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Multilayered polymeric film vibrations under un-symmetrical loading

E. Kibirkštis*, A. Dabkevičius, K. Ragulskis***, V. Miliūnas****, V. Bivainis*****, L. Ragulskis******

**Kaunas University of Technology, Studentų 56, 51424 Kaunas, Lithuania, E-mail: edmundas.kibirkstis@ktu.lt*
***Kaunas University of Technology, Studentų 56, 51424 Kaunas, Lithuania, E-mail: arturas.dabkevicius@ktu.lt*
****Lithuanian Academy of Sciences and Kaunas University of Technology, Lithuania,*
E-mail: kazimieras3@hotmail.com, kazimieras3@yahoo.com
*****Kaunas University of Technology, Studentų 56, 51424 Kaunas, Lithuania, E-mail: valdas.miliunas@ktu.lt*
******Kaunas University of Technology, Studentų 56, 51424 Kaunas, Lithuania, E-mail: vaidas.bivainis@ktu.lt*
*******Vytautas Magnus University, K. Donelaičio g. 58, 44248 Kaunas, Lithuania, E-mail: l.ragulskis@if.vdu.lt*

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1. Introduction

Multilayered materials can be used for producing different designs of packages applicable to particle-type and fluid-type food and industrial products. Graphical images are formed on multilayered materials by using flexographic printing. The combined material that finds widest application consists from paper which is coated with films produced from synthetic materials [1].

In this paper the expression of supplementary stiffness from static tension is taken into account, which is based on the expression for plate bending presented in [2]. That expression is transformed to the nodal displacements of the layers. Thus in this paper only the basic final relationships from the previous papers, published in journal “Mechanika” are presented and further development of the model described there is performed. This paper presents the development of finite element matrices of multilayered polymeric film when supplementary stiffness from static tension is taken into account.

In flexographic printing machines, when printing is performed on tensioned materials while the tape for printing is being transported between different printing sections, the printing material may be loaded un-symmetrically [3]. This may take place because of the incorrect mounting of supports of the directing circular elements of the polymeric film tape, the wear of bearings, rolls, necks and other reasons.

In the printing device often one side of the printing material experiences higher loading than the other [4]. During the process of package production, this has a negative influence on the quality of graphical image transmission, the strength, barrier and other qualities of produced packages. Therefore, it is important to investigate vibrations occurring in the process of un-symmetric loading as well as their eigenmodes, and to predict possible graphical defects occurring in the process of printing, thus enhancing the quality of off-prints and avoiding defected products [5].

Taking international experience into account for example on the basis of references [6 - 11] it can be concluded that the applied method of digital speckle photography is simple, fast and accurate.

By investigating the vibrations of polymeric films and their eigenmodes, it is possible to determine whether the printing material which is being transported to the printing device is loaded symmetrically or un-symmetrically. This method of investigation may also be applied for the identification and diagnostics of physical and mechanical properties of paper, paperboard or polymeric films [12].

The purpose of investigations presented in this paper is the analysis of vibrations as well as the eigenmodes of films produced from polymeric materials and having a number of layers when the loading of the material is un-symmetric. The shapes of eigenmodes of the film vibrations enable us to diagnose the character of the film tension, thus enhancing the quality of the process of printing.

2. Description of the problem

With the purpose of evaluating the mechanical qualities exhibited by the polymeric material in the process

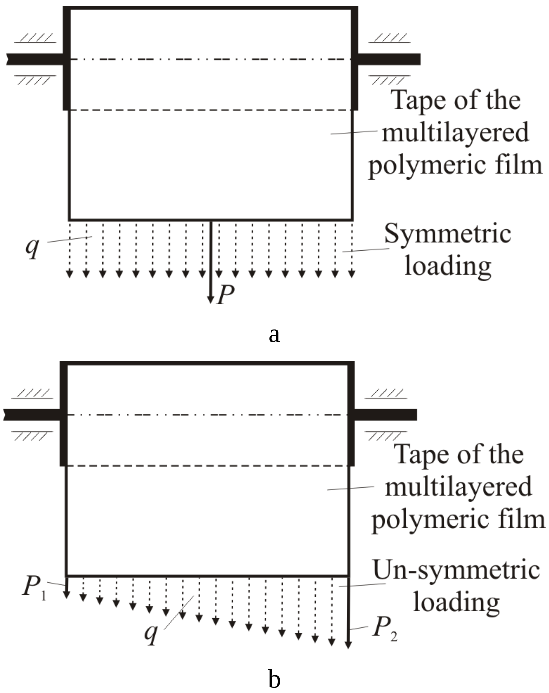

Fig. 1 Symmetric loading (a) and loading which is not symmetric (b) of the film produced from polymeric materials and having a number of layers in the experimental setup: q - distributed load, $P = P_1 = 22.5$ N and $P_2 = 27.5$ N - loading forces

