mark the number of layer in the cross-section, the number of strengthening (weakening), the number of reinforcement;

symbol f is used to mark the parameter of strengthening (weakening), and symbol s is used to mark the parameter of reinforcement;

regular crack is a crack caused by the forces considered;

random crack is an irregular crack caused not by the forces considered;

$h_{cr,0}$ is random crack depth (see Fig. 1);

h_{cr} is regular crack depth;

h is depth of the member;

$0-0$ is neutral axis;

$w-w$ is axis parallel to the neutral axis; it is expedient to place it on the edge of the cross-section with the lowest arithmetical value of strain;

$a-a$ is any axis parallel to the axis $0-0$;

$u-u$ is any layers of the material of the member

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* For the zone Z_c negative values are taken

that are parallel to the axis $0-0$ and whose strain ε_ε is assumed in the equations of static equilibrium;

x_w is distance* from the neutral axis $0-0$ to the axis $w-w$;

a_a is distance from the axis $w-w$ to the axis $a-a$;

a_ε is distance from the axis $w-w$ to the layers $u-u$ of the member;

symbols of other dimensions are shown in the Fig. 1;

E_i, E_{fi}, E_{si}, E are elasticity moduli of the layers of the main material in cross-section (e.g. layers of the concrete), strengthening (weakening) of the cross-section, reinforcement, selected materials of the equivalent cross-section; it is expedient to take E equal to the value E_i of the modulus of the main material at the most important layer;

$$\alpha_{ei}=\frac{E_i}{E};\ \alpha_{efi}=\frac{E_{fi}}{E};\ \alpha_{esi}=\frac{E_{si}}{E};$$

A_{fi} is cross-sectional area of strengthening (weakening), area of weakening is negative;

A_{si} is cross-sectional area of reinforcement (the reinforcement may be prestressed);

parts of cross-section is areas of the layers of the member, strengthening (weakening) and reinforcement in the cross-section;

equivalent cross-section is cross-section with areas of parts of its effective cross-section multiplied by respective coefficients α_e.

Strain diagrams (see Fig. 1):

straight line $\varepsilon_0-0-\varepsilon_0$ is diagram of linear strains for the material of transformed section (conforming to the hypothesis Bernoulli's that the plane sections remain plane);

line $\varepsilon_c-0-\varepsilon_t$ (not necessarily a straight line) is strain diagram for the main material of the cross-section; symbols *c* and *t* stand for compression and tension respectively;

line $\varepsilon_{sc}-0-\varepsilon_{st}$ (not necessarily a straight line) is reinforcement strain diagram.

ε_i, ε_f, ε_s ε_0 are strains* of layers of the main material of the cross-section, strengthening (weakening), reinforcement, selected material of transformed section respectively;

ε_m is absolute value of the strain* corresponding to the value of the maximum stress* σ_m (e.g. concrete strength* f_{cm}); linear strain* of the concrete layer i in compression $\varepsilon_m=\varepsilon_{0mi}$;

σ_u is stress* corresponding to the value of the limiting strain* ε_u (e.g. of concrete ε_{cu1});

ε_{pi} is prestrain of reinforcement;

σ_{pi} is prestress of reinforcement;

ε_{si} is strain* of reinforcement caused by external forces;

σ_{si} is stress* of reinforcement caused by external forces;

$\varepsilon_{Si}=\varepsilon_{pi}+\varepsilon_{si}$ is total strain of reinforcement measured from the initial (zero) state of the reinforcement;

$\sigma_{Si}=\sigma_{pi}+\sigma_{si}$ is total stress of the reinforcement measured from the zero state of the reinforcement;

$\nu_{pi}=\sigma_{pi}/\overline{E}_{si}$; $\nu_{Si}=\sigma_{Si}/\overline{E}_{si}$;

$\overline{E}_i$, $\overline{E}_{fi}$, $\overline{E}_{si}$ are secant elasticity (strain) moduli:

$$\overline{E}_i=\nu_i E_i=\nu_i\alpha_{ei}E,\ \overline{E}_{fi}=\nu_{fi}E_{fi}=\nu_{fi}\alpha_{efi}E,$$

$$\overline{E}=\nu_{Si}E_{Si}=\nu_{Si}\alpha_{esi}E;$$

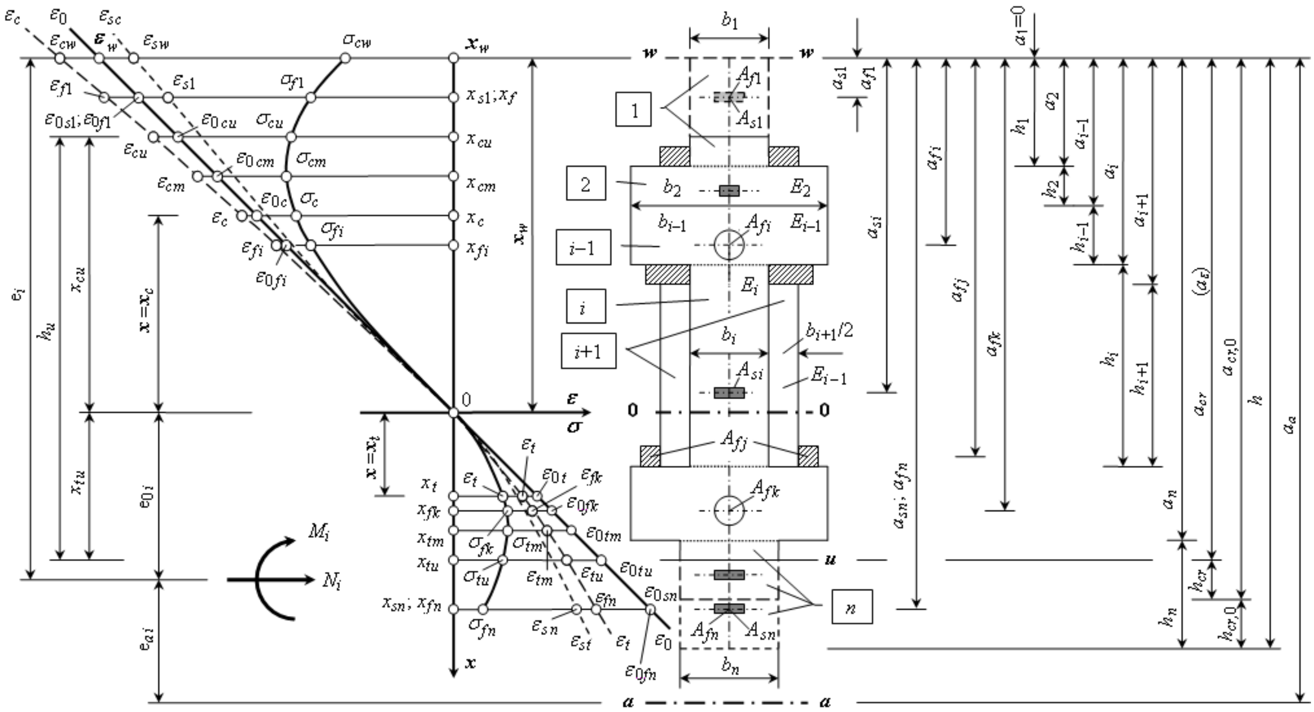

Fig. 1 Cross-section of the member and stress-strain diagrams assumed for analysis

