Investigation of Mechanical State of Reinforced Beams with Cracks by the ZI Method: Algorithm and Calculation Results

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1. Introduction

During designing of reinforced concrete beams, it is important to know formulas from design codes and also to understand the physical meaning of a phenomenon. The applying the real deformation laws of materials is unusual and irrational for an engineer. In this paper proposed calculation method is a compromise between high accuracy and volume of costs.

Generally, reinforced concrete structures at different deformation stages are considered: elastic, inelastic, when cracks occurred by using a cracked structural member at the destruction moment. On each of the stages it is important to properly determine the values of mechanical state parameters. Both in valid design codes of West Europe (EN 1992, etc.) and in East Europe (Russian SNiP, Ukrainian DBN, etc.) the stress distribution in a cross section is simplified and expressed by a triangle or rectangle shape. Such approximate is acceptable for many civil engineers, but it is non-universal and inaccurate, and sometimes may be incorrect.

Nowadays the finite element method is very popular in a structural analysis. However, it has some deficiencies: an exact model may be complex enough; an approximation level of results depends on skills of a designer; each next problem needs to be re-modelled, etc. In design and expertise companies, in addition to finite element software, to verify the given results, more accurate analytical engineering methods are applied, one of which is presented by this paper, i.e. the ZI method.

Ehsani et al. [1] experimentally investigated 4 reinforced concrete beams, two of which contained 2% of steel fibres. There were load-strain graphs presented. The authors compared the given results for structural members with different mechanical properties of materials. Slaitas et al. [2] during experiments tested 6 reinforced concrete beams. The flexural strengths were calculated by 7 various methods (one of which was the ZI method). There were bearing capacity reserves and crack depths investigated. A comparison was made. Some experimental investigations of this problem were early presented by Jokūbaitis et al. [3] and others [4]. In Golyshev et al. [5, 6] and Gvozdev's [7] a review of XX century analytical methods and experimental tests in East Europe is presented.

Filatov and Suvorov [8] presented a stepper-interactive algorithm, which considers the stress state of a normal section, by using diagrams σ-ε of real materials. The damaged zone with cracks was also investigated. Modern computer algebra software was applied. The proposed decision is precise in load-history aspects, but it is complicated enough for an engineer.

Raue and Timmler presented [9] a geometrically and physically non-linear model, which is based on the analysis of the potential energy. An example of a column under compression and bending was presented, taking into account the development of cracks. Such general model is difficult for the direct usage by engineers in practice, because of integration and optimization processes.

The presented paper is an extension in the applied direction of monographs [10, 11] about the ZI method, key solutions of which are based on the classic engineering theory of the beam section stress/strain state (Fig. 1).
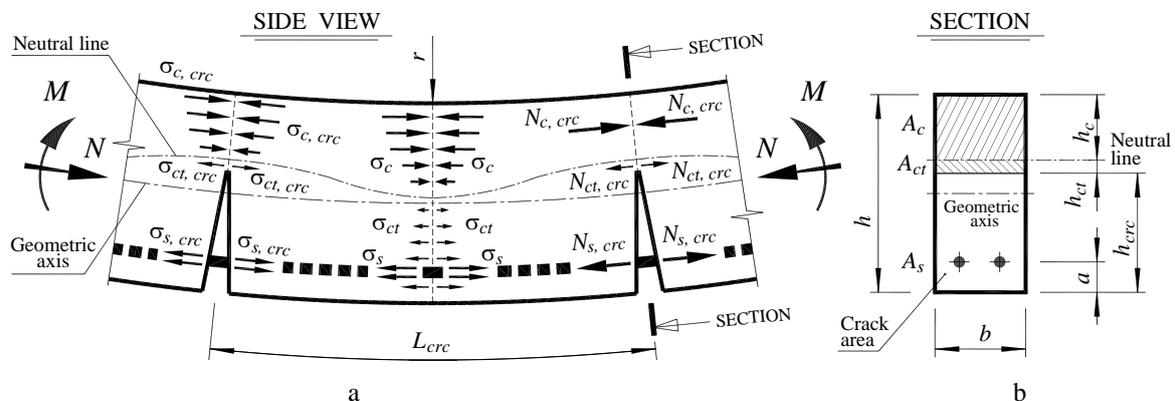
Fig. 1 A scheme of a beam with cracks under compression with bending: a) side view; b) section view

Here, cross sections are acting by a combination of an axial force and the bending moment, real σ-ε diagrams of using materials are directly applied. Such analysis is very actual one for reinforced concrete, because of the non-linear character of stress distribution and cracks.

The ZI method has many advantages: simple in

the usage, obvious for an engineer, universal, practical, continuous, using real properties of materials, etc. The results, determined by this method, were compared with experimental ones, and with calculations according the design codes EN 1992. A high compliance of the results was observed [12, 13].

On the other hand, the ZI method allows simply calculating with reliability 50, 95 or ~100% [14]. Next possibilities of the usage of the ZI method should be investigated.

The aim of the paper is to analyse the mechanical state of various sections of reinforced concrete beams with cracks. The more general aim is to explain the suitability and convenience of the ZI method, among the usage of other methods. A special attention has been paid to the description of the algorithm and to illustrate the calculation results of examples.