of non-continuous motion in the machine used for printing, when the polymeric film experiences uneven tension in its longitudinal direction, the method of experimental investigation was designed and applied in order to perform the analysis for the problem when the polymeric film is loaded non symmetrically. In this case the distribution of loading is uneven (the direction of loading of the film produced from polymeric materials is assumed longitudinal) and is achieved by adding a higher load on one of its ends. The diagram of symmetric loading is presented in Fig. 1, a and of un-symmetric loading in Fig. 1, b.

Layers of the analyzed multilayered polymeric film are shown in Fig. 2.

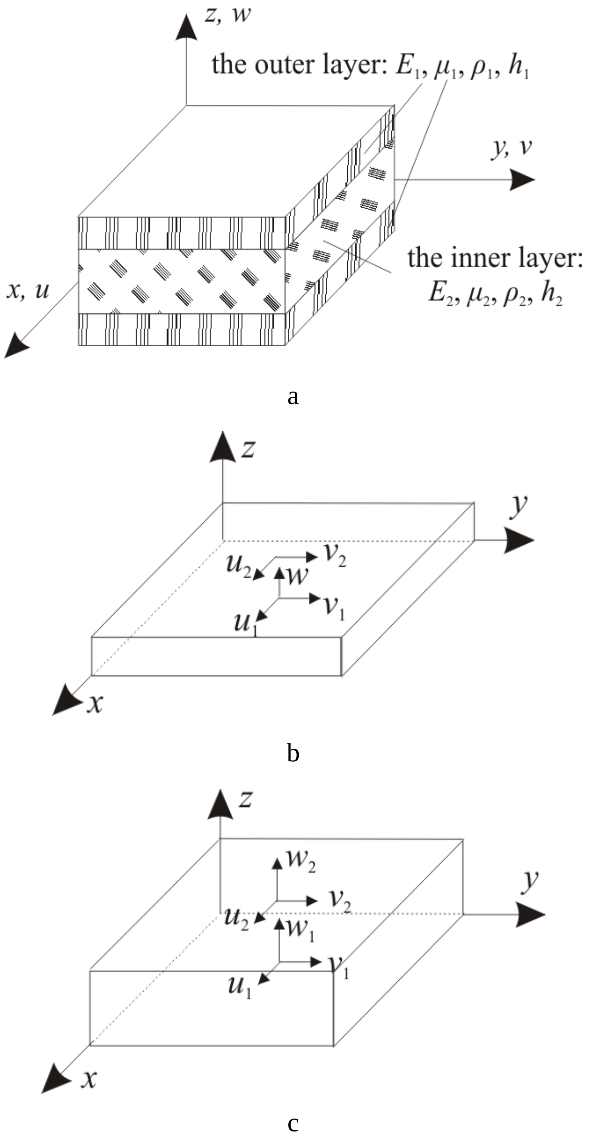

Fig. 2 Section of film produced from polymeric materials and having several layers (a), displacements of sub-elements are shown for the external layers (b) and for the internal layer (c)

The axes of coordinates (Fig. 2) are denoted as x, y, z; the corresponding displacements are denoted as u, v, w; E_1 denotes the modulus of elasticity of the outer layers, μ_1 - the Poisson's ratio of the outer layers, ρ_1 - the density of the material of the outer layers, h_1 - the thickness of the outer layers; E_2 - the modulus of elasticity of the inner layer, μ_2 - the Poisson's ratio of the inner layer, ρ_2 - the density of the material of the inner layer, h_2 - the thickness of the inner layer; subscripts on u, v, w denote the surface of the layer (1 corresponds to the lower surface and 2 corresponds to the upper surface of the layer).