$$\varphi = \frac{\varepsilon_w}{x_w} = \frac{\varepsilon_{0i}}{x_i} = \frac{\varepsilon_{0fi}}{x_{fi}} = \frac{\varepsilon_{0si}}{x_{si}} = \frac{\varepsilon_{0mi}}{x_{mi}} \text{ (see Fig. 1);}$$

$$k_i = \frac{\varepsilon_i}{\varepsilon_{0i}},\ k_{mi} = \frac{\varepsilon_{mi}}{\varepsilon_{0mi}},\ k_{fi} = \frac{\varepsilon_{fi}}{\varepsilon_{0fi}},\ k_{si} = \frac{\varepsilon_{si}}{\varepsilon_{0si}}.$$

3. The essence of the method

The paper deals with the case of the analysis of structural members subjected to the action of bending moments and axial forces in the symmetry plane of cross-section of the members.

Conditions of static equilibrium in respect to the axis $a-a$ of forces and bending moments, acting the effective cross-section (see Fig. 1) give (for simplicity, instead of $\sum_{i=1}^{n}$ the symbol Σ is written)

$$\Sigma N_{xi} + \Sigma N_{fi} + \Sigma N_{Si} + \Sigma N_i = 0 \quad (1)$$

$$\Sigma M_{axi} + \Sigma M_{afi} + \Sigma M_{aSi} + \Sigma N_i e_{ai} + \Sigma M_i = 0 \quad (2)$$

The following easy-to-integrate relationship $\sigma_i - \varepsilon_i$ for the layers of the main material is selected

$$\sigma_i = \overline{E}_i \varepsilon_i = E_i \varepsilon_i \nu_i = \alpha_{ei} E k_i \varepsilon_{0i} \nu_i = \varphi E k_i \alpha_{ei} \nu_i x_i \quad (3)$$

where

$$\varepsilon_i = k_i \varepsilon_{0i} = k_i \varphi \cdot x_i ;$$

$$\nu_i = 1 + c_{1i}\eta_i + c_{2i}\eta_i^2 + c_{3i}\eta_i^3 + c_{4i}\eta_i^4 + \cdots \quad (4)$$

For instance, ν_i for the concrete in compression zone can be taken from [9].

The resultant force of stresses σ_i of the cross-section layer i, which is of b_i in width and of h_i in depth (see Fig. 1), is

$$N_{xi} = \int_{\bar{x}_{i1}}^{\bar{x}_{i2}} \sigma_i b_i dx_i = \int_{\bar{x}_{i1}}^{\bar{x}_{i2}} \varphi E k_i \alpha_{ei} b_i \nu_i x_i dx_i = = \varphi E k_i \alpha_{ei} b_i \left(\omega_{i2}\bar{x}_{i2}^2 - \omega_{i1}\bar{x}_{i1}^2\right) \quad (5)$$

Here and hereafter the dash over x ($\bar{x}$) indicates that the effective part of the cross-section layer is taken, i.e. the part subjected to the action of stress σ_i; numbers $i1$ and $i2$ denote edges of the effective part of member layer i : 1 – the edge subjected mathematically to the lowest σ_i, and 2 – the edge subjected mathematically to the lighest σ_i.

Moment of force N_{xi} with respect to the neutral axis $0-0$ can be expressed by the formula

$$M_{xi} = \int_{\bar{x}_{i1}}^{\bar{x}_{i2}} \sigma_i b_i x_i dx_i = \int_{\bar{x}_{i1}}^{\bar{x}_{i2}} \varphi E k_i \alpha_{ei} b_i \nu_i x_i^2 dx_i = = \varphi E k_i \alpha_{ei} b_i \left(\varpi_{i2}\bar{x}_{i2}^3 - \varpi_{i1}\bar{x}_{i1}^3\right) \quad (6)$$

Eccentricity of force N_{xi} with respect to the neutral axis $0-0$ is as follows

$$e_{xi} = \frac{M_{xi}}{N_{xi}} = \frac{\varpi_{i2}\bar{x}_{i2}^3 - \varpi_{i1}\bar{x}_{i1}^3}{\omega_{i2}\bar{x}_{i2}^2 - \omega_{i1}\bar{x}_{i1}^2} \quad (7)$$

here

$$\omega_i = \frac{1}{2} + \frac{c_{1i}}{3}\eta_i + \frac{c_{2i}}{4}\eta_i^2 + \frac{c_{3i}}{5}\eta_i^3 + \frac{c_{4i}}{6}\eta_i^4 + \cdots \quad (8)$$

$$\varpi_i = \frac{1}{3} + \frac{c_{1i}}{4}\eta_i + \frac{c_{2i}}{5}\eta_i^2 + \frac{c_{3i}}{6}\eta_i^3 + \frac{c_{4i}}{7}\eta_i^4 + \cdots \quad (9)$$

$$\eta_i = \frac{\varepsilon_i}{\varepsilon_{mi}} = \frac{k_i \varepsilon_{0i}}{k_{mi}\varepsilon_{0mi}} = k_{ki}\frac{\bar{x}_i}{x_{mi}} \cong \frac{\bar{x}_i}{x_{mi}} \quad (10)$$