2. Main concept, assumptions

When a reinforced beam is loaded, the compressive stress diagram in the cross section changes the shape: at the elastic stage it is close to a triangular one; beyond the point of the strength limit it is becoming more and more like a rectangle one; before destruction the crack area decreases the compressive zone and change the stress distribution. In a general formulation the problem is non-linear. A practical engineer is interested in a simple result: the load-bearing capacity and the required reinforcement in a cross-section under a load combination of an axial force and bending moment. The question of the cracking history is irrelevant for him. Namely, such kind of the approach is considered in the design codes, in which non-linear diagrams σ-ε are presented (Fig. 2), but any exact method for applying of these general functions is not explained.
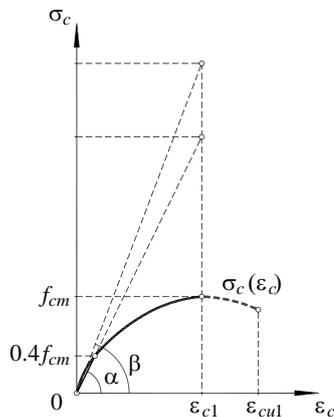
Fig. 2 The compressive concrete stress/strain relation from the design code EN 1992

Since a cross section is subjected only by normal stresses σ of an axial force N and bending moment M, the one-direction mechanical state is acted in each node of the cross section. Therefore, to use uniaxial test diagrams of materials it is very appropriate. Reinforced concrete is a non-homogeneous material, and additionally the cracks appear, so under loading a complex stress state is formed.

There are some assumptions in our investigation:
- the curve σ-ε (Fig. 2) is described by a polynomial;
- the polynomial of 3rd order is applied for concrete under compression and under tension;

- for reinforcement steel Prandtl's diagram is used;
- Bernoulli's hypothesis of plane sections is valid;
- maximal stresses of the compressive zone can be f_{cm};
- maximal strains of the compressive zone can be ε_{c1} for concrete or ε_y in the tensile zone for steel;
- Young's modulus is the same for compressive concrete and for tensile concrete [5].

As we can see (Fig. 1), when the cracks appear, two cross sections become the main ones: at the crack and in the middle between the cracks. Other neighbouring sections are in intermediate states.

Examples in the monograph [10, 11] consider two ways to calculate the cross sections in the middle between cracks of a beam:

I. Prof. A. Rozenblium's method [15], where the plane section hypothesis is applied to the whole cross section, and the strains of tensile concrete is determined by the limit strain, which saves even with increasing load. Bernoulli's hypothesis is not applied for tensile reinforcement steel.

II. The ZI author's method [10, 11], when the plane section hypothesis is applied to compressive concrete and tensile reinforcement, but it is not applied for tensile concrete. It is assumed, that for tensile concrete the limit stress (not limit strain) is used the same one, when even the load values are increasing.

In our opinion, the first method does not take into account the fact, that the tensile concrete is placed between two cracks, so the assumption of homogeneity is not valid for such case and the mechanical conditions are different (unlike the non-cracked reinforced concrete).

By solving the task by the second method, we consider, that the tensile concrete is represented by a tensile reinforcement, which ensures kinematic consistency through the whole cross section and saves the plane of the deformed section. This way is more appropriate to design code EN 1992, and it is a more logical one [10, 11].

In the beam cross section at the crack, Bernoulli's hypothesis applies to both compressive and tensile concrete together with tensile reinforcement.

In the paper, two problems are solved by the ZI method:
1. cross section at the crack (index crc is used);
2. cross section in the middle between two cracks.

In EN 1992 no recommendations are given to describe the relation σ-ε of tensile concrete.

3. Stress/strain equation in design code EN 1992

In the design code EN 1992 the stress/strain relation (Fig. 2) is presented by such function:

$$\sigma_c\left(\varepsilon_c\right) = \frac{k \cdot \eta\left(\varepsilon_c\right) - \eta^2\left(\varepsilon_c\right)}{1 + \left(k-2\right) \cdot \eta\left(\varepsilon_c\right)} \cdot f_{cm}, \quad (1)$$

when the strain influence is expressed as:

$$\eta\left(\varepsilon_c\right) = \frac{\varepsilon_c}{\varepsilon_{c1}}, \quad (2)$$

and the constant coefficient is:

$$k = \frac{1.05 \cdot E_{cm} \cdot \varepsilon_{c1}}{f_{cm}} . \tag{3}$$

Parameters f_{cm}, E_{cm}, ε_{c1} are chosen from EN 1992.