Multilayered structures consist from:

1) load carrying layers of high strength and stiffness which protect from thermal, chemical and other external actions;

2) interconnecting layers which are soft and they redistribute the load between the carrying layers.

Structures with layers having various qualities ensure reliable operation of systems under unfavourable environmental conditions. By the proper choice of layers structures with high strength and stiffness and at the same time with relatively small mass are designed. For stiff layers the assumptions of plate theory are valid. For soft layers those assumptions can not be applied.

Thus the models of the layer of multilayered polymeric films can be of two basic types:

1) the layer of plate type, in which the following assumption is made: the displacements of the lower surface of the layer in the direction of the z axis coincide with the displacements of the upper surface of the layer in the direction of the z axis (Fig. 2, b);

2) the layer of elastic body type, in which the following assumption is made: the displacements of the lower surface of the layer in the direction of the z axis may not coincide with the displacements of the upper surface of the layer in the direction of the z axis (Fig. 2, c).

Thus having the sub-element of the layer of plate type and the sub-element of the layer of elastic body type the finite element for multilayered material of desired type may be obtained.

Three layered structures consisting from two load carrying layers and the soft filler between them find wide application in engineering and packaging technology. Such structures are near to optimal in achieving minimum weight while at the same time satisfying the requirements for strength and stiffness. Thus further calculations are performed for the model consisting from the external layers of plate type and an internal layer of elastic body type (Fig. 2, a).

The node of the finite element consisting from three sub-elements has 10 degrees of freedom, which include: vertical displacement of the lower layer, displacements of the lower surface of the lower layer in the directions of the axes x and y, displacements of the upper surface of the lower layer in the directions of the axes x and y, vertical displacement of the upper layer, displacements of the lower surface of the upper layer in the directions of the axes x and y, displacements of the upper surface of the upper layer in the directions of the axes x and y.

2.1. Description of the model of the outer layer of a multilayered polymeric film

The rotations about the axes of coordinates x and y are denoted as Θ_x and Θ_y. The displacements because of bending are given by $u = z\Theta_y$ and $v = -z\Theta_x$. Thus $\Theta_y = \frac{u_2 - u_1}{h_1}$, $\Theta_x = \frac{v_1 - v_2}{h_1}$.

The mass matrix

$$[M]=\int\left(\left[\hat{N}\right]^{T}\begin{bmatrix}\rho_1 h_1 & 0 & 0\\ 0 & \frac{\rho_1 h_1^3}{12} & 0\\ 0 & 0 & \frac{\rho_1 h_1^3}{12}\end{bmatrix}\left[\hat{N}\right]+[N]^{T}\rho_1 h_1[N]\right)dxdy \tag{1}$$

where the density of the material is denoted as ρ_1 and

$$\left.\begin{aligned}\left[\hat{N}\right]&=\begin{bmatrix}N_1 & 0 & 0 & 0 & 0 & \dots\\ 0 & 0 & \frac{N_1}{h_1} & 0 & -\frac{N_1}{h_1} & \dots\\ 0 & -\frac{N_1}{h_1} & 0 & \frac{N_1}{h_1} & 0 & \dots\end{bmatrix}\\ [N]&=\begin{bmatrix}0 & \frac{N_1}{2} & 0 & \frac{N_1}{2} & 0 & \dots\\ 0 & 0 & \frac{N_1}{2} & 0 & \frac{N_1}{2} & \dots\end{bmatrix}\end{aligned}\right\} \tag{2}$$

where the shape functions as usual are denoted as N_i. The stiffness matrix

$$[K]=\int\left(\left[\hat{B}\right]^{T}\frac{h_1^3}{12}[D]\left[\hat{B}\right]+\left[\tilde{B}\right]^{T}\frac{E_1 h_1}{2(1+\mu_1)1.2}\left[\tilde{B}\right]+[B]^{T}h_1[D][B]+[G]^{T}\left[M_\sigma\right][G]\right)dxdy \tag{3}$$

where the modulus of elasticity is denoted as E_1, the Poisson's ratio is denoted as μ_1 and also