Values of coefficients c_i for the concrete in compression zone Z_c may be taken either from [9, 15, 16,]

$$c_{1i} = \left\{2(\eta_{ri}+1)(1-\eta_{ri})^3 + (\eta_{ri}-1)\alpha_{eri} + \left[(5\eta_{ri}-3)\alpha_{vi} + \eta_{ri}^3(3\eta_{ri}-5)\right]\nu_{mi}\right\} \Big/ \left\{\eta_{ri}(\eta_{ri}-1)^3\right\} \quad (11)$$

$$c_{2i} = \left\{(\eta_{ri}^2+4\eta_{ri}+1)(\eta_{ri}-1)^3 + (1+\eta_{ri}-2\eta_{ri}^2)\alpha_{eri} + 2\left[(1+\eta_{ri}-5\eta_{ri}^2)\alpha_{vi} + \eta_{ri}^3(5-\eta_{ri}-\eta_{ri}^2)\right]\nu_{mi}\right\} \Big/ \left\{\eta_{ri}^2(\eta_{ri}-1)^3\right\} \quad (12)$$

$$c_{3i} = \left\{2(\eta_{ri}+1)(1-\eta_{ri})^3 + (\eta_{ri}^2+\eta_{ri}-2)\alpha_{eri} + \left[(5\eta_{ri}^2+5\eta_{ri}-4)\alpha_{vi} + \eta_{ri}^2(4\eta_{ri}^2-5\eta_{ri}-5)\right]\nu_{mi}\right\} \Big/ \left\{\eta_{ri}^2(\eta_{ri}-1)^3\right\} \quad (13)$$

$$c_{4i} = \left\{(\eta_{ri}-1)^3 + (1-\eta_{ri})\alpha_{eri} + 2\left[(1-2\eta_{ri})\alpha_{vi} + \eta_{ri}^2(2-\eta_{ri})\right]\nu_{mi}\right\} \Big/ \left\{\eta_{ri}^2(\eta_{ri}-1)^3\right\} \quad (14)$$

Eqs. (11-14) are described in papers [9, 15, 16].

Symbols in Eqs. (11-14) (Fig. 2):
$E_{ci} = tan\beta$; $E_{ri} = tan(-\gamma)$; $\alpha_{eri} = E_{ri} / E_{ci}$; $\sigma_{cei} = E_{ci}\varepsilon_{ci}$; $\sigma_{ce1i} = E_{ci}\varepsilon_{c1i}$; $\sigma_{c1i} = f_{cmi}$; $\nu_{mi} = \nu_{c1i} = \sigma_{c1i} / \sigma_{ce1i}$; $\nu_{ri} = \sigma_{cri} / \sigma_{crei}$; $\alpha_{vi} = \nu_{ri} / \nu_{c1i} = \beta_{ri} / \eta_{ri}$; $\beta_{ri} = \sigma_{cri} / \sigma_{c1i}$; $\eta_{ri} = \varepsilon_{cri} / \varepsilon_{c1i}$.

It is assumed that within the limits of layer i $k_i = \frac{\varepsilon_i}{\varepsilon_{0i}} = \frac{\varepsilon_{i1}+\varepsilon_{i2}}{\varepsilon_{0i1}+\varepsilon_{0i2}} = const$ and $k_{ki} = \frac{k_i}{k_{mi}} \approx 1$.

For the analysis when $\eta_i = \eta_{i1}$, $\omega_i = \omega_{i1}$ and $\varpi_i = \varpi_{i1}$, it is assumed that $\bar{x}_i = \bar{x}_{i1}$; for the analysis when

$\eta_i = \eta_{i2}$, $\omega_i = \omega_{i2}$ and $\varpi_i = \varpi_{i2}$, it is assumed that $\bar{x}_i = \bar{x}_{i2}$. $x_{i1} = a_i + x_w$, $\bar{x}_{i1} = x_{i1}$, but $x_{cu} \le \bar{x}_{i1} \le x_{tu}$. $d_i = a_i + h_i$ (see Fig. 1); $x_{i2} = d_i + x_w$, $\bar{x}_{i2} = x_{i2}$, but $x_{cu} \le \bar{x}_{i2} \le x_{tu}$. $x_{cu} = \frac{\varepsilon_{0cu}}{\varphi}$; $x_{tu} = \frac{\varepsilon_{0tu}}{\varphi}$, but $x_{tu} \le a_{cr} - x_w$, where

$a_{cr} = a_{cr,0} - h_{cr} = h - h_{cr,0} - h_{cr}$;
$a_{min} = x_{cu} - x_w$, $a_{max} = x_{tu} - x_w$;
$a_{iu} = a_i$, but $a_{min} \le a_{iu} \le a_{max}$;
$d_{iu} = d_i$, but $a_{min} \le d_{iu} \le a_{max}$;
$\bar{x}_{i1} = a_{iu} + x_w$. $\bar{x}_{i2} = d_{iu} + x_w$.

When values of $\bar{x}_{i1}$ and $\bar{x}_{i2}$ are put in the Eqs. (5) and (6), Eqs. (15) and (16) are obtained

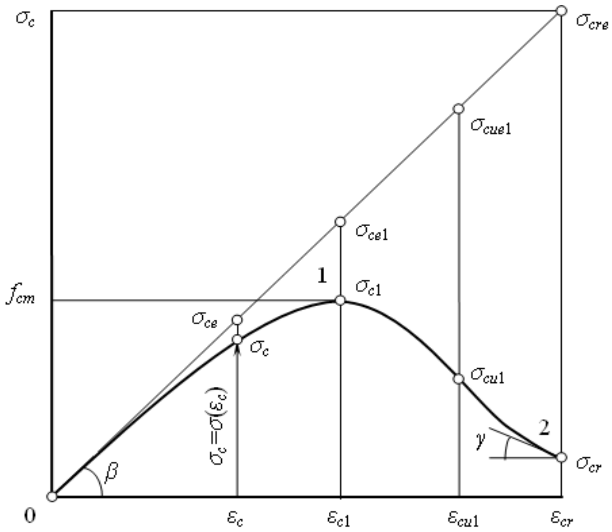

Fig. 2 Stress-strain relationship for concrete (Eq. (3))

$$N_{xi} = \varphi E k_i \alpha_{ei} b_i \left(\omega_{i2}\bar{x}_{i2}^2 - \omega_{i1}\bar{x}_{i1}^2\right) = \varphi E k_i \alpha_{ei} b_i \left[\left(\omega_{i2} - \omega_{i1}\right)x_w^2 + 2\left(\omega_{i2} d_{iu} - \omega_{i1} a_{iu}\right)x_w + \left(\omega_{i2} d_{iu}^2 - \omega_{i1} a_{iu}^2\right)\right] \quad (15)$$