4. An alternative stress/strain equation, applying to the cross-sectional distribution of the stresses

In the ZI method, a below-presented 3rd order polynomial non-dimensional function is used as a correction factor $p < 1$ for the Young's law:

$$p(\varepsilon_c) = 1 + c_1 \cdot \eta(\varepsilon_c) + c_2 \cdot \eta^2(\varepsilon_c) . \tag{4}$$

It is applied for the curved stress/strain relation $\sigma(\varepsilon)$ during the compressive stress interval from 0 to f_{cm} (Fig. 2):

$$\sigma_c(\varepsilon_c) = E_c \cdot \varepsilon_c \cdot p(\varepsilon_c) , \tag{5}$$

where: $E_c = 1.05 \cdot E_{cm}$ is tangent modulus (angle β on Fig. 2) of elasticity of normal concrete. The coefficients c_1 and c_2 in Eq. (4) are expressed by formulas:

$$c_1 = 3 \cdot \frac{1}{k} - 2 \text{ and } c_2 = 1 - 2 \cdot \frac{1}{k} . \tag{6}$$

In the cross-sectional x-direction of the height h, the strain function $\varepsilon_c = \varepsilon_c(x)$ is variable, because it depends on co-ordinate x. Hence, the stresses depend on x also:

$$\sigma_c(x) = E_c \cdot \varepsilon_c(x) \cdot p(x) . \tag{7}$$

An internal force and bending moment are integrated through the height of the cross-sectional compressive zone (Fig. 1):

$$N_c(x) = \int_0^{x_w} \sigma_c(x) \cdot b \cdot dx , \tag{8 a}$$

$$M_c(x) = \int_0^{x_w} \sigma_c(x) \cdot b \cdot x \cdot dx . \tag{8 b}$$

Here, the co-ordinate x_w describes the top of the cross section. In fact, a distance from 0 to x_w is the height h_c of the concrete compressive zone. After mathematical operations the internal actions are found:

$$N_c(x) = \omega_{nc}(x) \cdot \varepsilon_w \cdot E_c \cdot b \cdot x_w , \tag{9 a}$$

$$M_c(x) = \omega_{mc}(x) \cdot \varepsilon_w \cdot E_c \cdot b \cdot x_w^2 . \tag{9 b}$$

Here, the compressive strain ε_w describes the maximal value at the top of the cross section. Results of integration operations are expressed by following formulas:

$$\omega_{nc}(x) = \frac{1}{2} + \frac{1}{3} c_1 \cdot \eta(x) + \frac{1}{4} c_2 \cdot \eta^2(x) , \tag{10 a}$$

$$\omega_{mc}(x) = \frac{1}{3} + \frac{1}{4} \cdot c_1 \cdot \eta(x) + \frac{1}{5} \cdot c_2 \cdot \eta^2(x) . \tag{10 b}$$

where: coefficients c_1 and c_2 have been at Eq. (6).

5. Solution algorithm

In the classic manner, this algorithm is presented by three stages (Fig. 3): input, main stage of an iterative solving, output. For realisation the computer algebra software Maple has been used.

At the prior stage, information about key parameters, assumptions and initial technical data (geometrical characteristics, forces, materials and so on) are stated. For the first step an unknown maximal strain ε_w in the cross-sectional compressive zone is preliminary set.

During the iterative stage, above-described by Eqs. (4 – 9), the integral force N_c and moment M_c are determined by using algebraic expressions Eq. (10). In the same way N_t and M_t for the tensile zone are given, actions N_s and M_s for reinforcement are solved by analytic formulas. Later, the formation of equilibrium equations and formulation of a polynomial for x_w are followed. After solving, we should choose a real value and re-calculate $\varepsilon_w(x_w)$. Next, a tolerance $\Delta\varepsilon_w$ of the strain is checked. In case of a false result, we repeat the calculation. In case of a true one, static equilibrium equations $\Sigma(N)$ and $\Sigma(M)$ are checked. The main accent of the cycle calculations is to determinate a location of the neutral line, where $\sigma = 0$.

The last stage is the presentation of results and comparison with the experimental ones.

6. Calculations examples using the ZI method

In experiments the used beams [16, 17] were selected for calculation by the ZI method. Three groups No. 7, 8 and 9 (Table 1) of reinforced concrete beams were tested to the destruction (2 beams in each group).

The elastic modulus of reinforcement was $E_s = 205$ GPa, the yield strength was $\sigma_y = 428$ MPa, for all tested reinforced concrete beams. The calculations have considered the stage of maintenance of cracked structures. During the testing, the middle part of the beam, subjected only to increasing bending moment and constant axial force was checked (Fig. 4). The destruction moment 26.0 kNm was fixed for the beams of the 7th group, 30.8 kNm – for the 8th group, 30.0 kNm – for the 9th group.

Table 1

Initial data from experiments

Group No.	b, mm	h, mm	a, mm	A_s, cm^2	f_{cm}, MPa	N_{Em}, kNm
7	112	244	37.5	3.2	25.6	+7.9
8	110	242	32.0	3.3	34.8	+0.7
9	111	239	32.0	3.3	42.0	–2.9
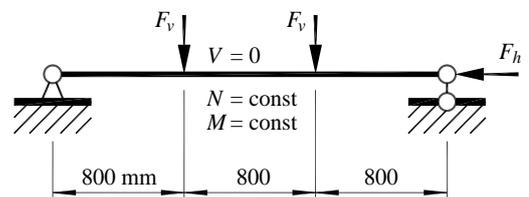
Fig. 4 The testing scheme of beams

START

Initial governing data

Main assumptions of the task

| Cross-sectional geometrical characteristics | Axial forces and bending moments in the cross section | Technical characteristics of materials |

All required other derived parameters

Set an initial value of the strain: $\varepsilon_w = \varepsilon_{w,initial}$

$\varepsilon_{w,start} = \varepsilon_w$

The integral force of the compression stresses: Eq. (9a), (10a)