$$\left.\begin{aligned}&\left[\hat{B}\right]=\begin{bmatrix}0 & -\frac{\frac{\partial N_1}{\partial x}}{h_1} & 0 & \frac{\frac{\partial N_1}{\partial x}}{h_1} & 0 & \dots\\ 0 & 0 & -\frac{\frac{\partial N_1}{\partial y}}{h_1} & 0 & \frac{\frac{\partial N_1}{\partial y}}{h_1} & \dots\\ 0 & -\frac{\frac{\partial N_1}{\partial y}}{h_1} & -\frac{\frac{\partial N_1}{\partial x}}{h_1} & \frac{\frac{\partial N_1}{\partial y}}{h_1} & \frac{\frac{\partial N_1}{\partial x}}{h_1} & \dots\end{bmatrix},\ \left[\tilde{B}\right]=\begin{bmatrix}\frac{\partial N_1}{\partial y} & 0 & -\frac{N_1}{h_1} & 0 & \frac{N_1}{h_1} & \dots\\ \frac{\partial N_1}{\partial x} & -\frac{N_1}{h_1} & 0 & \frac{N_1}{h_1} & 0 & \dots\end{bmatrix},\ \left[M_\sigma\right]=h_1\begin{bmatrix}\sigma_x & \tau_{xy}\\ \tau_{xy} & \sigma_y\end{bmatrix},\\ &[G]=\begin{bmatrix}\frac{\partial N_1}{\partial x} & 0 & 0 & 0 & 0 & \dots\\ \frac{\partial N_1}{\partial y} & 0 & 0 & 0 & 0 & \dots\end{bmatrix},\ [B]=\begin{bmatrix}0 & \frac{\frac{\partial N_1}{\partial x}}{2} & 0 & \frac{\frac{\partial N_1}{\partial x}}{2} & 0 & \dots\\ 0 & 0 & \frac{\frac{\partial N_1}{\partial y}}{2} & 0 & \frac{\frac{\partial N_1}{\partial y}}{2} & \dots\\ 0 & \frac{\frac{\partial N_1}{\partial y}}{2} & \frac{\frac{\partial N_1}{\partial x}}{2} & \frac{\frac{\partial N_1}{\partial y}}{2} & \frac{\frac{\partial N_1}{\partial x}}{2} & \dots\end{bmatrix},\ [D]=\begin{bmatrix}\frac{E_1}{1-\mu_1^2} & \frac{E_1\mu_1}{1-\mu_1^2} & 0\\ \frac{E_1\mu_1}{1-\mu_1^2} & \frac{E_1}{1-\mu_1^2} & 0\\ 0 & 0 & \frac{E_1}{2(1+\mu_1)}\end{bmatrix}\end{aligned}\right\} \tag{4}$$

where stresses σ_x, σ_y, τ_{xy} are determined from

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=[D][B]\{\delta\} \tag{5}$$

where the solution of the static problem is denoted as $\{\delta\}$.

2.2. Description of the model of the inner layer of a multilayered polymeric film

The mass matrix

$$[M]=\int\left(\begin{array}{l}[\bar{N}]^T\rho_2\frac{h_2}{6}[\bar{\bar{N}}]+[\bar{\bar{N}}]^T\rho_2\frac{h_2}{6}[\bar{N}]+\\+[\bar{N}]^T\rho_2\frac{h_2}{3}[\bar{N}]+[\bar{\bar{N}}]^T\rho_2\frac{h_2}{3}[\bar{\bar{N}}]\end{array}\right)dxdy \quad (6)$$

where the density of the material of the layer is denoted as ρ_2, the thickness of the layer is denoted as h_2 and

$$\left.\begin{array}{l}[\bar{N}]=\begin{bmatrix}N_1 & 0 & 0 & 0 & 0 & 0 & \dots\\0 & N_1 & 0 & 0 & 0 & 0 & \dots\\0 & 0 & N_1 & 0 & 0 & 0 & \dots\end{bmatrix}\\[\bar{\bar{N}}]=\begin{bmatrix}0 & 0 & 0 & N_1 & 0 & 0 & \dots\\0 & 0 & 0 & 0 & N_1 & 0 & \dots\\0 & 0 & 0 & 0 & 0 & N_1 & \dots\end{bmatrix}\end{array}\right\} \quad (7)$$