$$M_{axi} = N_{xi} e_{axi} = \varphi E k_i \alpha_{ei} b_i \left(\omega_{i2}\bar{x}_{i2}^2 - \omega_{i1}\bar{x}_{i1}^2\right)\left(a_a + x_w - e_{xi}\right) = \varphi E k_i \alpha_{ei} b_i \left(\omega_{i2}\bar{x}_{i2}^2 - \omega_{i1}\bar{x}_{i1}^2\right)\left(a_a + x_w - \frac{\varpi_{i2}\bar{x}_{i2}^3 - \varpi_{i1}\bar{x}_{i1}^3}{\omega_{i2}\bar{x}_{i2}^2 - \omega_{i1}\bar{x}_{i1}^2}\right) =$$
$$= \varphi E k_i \alpha_{ei} b_i \begin{Bmatrix} \left[\left(\omega_{i2} - \omega_{i1}\right) - \left(\varpi_{i2} - \varpi_{i1}\right)\right]x_w^3 + \left[a_a\left(\omega_{i2} - \omega_{i1}\right) + 2\left(\omega_{i2} d_{iu} - \omega_{i1} a_{iu}\right) - 3\left(\varpi_{i2} d_{iu} - \varpi_{i1} a_{iu}\right)\right]x_w^2 + \\ + \left[2a_a\left(\omega_{i2} d_{iu} - \omega_{i1} a_{iu}\right) + \left(\omega_{i2} d_{iu}^2 - \omega_{i1} a_{iu}^2\right) - 3\left(\varpi_{i2} d_{iu}^2 - \varpi_{i1} a_{iu}^2\right)\right]x_w + \left[a_a\left(\omega_{i2} d_{iu}^2 - \omega_{i1} a_{iu}^2\right) - \left(\varpi_{i2} d_{iu}^3 - \varpi_{i1} a_{iu}^3\right)\right] \end{Bmatrix} \quad (16)$$

Normal force acting the area A_{fi}

$$\begin{aligned} N_{fi} &= \sigma_{fi} A_{fi} = \bar{E}_{fi}\varepsilon_{fi} A_{fi} = \nu_{fi} E_{fi} \varepsilon_{fi} A_{fi} = \\ &= \nu_{fi}\alpha_{efi} E k_{fi} \varepsilon_{0fi} A_{fi} = \varphi E k_{fi}\alpha_{efi} A_{fi} \nu_{fi} \bar{x}_{fi} = \\ &= \varphi E k_{fi}\alpha_{efi} A_{fi}\nu_{fi}\left(a_{fiu} + x_w\right) = \\ &= \varphi E k_{fi}\alpha_{efi} A_{fi}\nu_{fi} a_{fiu} + \varphi E k_{fi}\alpha_{efi} A_{fi}\nu_{fi} x_w \end{aligned} \quad (17)$$

Moment of force N_{fi} with respect to the neutral axis

$$\begin{aligned} M_{fi} &= N_{fi}\bar{x}_{fi} = \varphi E k_{fi}\alpha_{efi} A_{fi}\nu_{fi}\bar{x}_{fi}^2 = \\ &= \varphi E k_{fi}\alpha_{efi} A_{fi}\nu_{fi}\left(a_{fiu} + x_w\right)^2 \end{aligned} \quad (18)$$

When $x_{cu} \le x_{fi} \le x_{tu}$ or $a_{min} \le a_{fi} \le a_{max}$, then $\bar{x}_{fi} = x_{fi}$ and $a_{fiu} = a_{fi}$;

when $x_{cu} > x_{fi}$ or $x_{fi} > x_{tu}$ or $a_{min} > a_{fi}$ or $a_{fi} > a_{max}$, then $A_{fi} = N_{fi} = M_{fi} = 0$.

By analogy one can determine normal force N_{Si} acting in the area A_{si}. Strains of reinforcement here are always measured from the value of 0, i.e. from the beginning of loading, thus:

$$\varepsilon_{Si} = \varepsilon_{pi} + \varepsilon_{si} = \varepsilon_{pi} + k_{si}\varepsilon_{0si} = \varepsilon_{pi} + k_{si}\varphi\bar{x}_{si} \quad (19)$$

$$\begin{aligned} \sigma_{Si} &= \bar{E}_{si}\varepsilon_{Si} = \nu_{Si} E_{si}\varepsilon_{Si} = \alpha_{esi} E \nu_{Si}\varepsilon_{Si} = \\ &= \alpha_{esi} E \nu_{Si}\left(\varepsilon_{pi} + k_{si}\varphi\bar{x}_{si}\right) = \\ &= \alpha_{esi} E \nu_{pi}\varepsilon_{pi}\frac{\nu_{Si}}{\nu_{pi}} + \varphi k_{si}\alpha_{esi} E \nu_{Si}\bar{x}_{si} \end{aligned} \quad (20)$$

Prestressing force

$$\begin{aligned} P_i &= \sigma_{pi} A_{si} = \bar{E}_{si} A_{si}\varepsilon_{pi} = \nu_{pi} E_{si} A_{si}\varepsilon_{pi} = \\ &= \alpha_{esi} E A_{si}\nu_{pi}\varepsilon_{pi} = \alpha_{esi} E A_{si}\nu_{Si}\varepsilon_{pi}\frac{\nu_{pi}}{\nu_{Si}} \end{aligned} \quad (21)$$

$$\alpha_{esi} E A_{si}\nu_{Si}\varepsilon_{pi} = P_i\frac{\nu_{Si}}{\nu_{pi}}$$

$$\begin{aligned} \varepsilon_w &= \frac{\varepsilon_{0s}}{x_s} x_w = \frac{\varepsilon_{0s}}{a_s + x_w} x_w = \frac{\varepsilon_s}{k_s\left(a_s + x_w\right)} x_w = \\ &= \frac{\varepsilon_{su}/k_s}{a_s + x_w} x_w = \frac{\varepsilon_\varepsilon/k_\varepsilon}{a_\varepsilon + x_w} x_w = \frac{\varepsilon_\varepsilon x_w/k_\varepsilon}{a_\varepsilon + x_w} \end{aligned} \quad (22)$$