The integral moment of the compression stresses: Eq. (9b), (10b)

The equilibrium equation for forces: $\sum(N) = N_c + N_t + N_s + N_{Em} = 0$

The equilibrium equation for moments: $\sum(M) = M_c + M_t + M_s + M_{Em} = 0$

$b_{x3} \cdot x_w^3 + b_{x2} \cdot x_w^2 + b_{x1} \cdot x_w + b_{x0} = 0$

$x_{w1}, \ x_{w2}, \ x_{w3}$

$x_w \rightarrow$ real

$\varepsilon_w = \varepsilon_w(x_w)$

TRUE $\Delta\varepsilon_w < 0.5\%$ **FALSE**

FALSE   Check: $\Sigma(N) = 0$ **TRUE**

FALSE   Check: $\Sigma(M) = 0$

TRUE

Results: $x_w, \varepsilon_w, \xi_w, \varepsilon_s, \sigma_s, \rho, r$, and other

Comparison with results of experiments

Data for designers

END

Fig. 3 The algorithm of the mechanical state analysis of a reinforced beam by the ZI method

In the cross section at the crack, the reinforced concrete beam becomes a steel-concrete complex structure, in which steel bars have no direct contact with the concrete and work mechanically separately (Fig. 1). Immediately after the crack formation, the redistribution of stresses occurs. Due to the cracks, the kinematic boundary conditions of the Theory of the Deformable Body are changed: the tensile concrete at the cracks becomes loose, and the tensile concrete in the middle between cracks still remains tensed together with the reinforcement steel. When the concrete cracks, the steel itself reacts in its own manner – the strains increase at the free space in the crack.

In the cross section between the cracks, the forces of the direct contact between reinforcement and concrete act. The distance between the cracks (Fig. 1) is usually expressed by 2 anchoring lengths of the reinforcement, so the strains are accumulated from crack to crack. When the cracks open, the cracking process is stabilized. The states of all other adjacent sections are intermediate ones.

To explain the results according to the calculation by the ZI method of the 7th group of the experimental beams, when the moment 14.0 kNm is acted, equilibrium of cross-sectional forces, strain and stress diagrams are presented for the cross section at the crack (Fig. 5) and for the cross section between cracks (Fig. 6).

The relation of parameters, depending of changing of the bending moment, has been investigated, starting from ~1/5 (when cracks occurred) of the limit moment M_{test} to the destruction of the beam.

In the formed Soviet Union in 1965 according to prof. A. Gvozdev [18], there is no method to link cracks in a reinforced concrete beam and tensile concrete layer with beam deflections. Such kind of problems was solved by prof. A. Rozenblium (Lithuania), and the ZI method extends this calculation.
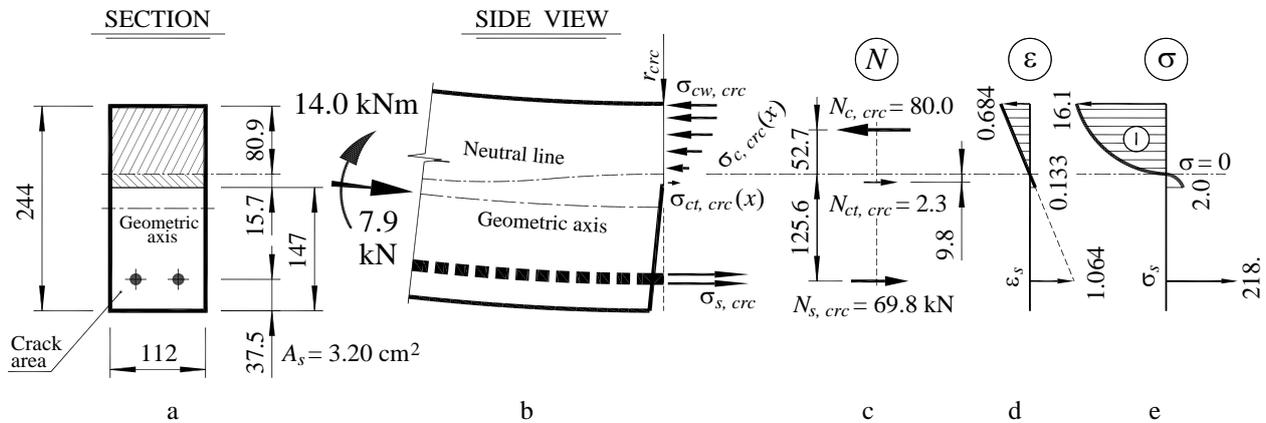
Fig. 5 The calculation scheme and results of the beam cross section at the crack: a) section view; b) side view; c) cross-sectional forces; d) diagram of strains; e) diagram of stresses
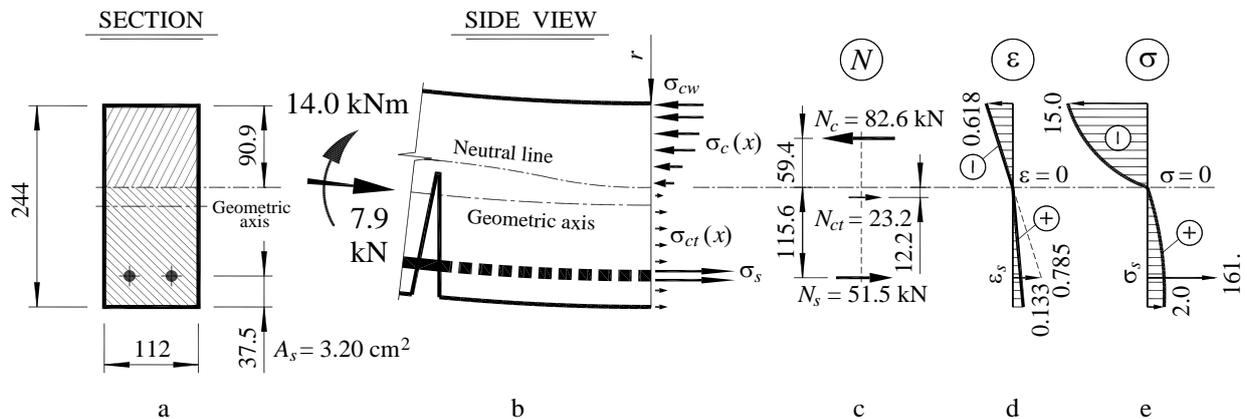
Fig. 6 The calculation scheme and results of the beam cross section in the middle between cracks: a) section view; b) side view; c) cross-sectional forces; d) diagram of strains; e) diagram of stresses