The derivatives of w with respect to x and y are expressed as

$$\begin{Bmatrix}\frac{\partial w}{\partial x}\\\frac{\partial w}{\partial y}\end{Bmatrix}=\frac{h_2-z}{h_2}[\bar{G}]\{\delta\}+\frac{z}{h_2}[\bar{\bar{G}}]\{\delta\} \quad (8)$$

where the vector of nodal displacements is denoted as $\{\delta\}$ and

$$\left.\begin{array}{l}[\bar{G}]=\begin{bmatrix}0 & 0 & \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & \dots\\0 & 0 & \frac{\partial N_1}{\partial y} & 0 & 0 & 0 & \dots\end{bmatrix}\\[\bar{\bar{G}}]=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial x} & \dots\\0 & 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & \dots\end{bmatrix}\end{array}\right\} \quad (9)$$

The stiffness matrix

$$[K]=\int\left(\begin{array}{l}[B]^T[D]\frac{1}{2}[\bar{B}]+[\bar{B}]^T[D]\frac{1}{2}[B]+[B]^T[D]\frac{1}{2}[\bar{\bar{B}}]+[\bar{\bar{B}}]^T[D]\frac{1}{2}[B]+[\bar{B}]^T[D]\frac{h_2}{6}[\bar{\bar{B}}]+\\+[\bar{\bar{B}}]^T[D]\frac{h_2}{6}[\bar{B}]+[\bar{B}]^T[D]\frac{h_2}{3}[\bar{B}]+[\bar{\bar{B}}]^T[D]\frac{h_2}{3}[\bar{\bar{B}}]+[B]^T[D]\frac{1}{h_2}[B]+\\+[\bar{G}]^T[M_\sigma]\frac{h_2}{6}[\bar{\bar{G}}]+[\bar{\bar{G}}]^T[M_\sigma]\frac{h_2}{6}[\bar{G}]+[\bar{G}]^T[M_\sigma]\frac{h_2}{3}[\bar{G}]+[\bar{\bar{G}}]^T[M_\sigma]\frac{h_2}{3}[\bar{\bar{G}}]\end{array}\right)dxdy \quad (10)$$

where

$$\left.\begin{array}{l}[B]=\begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & \dots\\0 & 0 & 0 & 0 & 0 & 0 & \dots\\0 & 0 & -N_1 & 0 & 0 & N_1 & \dots\\0 & 0 & 0 & 0 & 0 & 0 & \dots\\0 & -N_1 & 0 & 0 & N_1 & 0 & \dots\\-N_1 & 0 & 0 & N_1 & 0 & 0 & \dots\end{bmatrix},[\bar{B}]=\begin{bmatrix}\frac{\partial N_1}{\partial x} & 0 & 0 & 0 & 0 & 0 & \dots\\0 & \frac{\partial N_1}{\partial y} & 0 & 0 & 0 & 0 & \dots\\0 & 0 & 0 & 0 & 0 & 0 & \dots\\\frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & 0 & \dots\\0 & 0 & \frac{\partial N_1}{\partial y} & 0 & 0 & 0 & \dots\\0 & 0 & \frac{\partial N_1}{\partial x} & 0 & 0 & 0 & \dots\end{bmatrix},\\[\bar{\bar{B}}]=\begin{bmatrix}0 & 0 & 0 & \frac{\partial N_1}{\partial x} & 0 & 0 & \dots\\0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & 0 & \dots\\0 & 0 & 0 & 0 & 0 & 0 & \dots\\0 & 0 & 0 & \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & 0 & \dots\\0 & 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial y} & \dots\\0 & 0 & 0 & 0 & 0 & \frac{\partial N_1}{\partial x} & \dots\end{bmatrix},[D]=\begin{bmatrix}K+\frac{4}{3}G & K-\frac{2}{3}G & K-\frac{2}{3}G & 0 & 0 & 0\\K-\frac{2}{3}G & K+\frac{4}{3}G & K-\frac{2}{3}G & 0 & 0 & 0\\K-\frac{2}{3}G & K-\frac{2}{3}G & K+\frac{4}{3}G & 0 & 0 & 0\\0 & 0 & 0 & G & 0 & 0\\0 & 0 & 0 & 0 & \frac{G}{1.2} & 0\\0 & 0 & 0 & 0 & 0 & \frac{G}{1.2}\end{bmatrix},[M_\sigma]=\begin{bmatrix}\sigma_x & \tau_{xy}\\\tau_{xy} & \sigma_y\end{bmatrix}\end{array}\right\} \quad (11)$$

where the bulk modulus is denoted as $K=\frac{E_2}{3(1-2\mu_2)}$, the shear modulus is denoted as $G=\frac{E_2}{2(1+\mu_2)}$. And the stresses in the middle of the layer are determined from the static problem:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix} = [D]\left(\frac{1}{h_2}[B] + \frac{1}{2}\left[\overline{B}\right] + \frac{1}{2}\left[\overline{\overline{B}}\right] \right)\{\delta\} . \qquad (12)$$