Total force acting in the tension reinforcement is determined by (23)

$$N_{Si} = \sigma_{Si} A_{si} = \bar{E}_{si}\varepsilon_{Si} A_{si} = \alpha_{esi} E A_{si}\nu_{Si}\left(\varepsilon_{pi} + \varepsilon_{si}\right) = P_i\frac{\nu_{Si}}{\nu_{pi}} + \varphi E k_{si}\alpha_{esi} A_{si}\nu_{Si}\bar{x}_{si} = \varphi E\left(\frac{\nu_{Si}}{\nu_{pi}}\frac{P_i}{\varphi E} + k_{si}\alpha_{esi} A_{si}\nu_{Si}\bar{x}_{si}\right) =$$

$$= \varphi E\left[\frac{\nu_{Si}}{\nu_{pi}}\frac{P_i}{E}\frac{x_w}{\varepsilon_w} + k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_{siu} + x_w\right)\right] = \varphi E\left(\frac{\nu_{Si}}{\nu_{pi}}\frac{P_i}{E}\frac{1}{\frac{\varepsilon_\varepsilon x_w / k_\varepsilon}{a_\varepsilon + x_w}}x_w + k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu} + k_{si}\alpha_{esi}A_{si}\nu_{Si}x_w\right) =$$

$$= \varphi E\left[\left(k_{si}\alpha_{esi}A_{si}\nu_{Si} + \frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon / k_\varepsilon}\right)x_w + k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu} + \frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon\right] \tag{23}$$

Moment of the force N_{Si} around the axis $a - a$

$$M_{aSi} = N_{Si}\left(a_a - a_{siu}\right) = \varphi E\left[\left(k_{si}\alpha_{esi}A_{si}\nu_{Si} + \frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon / k_\varepsilon}\right)x_w + k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu} + \frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon\right]\left(a_a - a_{siu}\right) =$$

$$= \varphi E\left(k_{si}\alpha_{esi}A_{si}\nu_{Si}x_w + \frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon / k_\varepsilon}x_w + k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu} + \frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon\right)\left(a_a - a_{siu}\right) =$$

$$= \varphi E\left[k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_{siu} + x_w\right) + \frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_\varepsilon + x_w\right)\right]\left(a_a - a_{siu}\right) \tag{24}$$

When $x_{cu} \le x_{si}$, or $a_{min} \le a_{si}$, then $\bar{x}_{si} = x_{si}$ and $a_{siu} = a_{si}$; $x_{si} = a_{siu} + x_w$; when $x_{cu} > x_{si}$ or $a_{min} > a_{si}$, then $A_{si} = N_{Si} = M_{aSi} = 0$.

$$N_i = \varphi E\frac{N_i}{\varphi E} = \varphi E\frac{N_i}{E\varepsilon_w}x_w = \varphi E\frac{N_i}{E\frac{\varepsilon_\varepsilon x_w / k_\varepsilon}{a_\varepsilon + x_w}}x_w = \varphi E\frac{N_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_\varepsilon + x_w\right) = \varphi E\left(\frac{N_i}{E\varepsilon_\varepsilon / k_\varepsilon}x_w + \frac{N_i}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon\right) \tag{25}$$

$$M_i = \varphi E\frac{M_i}{\varphi E} = \varphi E\frac{M_i}{E\varepsilon_w}x_w = \varphi E\frac{M_i}{E\frac{\varepsilon_\varepsilon x_w / k_\varepsilon}{a_\varepsilon + x_w}}x_w = \varphi E\frac{M_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_\varepsilon + x_w\right) = \varphi E\left(\frac{M_i}{E\varepsilon_\varepsilon / k_\varepsilon}x_w + \frac{M_i}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon\right) \tag{26}$$

$$\Sigma M_i + \Sigma N_i e_{ai} = \Sigma M_i + \Sigma N_i\left(a_a - e_i\right) = \Sigma\varphi E\frac{M_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_\varepsilon + x_w\right) + \Sigma\varphi E\frac{N_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_\varepsilon + x_w\right)\left(a_a - e_i\right) =$$

$$= \varphi E\left[\frac{\Sigma M_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_\varepsilon + x_w\right) + \frac{\Sigma N_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_a - e_i\right)\left(a_\varepsilon + x_w\right)\right] = \varphi E\frac{\Sigma M_i + \Sigma N_i\left(a_a - e_i\right)}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_\varepsilon + x_w\right) =$$

$$= \varphi E\left[\frac{\Sigma M_i}{E\varepsilon_\varepsilon / k_\varepsilon}x_w + \frac{\Sigma N_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_a - e_i\right)x_w + \frac{\Sigma M_i}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon + \frac{\Sigma N_i}{E\varepsilon_\varepsilon / k_\varepsilon}\left(a_a - e_i\right)a_\varepsilon\right] =$$

$$= \varphi E\left[\frac{\Sigma M_i + \Sigma N_i\left(a_a - e_i\right)}{E\varepsilon_\varepsilon / k_\varepsilon}x_w + \frac{\Sigma M_i + \Sigma N_i\left(a_a - e_i\right)}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon\right] \tag{27}$$

After inserting the respective values in Eq. (1) the following conditions of static equilibrium are arrived at

$$\Sigma\varphi E k_i\alpha_{ei}b_i\left(\omega_{i2}\bar{x}_{i2}^2 - \omega_{i1}\bar{x}_{i1}^2\right) + \Sigma\varphi E k_{fi}\alpha_{efi}A_{fi}\nu_{fi}\bar{x}_{fi} + \Sigma\varphi E\left(\frac{\nu_{Si}}{\nu_{pi}}\frac{P_i}{\varphi E} + k_{si}\alpha_{esi}A_{si}\nu_{Si}\bar{x}_{si}\right) + \Sigma\varphi E\frac{N_i}{\varphi E} = 0 \tag{28}$$

$$\Sigma k_i\alpha_{ei}b_i\left(\omega_{i2} - \omega_{i1}\right)x_w^2 + \left[2\Sigma k_i\alpha_{ei}b_i\left(\omega_{i2}d_{iu} - \omega_{i1}a_{iu}\right) + \Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi} + \Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si} + \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right) + \Sigma N_i}{E\varepsilon_\varepsilon / k_\varepsilon}\right]x_w +$$

$$+ \Sigma k_i\alpha_{ei}b_i\left(\omega_{i2}d_{iu}^2 - \omega_{i1}a_{iu}^2\right) + \Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu} + \Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu} + \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right) + \Sigma N_i}{E\varepsilon_\varepsilon / k_\varepsilon}a_\varepsilon = 0 \tag{29}$$