At the first, for above-described beam groups No. 7, 8 and 9 the variation of the relative cross-sectional height h_c/d of the compressive zone, the relative height h_{ct}/d of the tensile zone and the relative height h_{crc}/d of the crack deep, depending on the relative bending moment M_{Em}/M_{test}, have been considered (Fig. 7). To compare the values, the "working" cross-sectional height $d = h - a$ of the reinforced concrete beam (Fig. 1) is chosen, which is an ordinary and acceptable for a civil engineer.

On the graph (Fig. 7), the continuous lines show the results of the 7th beam group, the dashed lines are for

8th group, and the dotted ones are for 9th group. The same marking has been used for lines on Figs. 8, 9 and 12.

The graph (Fig. 7) points, that the h_c/d value is the highest 0.422–0.469 at the prime relative bending moment 0.15–0.20. As the moment increases to 0.5, the ratio h_c/d decreases to 0.348–0.392, and near the relative moment 1.0 it slowly increases again to 0.362–0.413. When the crack occurs at the moment 0.15–0.20, the tensile layer is the thickest of 0.302–0.318, and the crack is the smallest, i.e. 0.410–0.438. Near the middle of the graph, when the moment is about 0.6, the value h_{ct}/d decreases rapidly to

0.065–0.078, and the crack h_{crc}/d increases to 0.715–0.727. At the moment 1.0, the ratio h_{ct}/d is only 0.036–0.046, and the crack increases to 0.729–0.748. The curvature of all lines shows, that as the bending moment grows, the distribution of forces is non-linearly changed. The most important influence on the process has the decrease of the thickness of the tensile concrete layer.
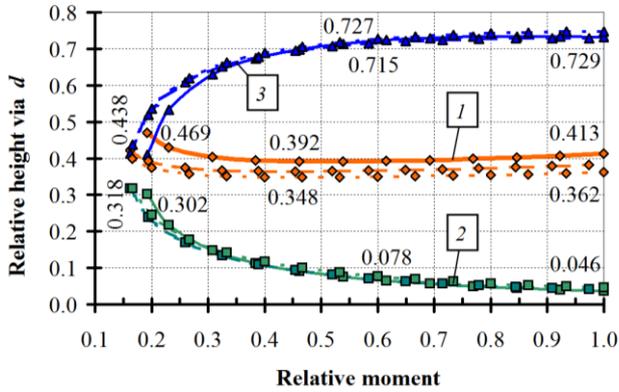
Fig. 7 The variation of relative cross-sectional height of zones, depending on the relative bending moment: *1* – compressive layer; *2* – tensile layer; *3* – crack deep

Second, the relation of relative strains $\varepsilon_c/\varepsilon_{c1}$ at the top on the cross section at the crack of beams for the 7, 8 and 9th groups, depending of the relative moment M_{Em}/M_{test}, has been investigated (Fig. 8). The value of concrete strains $\varepsilon_c/\varepsilon_{c1}$ at the relative bending moment 0.15–0.20 is 10–14% and during the loading process increases evenly with lowly curvature to 63, 71 ir 79%, depending of the strength of concrete (for stronger concrete, the value of $\varepsilon_c/\varepsilon_{c1}$ is lower). The relative strains $\varepsilon_s/\varepsilon_y$ of the reinforcement increase almost linearly from 11–16 to 103–119%. As we can see, the relative values of concrete are lower in comparison with the relative values of reinforcement, so the limit state of the reinforcement steel is reached earlier than in concrete, for such kind of beams.
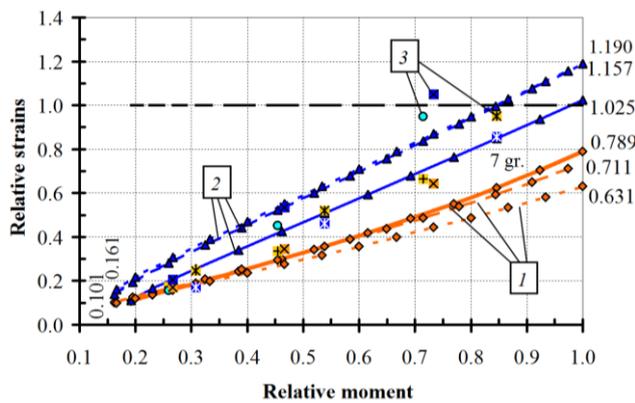
Fig. 8 The variation of relative strains, depending on the relative bending moment *1* – the top point of concrete; *2* – reinforcement; *3* – experimental data