3. Analysis of the vibrations of the multilayered polymeric film

The length and width of the polymeric film are chosen equal to 0.2 m. The following boundary conditions are assumed: all displacements are zero on the lower and upper boundaries, only on the upper boundary special conditions are assumed such as $v = 0$ on the left side and $v = 0.02$ on the right side with linear variation between them.

For the outer layer the following data are assumed: Poisson's ratio $\mu_1 = 0.3$, Young's modulus $E_1 = 0.28 \times 10^9$ Pa, density $\rho_1 = 800$ kg/m^3, thickness $h_1 = 10$ μm.

For the inner layer the following data are assumed: Poisson's ratio $\mu_2 = 0.3$, Young's modulus $E_2 = 2.8 \times 10^9$ Pa, density $\rho_2 = 800$ kg/m^3, thickness $h_2 = 100$ μm.

There are rather great uncertainties in the determination of the physical parameters of the layers of the multilayered polymeric film. They have direct influence to the obtained results and thus the correspondence of the experimentally and numerically obtained eigenmodes. The performed investigations [3-4] showed that the correspondence between the shapes of the experimental and numerical eigenmodes is satisfactory, but in order to achieve desirable accuracy between the eigenfrequencies special experiments for the determination of precise physical parameters of the layers are required.

Transverse displacement of the lower and upper planes is investigated. In Figs. 3-9 the obtained results are presented.

The numerical results presented in the figure are used in the process of interpretation of the experimentally obtained eigenmodes as described further in this paper. They enable to determine which eigenmode is obtained in the process of experimental investigations.