When $a_\varepsilon = 0$ and $\varepsilon_\varepsilon / k_\varepsilon = \varepsilon_w$, then the plane equation is

$$\Sigma k_i\alpha_{ei}b_i\left(\omega_{i2} - \omega_{i1}\right)x_w^2 + \left[2\Sigma k_i\alpha_{ei}b_i\left(\omega_{i2}d_{iu} - \omega_{i1}a_{iu}\right) + \Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi} + \Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si} + \frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right) + \Sigma N_i}{E\varepsilon_w}\right]x_w +$$

$$+\Sigma k_i\alpha_{ei}b_i\left(\omega_{i2}d_{iu}^2-\omega_{i1}a_{iu}^2\right)+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}=0 \qquad (30)$$

After inserting the respective values in to Eq. (2) the following conditions of static equilibrium are arrived at

$$\Sigma k_i\alpha_{ei}b_i\left[\left(\omega_{i2}\bar{x}_{i2}^2-\omega_{i1}\bar{x}_{i1}^2\right)\left(a_a+x_w\right)-\left(\varpi_{i2}\bar{x}_{i2}^3-\varpi_{i1}\bar{x}_{i1}^3\right)\right]+\Sigma\left(k_{fi}\alpha_{efi}A_{fi}\nu_{fi}x_w+k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}\right)\left(a_a-a_{fiu}\right)+$$
$$+\Sigma\left\{\left[\left(k_{si}\alpha_{esi}A_{si}\nu_{Si}+\frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon/k_\varepsilon}\right)x_w+k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}+\frac{P_i\nu_{Si}/\nu_{pi}}{E\varepsilon_\varepsilon/k_\varepsilon}a_\varepsilon\right]\left(a_a-a_{siu}\right)\right\}+$$
$$+\frac{\Sigma M_i+\Sigma N_i\left(a_a-e_i\right)}{E\varepsilon_\varepsilon/k_\varepsilon}x_w+\frac{\Sigma M_i+\Sigma N_i\left(a_a-e_i\right)}{E\varepsilon_\varepsilon/k_\varepsilon}a_\varepsilon=0 \qquad (31)$$

$$\Sigma k_i\alpha_{ei}b_i\left[\left(\omega_{i2}-\omega_{i1}\right)-\left(\varpi_{i2}-\varpi_{i1}\right)\right]x_w^3+\Sigma k_i\alpha_{ei}b_i\left[\left(\omega_{i2}-\omega_{i1}\right)a_a+2\left(\omega_{i2}d_{iu}-\omega_{i1}a_{iu}\right)-3\left(\varpi_{i2}d_{iu}-\varpi_{i1}a_{iu}\right)\right]x_w^2+$$
$$+\left\{\begin{aligned}&\Sigma k_i\alpha_{ei}b_i\left[2\left(\omega_{i2}d_{iu}-\omega_{i1}a_{iu}\right)a_a+\left(\omega_{i2}d_{iu}^2-\omega_{i1}a_{iu}^2\right)-3\left(\varpi_{i2}d_{iu}^2-\varpi_{i1}a_{iu}^2\right)\right]+\\&+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}\left(a_a-a_{fiu}\right)+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}\left(a_a-a_{siu}\right)+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)\left(a_a-a_{siu}\right)+\Sigma N_i\left(a_a-e_i\right)+\Sigma M_i}{E\varepsilon_\varepsilon/k_\varepsilon}\end{aligned}\right\}x_w+$$
$$+\Sigma k_i\alpha_{ei}b_i\left[\left(\omega_{i2}d_{iu}^2-\omega_{i1}a_{iu}^2\right)a_a-\left(\varpi_{i2}d_{iu}^3-\varpi_{i1}a_{iu}^3\right)\right]+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}\left(a_a-a_{fiu}\right)+$$
$$+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}\left(a_a-a_{siu}\right)+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)\left(a_a-a_{siu}\right)+\Sigma N_i\left(a_a-e_i\right)+\Sigma M_i}{E\varepsilon_\varepsilon/k_\varepsilon}a_\varepsilon=0 \qquad (32)$$

When $a_a=-x_w$, then moment of forces in respect to the neutral axis $0-0$

$$\Sigma k_i\alpha_{ei}b_i\left(\varpi_{i2}-\varpi_{i1}\right)x_w^3+\left[3\Sigma k_i\alpha_{ei}b_i\left(\varpi_{i2}d_{iu}-\varpi_{i1}a_{iu}\right)+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)+\Sigma N_i}{E\varepsilon_\varepsilon/k_\varepsilon}\right]x_w^2+$$
$$+\left[3\Sigma k_i\alpha_{ei}b_i\left(\varpi_{i2}d_{iu}^2-\varpi_{i1}a_{iu}^2\right)+2\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}+2\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)\left(a_{siu}+a_\varepsilon\right)+\Sigma N_i\left(e_i+a_\varepsilon\right)-\Sigma M_i}{E\varepsilon_\varepsilon/k_\varepsilon}\right]x_w+$$
$$+\Sigma k_i\alpha_{ei}b_i\left(\varpi_{i2}d_{iu}^3-\varpi_{i1}a_{iu}^3\right)+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}^2+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}^2+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)a_{siu}+\Sigma N_ie_i-\Sigma M_i}{E\varepsilon_\varepsilon/k_\varepsilon}a_\varepsilon=0 \qquad (33)$$

When $a_a=0$, then moment of forces in respect to the axis $w-w$

$$\Sigma k_i\alpha_{ei}b_i\left[\left(\varpi_{i2}-\varpi_{i1}\right)-\left(\omega_{i2}-\omega_{i1}\right)\right]x_w^3+\Sigma k_i\alpha_{ei}b_i\left[3\left(\varpi_{i2}d_{iu}-\varpi_{i1}a_{iu}\right)-2\left(\omega_{i2}d_{iu}-\omega_{i1}a_{iu}\right)\right]x_w^2+$$
$$+\left\{\Sigma k_i\alpha_{ei}b_i\left[3\left(\varpi_{i2}d_{iu}^2-\varpi_{i1}a_{iu}^2\right)-\left(\omega_{i2}d_{iu}^2-\omega_{i1}a_{iu}^2\right)\right]+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)a_{siu}+\Sigma N_ie_i-\Sigma M_i}{E\,\varepsilon_\varepsilon/k_\varepsilon}\right\}x_w+$$
$$+\Sigma k_i\alpha_{ei}b_i\left(\varpi_{i2}d_{iu}^3-\varpi_{i1}a_{iu}^3\right)+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}a_{fiu}+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}a_{siu}+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)a_{siu}+\Sigma N_ie_i-\Sigma M_i}{E\,\varepsilon_\varepsilon/k_\varepsilon}a_\varepsilon=0 \qquad (34)$$