Comparison of concrete relative strains $\varepsilon_c/\varepsilon_{c1}$ obtained from calculations with experimentally given $\varepsilon_{c,\,test}/\varepsilon_{c1}$ shows (Fig. 8), that the differences are the follow (Table 2). Comparison of the relative reinforcement strains $\varepsilon_s/\varepsilon_y$ with experimental ones $\varepsilon_{s,\,test}/\varepsilon_y$ shows the following differences (Table 3). Thus, in the cross-sectional calcula-

tion, the distribution of strains between concrete and reinforcement steel deviates by 16–25%. Probably, a cross-sectional distortion appears, because strains deviate from the plane section, although this hypothesis is considered as an underlying one in the ZI method. Of course, there may be other reasons, such as scale factor, etc.

Table 2

Differences of relative strains of concrete

Group No.	$\dfrac{M}{M_{test}}$	$\Delta\varepsilon_c,$ %	$\dfrac{M}{M_{test}}$	$\Delta\varepsilon_c,$ %	$\dfrac{M}{M_{test}}$	$\Delta\varepsilon_c,$ %
7	0.308	+30	0.538	+45	0.846	+52
8	0.260	+3	0.455	+13	0.714	+36
9	0.267	+7	0.467	+25	0.733	+45

Table 3

Differences of relative strains of reinforcement

Group No.	$\dfrac{M}{M_{test}}$	$\Delta\varepsilon_s,$ %	$\dfrac{M}{M_{test}}$	$\Delta\varepsilon_s,$ %	$\dfrac{M}{M_{test}}$	$\Delta\varepsilon_s,$ %
7	0.308	−33	0.538	−10	0.846	+1
8	0.260	−44	0.455	−13	0.714	+14
9	0.267	−33	0.467	−3	0.733	+21

Third, variation of the relative stresses σ_c/f_{cm} at the top cross-sectional layer and the relative stresses in the reinforcement steel σ_s/σ_y, depending of changing of the relative bending moment M_{Em}/M_{test} have been investigated (Fig. 9). The stress analysis shows, that the value of relative concrete stresses σ_c/f_{cm} at the relative bending moment 0.15–0.20 is 18–23% and all the loading time increases evenly to 85, 92 ir 97% with insignificant deflection, depending of the concrete strength (for stronger concrete, the value σ_c/f_{cm} is lower). The relative stresses σ_s/σ_y of the reinforcement steel increase by the almost linear dependence from 11–16 to 103–119%.
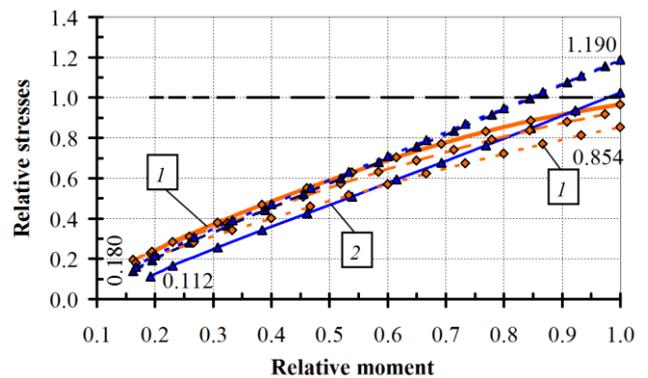
Fig. 9 The variation of relative stresses, depending on the relative bending moment: *1* – the top point of concrete; *2* – reinforcement

Similarly, like the cross section at the crack, the cross section at the middle between two cracks on each of modelled beams (Table 1) has been calculated. For example, when the beams of the 7th group have been acted by the bending moment from 5.0 to 26.0 kNm, the change of heights for compressive and tensile cross-sectional zones as well as values of strains and stresses have been analysed. Considering of the distribution (Fig. 10) of the relative stresses $\sigma(x)/f_{cm}$ trough the relative cross-sectional height x/h, we see, that as the bending moment increases,

the diagram line bends and deviates from the forward direction, i.e. non-linear properties have been more and more developed in concrete. At the same time, with the growing bending moment, we see a change of the position of the cross-sectional neutral line.
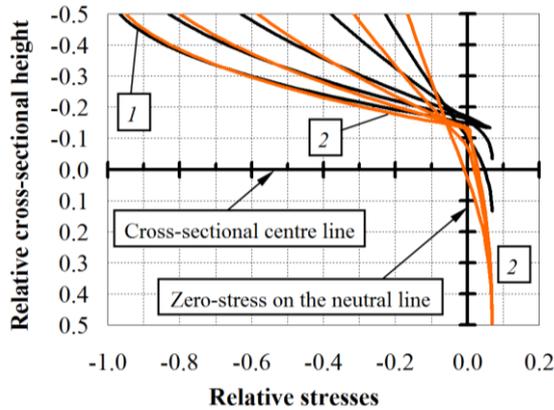
Fig. 10 The distribution of relative stresses in the direction of the cross-sectional height, when the bending moment 5.0, 8.0, 14.0, 20.0 and 26.0 kNm: *1* – black lines mark the section with cracks, *2* – gray lines denote the whole section