When $a_a=0$ and $a_\varepsilon=0$ and $\varepsilon_\varepsilon/k_\varepsilon=\varepsilon_w$, then moment of forces in respect to the axis $w-w$

$$\Sigma k_i\alpha_{ei}b_i\left[\left(\varpi_{i2}-\varpi_{i1}\right)-\left(\omega_{i2}-\omega_{i1}\right)\right]x_w^3+\Sigma k_i\alpha_{ei}b_i\left[3\left(\varpi_{i2}d_{iu}-\varpi_{i1}a_{iu}\right)-2\left(\omega_{i2}d_{iu}-\omega_{i1}a_{iu}\right)\right]x_w^2+$$
$$+\left\{\Sigma k_i\alpha_{ei}b_i\left[3\left(\varpi_{i2}d_{iu}^2-\varpi_{i1}a_{iu}^2\right)-\left(\omega_{i2}d_{iu}^2-\omega_{i1}a_{iu}^2\right)\right]+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}+\frac{\Sigma\left(P_i\nu_{Si}/\nu_{pi}\right)a_{siu}+\Sigma N_ie_i-\Sigma M_i}{E\,\varepsilon_w}\right\}x_w+$$
$$+\Sigma k_i\alpha_{ei}b_i\left(\varpi_{i2}d_{iu}^3-\varpi_{i1}a_{iu}^3\right)+\Sigma k_{fi}\alpha_{efi}A_{fi}\nu_{fi}a_{fiu}^2+\Sigma k_{si}\alpha_{esi}A_{si}\nu_{Si}a_{siu}^2=0 \qquad (35)$$

For example, when in the equations of static equilibrium strains of the compressed edge are assumed, then in Eqs. (22)-(27) and (29) and in Eqs. (31)-(34) $a_\varepsilon=0$ and $\varepsilon_\varepsilon/k_\varepsilon=\varepsilon_c/k_c=\varepsilon_w$; and when the equations of static equilibrium correspond to the strains of the member reinforcement group i then in Eqs. (22)-(27) and (29) and in Eqs. (31)-(34) $a_\varepsilon=a_{si}$ and $\varepsilon_\varepsilon/k_\varepsilon=\varepsilon_{si}/k_{si}$, etc.

4. Case of singly reinforced (nonprestressed) beam of rectangular cross-section

The case when $N_i=P_i=A_{fi}=0$, $b_i=b$, $n=1$, $a_{iu}=0$, $d_{iu}=h$, $i=1$ (further on this unity is omitted) and the strains of the concrete in tension and the compression zones of the beam do not exceed the limiting values of strains, i.e. $\varepsilon_{cu} < \varepsilon_c < \varepsilon_{ctu}$ (Fig. 3) is presented

$$(k_2\omega_2 - k_1\omega_1)bx_w^2 + (2k_2\omega_2 bh + k_s\alpha_{es}A_s\nu_S)x_w + + k_2\omega_2 bh^2 + k_s\alpha_{es}A_s\nu_S a_s = 0 \quad (36)$$

when $a_a = a_s$, $a_\varepsilon = a_s$ and $\varepsilon_\varepsilon / k_\varepsilon = \varepsilon_s / k_s$, then

$$\left[k_2(\omega_2 - \varpi_2) - k_1(\omega_1 - \varpi_1)\right]x_w^3 + +\left[(k_2\omega_2 - k_1\omega_1)a_s + k_2(2\omega_2 - 3\varpi_2)h\right]x_w^2 + +\left\{k_2\left[2\omega_2 a_s + (\omega_2 - 3\varpi_2)h\right]h + \frac{M}{bE\varepsilon_s / k_s}\right\}x_w + + k_2(\omega_2 a_s - \varpi_2 h)h^2 + \frac{Ma_s}{bE\varepsilon_s / k_s} = 0 \quad (37)$$

when $a_a = a_s$, $a_\varepsilon = 0$ and $\varepsilon_\varepsilon / k_\varepsilon = \varepsilon_w$, then

$$k_2\left[(\omega_2 - \varpi_2) - k_1(\omega_1 - \varpi_1)\right]x_w^3 + +\left[(k_2\omega_2 - k_1\omega_1)a_s + k_2(2\omega_2 - 3\varpi_2)h\right]x_w^2 + +\left\{k_2\left[2\omega_2 a_s + (\omega_2 - 3\varpi_2)h\right]h + \frac{M}{bE\varepsilon_w}\right\}x_w + +k_2(\omega_2 a_s - \varpi_2 h)h^2 = 0 \quad (38)$$

When the stresses of concrete in the tension zone are ignored (Fig. 3), in Eqs. (36)-(38) $\omega_2 = \varpi_2 = 0$

$$k_1\omega_1 bx_w^2 - k_s\alpha_{es}A_s\nu_S x_w - k_s\alpha_{es}A_s\nu_S a_{su} = 0 \quad (39)$$

$$k_1(\omega_1 - \varpi_1)x_w^3 + k_1\omega_1 a_s x_w^2 - -\frac{M}{bE\varepsilon_s / k_s}x_w - \frac{Ma_s}{bE\varepsilon_s / k_s} = 0 \quad (40)$$

$$k_1(\omega_1 - \varpi_1)x_w^2 + k_1\omega_1 a_s x_w - \frac{M}{bE\varepsilon_w} = 0 \quad (41)$$

In Eqs. (37), (38), (40) and (41) the moments of forces were taken in respect to the axis $s-s$.

When the member is not reinforced, from Eq. (36) we get

$$(k_2\omega_2 - k_1\omega_1)x_w^2 + 2k_2\omega_2 hx_w + k_2\omega_2 h^2 = 0 \quad (42)$$

and from Eq. (33) we get

$$(k_2\varpi_2 - k_1\varpi_1)x_w^3 + 3k_2\varpi_2 hx_w^2 + +\left(3k_2\varpi_2 h^2 - \frac{M}{bE\varepsilon_1 / k_1}\right)x_w + k_2\varpi_2 h^3 = 0 \quad (43)$$

$$(k_2\varpi_2 - k_1\varpi_1)x_w^3 + 3k_2\varpi_2 hx_w^2 + +\left(3k_2\varpi_2 h^2 - \frac{M}{bE\varepsilon_2 / k_2}\right)x_w + +k_2\varpi_2 h^3 - \frac{Mh}{bE\varepsilon_2 / k_2} = 0 \quad (44)$$

In Eqs. (43) and (44) the moments of forces were taken in respect to the axis $0-0$.