On the stress graph (Fig. 10), the stresses of the whole cross section "delay" in comparison with the stresses in the cracked cross section. At the same time, it can be seen, that the compressive zone of the whole cross section is always higher than the compressive zone in the cross section with the crack.

7. The plane section hypothesis in the ZI method

The deviation from the plane section of the real strains in the tensile reinforcement steel in a cross section of a bending reinforced concrete beam is expressed by using the correction factor k_s. As it was discussed early, in the cross section at the crack, the assumption of the plane section has been applied to all strains: ε_c, ε_{ct} and ε_s. In the cross section between the cracks this assumption has not been used for strains ε_{ct} of the tensile concrete. That the tensile stresses $\sigma_{ct}(x)$ are close to the limit tensile strength σ_{ctu} of the concrete, is evidenced by the appearance of new cracks. It is assumed, that strains in the tensile concrete are geometrically described by another plane section, which does not coincide with the plane section of the whole cross section.

The main role in describing of beam cross-sectional strains is played by the concrete compressive zone, because there the strains are completely corresponding to the plane section hypothesis. In the cross-sectional tensile zone, the assumption of material homogeneity is not valid near cracks. Stress and strain fields are also deflected under local laws, taking into account the adhesion contact between concrete and reinforcement steel. Therefore, in the cross section with the crack, a strain lag from the plane section is evaluated by the coefficient $k_{s,\,crc} < 1$, and in the cross section at the middle between the cracks, we increase the coefficient $k_{s,\,mid} > 1$ accordingly. From the point of view of engineering accuracy, it is sufficient to use the linear relation:

$$k_{s,crc} + k_{s,\mathrm{mid}} = 2 \; . \tag{11}$$

Experimental results show [19], that values $k_{s,\,crc} < 0.6$ and respectively $k_{s,\,mid} > 1.4$ can occur only during the slip of the reinforcement, which also can appear.

Thus, the change of the beam mechanical state parameters, depending on the coefficient k_s, has been investigated. Values of this coefficient for the cross section at the crack from 0.5 to 1.0 and for cross section at the middle between cracks from 1.0 to 1.5 have been chosen. It has been calculated for moments 8.0 kNm (continuous lines), 14.0 kNm (dashed lines) and 22.0 kNm (dotted lines). The graph (Fig. 11) shows the results of the 7th beam group.

The calculation results of the cross section at the crack (k_s from 0.5 to 1.0) show, that the relative height h_c/d of the compressive concrete layer changes from 31–32 to 39–40%, the relative height h_{ct}/d of the tensile concrete zone increases from 3–9 to 5–15%, the crack relative depth decreases from 78–83 to 63–74%. So, the height of the compressive concrete zone almost does not depend on the value of the bending moment. The height of the tensile concrete zone and the corresponding depth of the crack depend on the acting moment and therefore change.

The results of the calculation of the cross section at the middle between cracks (k_s from 1.0 to 1.5) show, that h_c/d increases from 43–50 to 49–55%, the values h_{ct}/d decreased from 68–75 to 63–70%. Both h_c/d and h_{ct}/d depends on the values of the bending moment as they change. All graph lines are closed straight lines. A separate paper is needed to describe the effect of the coefficient k_s.
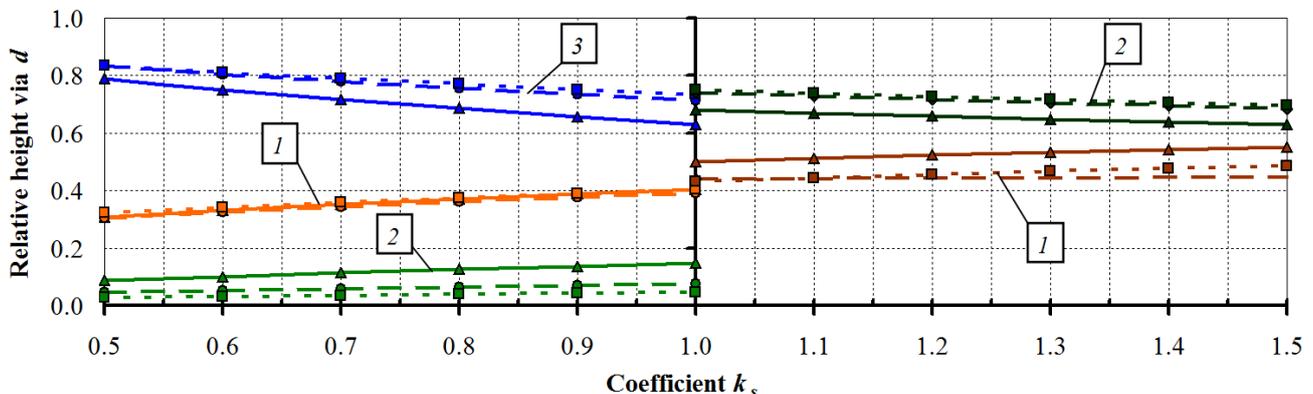
Fig. 11 The variation of the relative cross-sectional height, depending on the coefficient k_s (7th beam group): *1* – height of the compressive layer; *2* – height of the tensile layer; *3* – the crack deep

The beam cross section at the crack (Fig. 5) and the cross section at the middle between two cracks (Fig. 6) have been curved different, even in our case of pure bending without shear (Fig. 4). To determine the stiffness of a beam, the principle of the average of curvatures $(\chi + \chi_{crc})/2$ has been applied, which is sufficient from the point of view of technical accuracy. The dependence of the averaged $(\sigma_c + \sigma_{c, crc})/2f_{cm}$ relative stresses on the top of the cross section for the 7, 8 and 9th beam groups via relative curvature χ_{mid}/χ_{max} has been investigated. Here, χ_{max} has been chosen, when the maximum moment M_{test} has been.
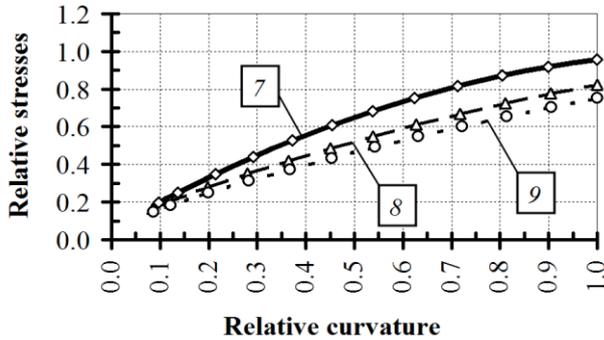
Fig. 12 The variation of relative stresses via the relative curvature (the marks show the numbers of beam groups)

The results indicate (Fig. 12), that the values of the relative stresses on the top of cross-sectional compressive concrete for all groups of beams, when the primary relative bending moment is acting, are 15–20% and locate near each other. As the moment is increasing to the maximal value, the curves are moving away from each other and at the end the values are 96, 82 and 75%, respectively. The line of the 7th beam group is more distorted than the others. It indicates, that the concrete stresses approach the strength limit f_{cm} faster, the curves are not coincidentally close to diagram σ-ε (Fig. 2).

8. Conclusions and perspectives

Based on the presented investigation of the mechanical state of the reinforced concrete beams the following conclusions have been made:

1. The ZI method can be developed to solve various problems for different cross sections. This method is a convenient practical tool for investigation of the mechanical state of beams.

2. The relations of the height h_c of the compressive zone, the height h_{ct} of the tensile zone and the depth of the opened crack h_{crc} of the cross section at the crack of three groups of reinforced concrete beams under bending and axial force have been presented.

3. The relations of the maximal compressive strains on the top of the beam cross section and strains in the reinforcement steel, depending of the bending moment, have been investigated. The calculation results have been compared with laboratory tests of such beams.

4. The relations of the maximal compressive stresses on the top of the beam cross section and stresses in the reinforcement steel, depending of the bending moment, have been described. The calculation results of the cross section at the crack (cross-sectional zones, strains, stresses, curvatures, etc.) have been compared with the results, given for the cross section between cracks.

5. The possibility to consider the problem of strain deviation from the plane section for the reinforcement steel has been illustrated by the correction factor k_s.

6. The considered ZI method is especially effective when it is necessary to exactly determine the strain state parameters as curvatures, deflections, etc., because the accuracy of this method is higher than the engineering methods currently used in design codes, mainly based on experimental tests.

7. The ZI method lets make a more accurate analysis of the mechanical state of a cross section, when diagrams of materials are non-linear. In structural designing, it is useful to use the ZI method as an alternative to the currently used method in design codes, and it would be especially convenient for an engineer to automate calculations, of course.

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INVESTIGATION OF MECHANICAL STATE OF REINFORCED BEAMS WITH CRACKS BY THE ZI METHOD: ALGORITHM AND CALCULATION RESULTS

S u m m a r y

The mechanical state of reinforcement concrete beams with cracks, which are acted by axial force and bending moment, is investigated. The considered ZI method is a compromise between high accuracy and volume of costs, when real curve diagrams are applied to describe the properties of materials and solution is rationally simplified by engineering assumptions. The paper presents the main formulas and algorithm for solving of engineering problems by using the ZI method. Three groups of beams have been calculated to represent the results. Two cross sections of cracked reinforced concrete beams have been analysed: at the crack and in the middle between cracks. Depending on increasing of the bending moment, the change of the location of the neutral line has been determined, diagrams of strains and stresses have been made. The possibility to take into account the deviations of the reinforcement strains from the plane section has been illustrated. The values of the bending curvatures of the beams have been also determined. The calculation results have been compared with the results of laboratory experiments, conclusions have been presented.