In Eqs. (36)-(44), coefficient of the compressive zone $k_i = k_1$, and coefficient of the tensile zone $k_i = k_2$.

An example of drawing up the equations of equilibrium and their use for the calculation of the member strength when the plane sections hypothesis is not applied is presented in the work [16].

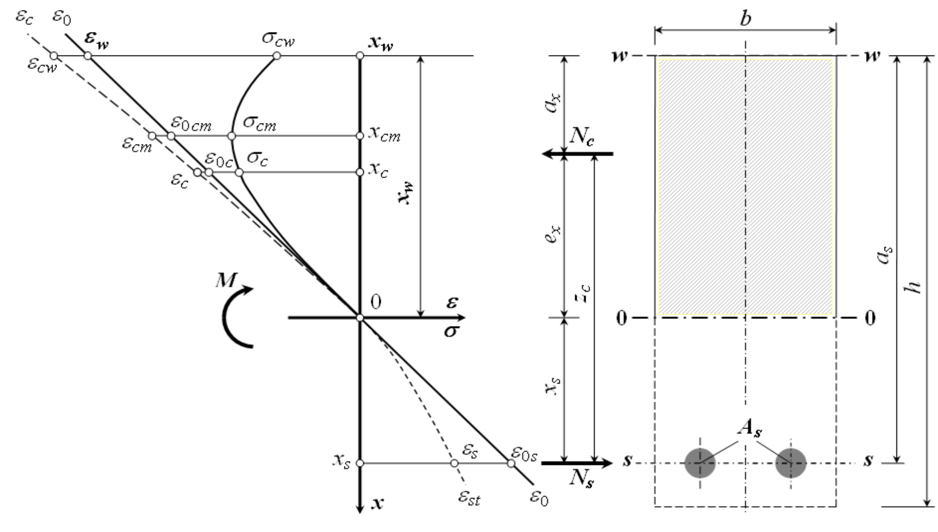

Fig. 3 Cross-section of the member and stress-strain diagrams assumed for calculation

5. Conclusions

1. At present, we still do not have a simple and practical method for calculating stress-strain state parameters at normal sections of members that would rely on nonlinear stress-strain relations of materials presented by solid curves without replacing them by broken lines comprised of segments of a straight line; and that would take into account the deviations of the strains from the ones defined by the plain sections hypothesis.

2. The paper presents a method for calculating stress-strain state parameters at normal sections when outer forces (bending moment and/or axial force) act on the plane of the cross-section symmetry axis of the member. Material strain diagrams can be linear as well. Using the proposed method and relying on the strains of concrete or reinforcement of compressed or tension zone it is possible to calculate bending moment or longitudinal force. The method is also suitable for making calculations for variously reinforced members of different materials including layered members. With the help of the presented method, any stage of loading can be analyzed.

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I. Židonis

ALTERNATYVUS METODAS STRYPINIŲ ELEMENTŲ ĮTEMPIŲ-DEFORMACIJŲ BŪVIUI NORMALINIUOSE PJŪVIUOSE APSKAIČIUOTI

R e z i u m ė

Pateikiamas gana universalus, vientisas nuoseklaus artėjimo (iteracinis) metodas konstrukcinių elementų įtempių-deformacijų būvio parametrams apskaičiuoti pagal medžiagų įtempių diagramas elementų ašiai statmenuose (normaliniuose) pjūviuose be plyšių, ties plyšiu, tarp plyšių, kai lenkimo momentas ir (arba) išilginė jėga veikia elemento skerspjūvio simetrijos ašies plokštumoje. Galima atsižvelgti į medžiagų deformacijų nukrypimą nuo plokščių pjūvių hipotezės, į skirtingas elemento sluoksnių medžiagų charakteristikas. Metodas tinka įvairiai armuotiems įvairių medžiagų, taip pat ir sluoksniuotiems, elementams skaičiuoti. Galima nagrinėti bet kurią apkrovimo stadiją nuo pradžios iki elemento suirimo, net ir elemento stiprumo mažėjimo stadiją. Gali būti imamos ir nekreivinės medžiagų įtempių diagramos.

I. Židonis

ALTERNATIVE METHOD FOR THE CALCULATION OF STRESS-STRAIN STATE PARAMETERS IN NORMAL SECTIONS OF STRUCTURAL MEMBERS

S u m m a r y

The paper presents a rather universal, integral method of sequential approximation (iterative method) for the calculation of parameters of stress-strain state in structural components at normal sections before cracking, at the crack, between the cracks when bending moment and (or) longitudinal force act on the plane of the cross-section symmetry axis of structural components. It is possible to take into account the deviation of the strain of materials from the plane sections hypothesis and different characteristics of the materials of the layers of the member. The method can be also applied for variously reinforced members made of different materials including layered members. Using this method, any stage of loading can be analyzed, even the stage of the weakening of the bearing force of the member. Employing the presented method, it is also possible to use linear stress diagrams of materials.

И. Жидонис

АЛЬТЕРНАТИВНЫЙ МЕТОД РАСЧЕТА ПАРАМЕТРОВ НАПРЯЖЕННО-ДЕФОРМИРОВАННОГО СОСТОЯНИЯ ЭЛЕМЕНТОВ КОНСТРУКЦИЙ ПО НОРМАЛЬНЫМ СЕЧЕНИЯМ

Р е з ю м е

Представляется довольно универсальный и единый (обобщенный) метод последовательных приближений (метод итераций) для вычисления параметров напряженно-деформированного состояния по нормальным сечениям элементов конструкций без трещин, по трещине, между трещинами при криволинейных эпюрах напряжений, когда внешние усилия (изгибающий момент и/или продольная сила) действуют в плоскости оси симметрии сечения элемента. Есть возможность учета отклонений деформаций материалов от гипотезы плоских сечений, учета неодинаковых характеристик материалов слоев элемента. Метод применим для расчета различно армированных элементов из различных материалов, также для слоистых элементов. Можно исследовать любую стадию нагружения от начала до разрушения элемента, даже стадию снижения несущей способности элемента. Могут приниматься и некриволинейные эпюры напряжений материалов.

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