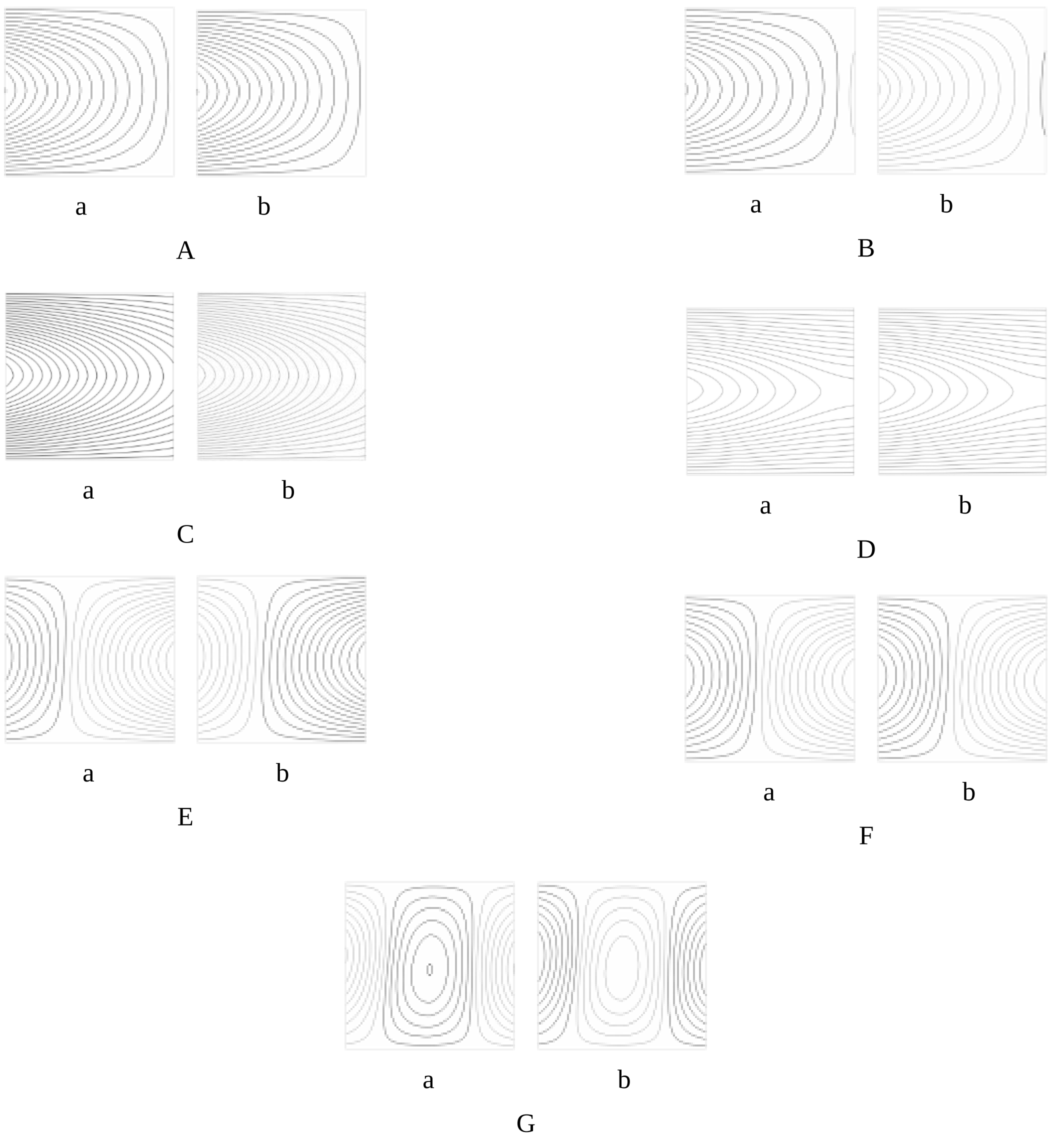

Fig. 3 Contour plots for the lower (a) and upper (b) surfaces of the (A) first, (B) second, …, (G) seventh eigenmode

4. Description of the method and experimental testing devices of multilayered materials

In order to determine the character of stress distribution in a multilayered material in the dynamic motion modes when the loading of the ` polymeric film tape is un-symmetric, a special setup for experimental investigation was developed (Figs. 4-5).

In the process of experimental tests, one side of the investigated polymeric material (Fig. 4) was fastened between the fastening devices in the electro-dynamical exciter of vibrations. At the same time, weight load *P* was un-symmetrically fastened to the other side of the polymeric film (which was also glued to the fastening element), see Figs. 1, b and 4.

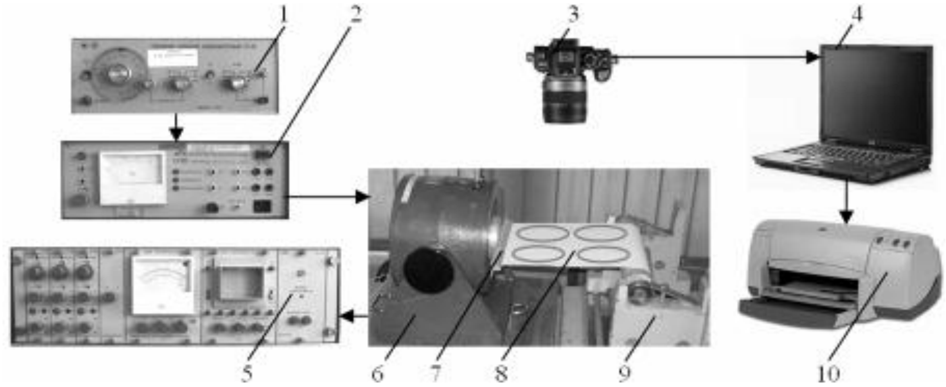

Fig. 4 General view of the setup for experimental tests: *1* – generator of signals ГЗ-111, *2* – amplifier LV-103, *3* – digital photo camera Canon Power Shot A700, *4* – carried personal computer ATOMIK Action 8050i, *5* – device for measurement of vibrations – oscilloscope VEB Robotron Messelektronik RFT00033, *6* – electro-dynamical exciter of vibrations VEB Robotron Messelektronik 11076, *7* – aluminium elbows, *8* – the investigated polymeric material, *9* – stand, *10* – printer HP DeskJet 920c

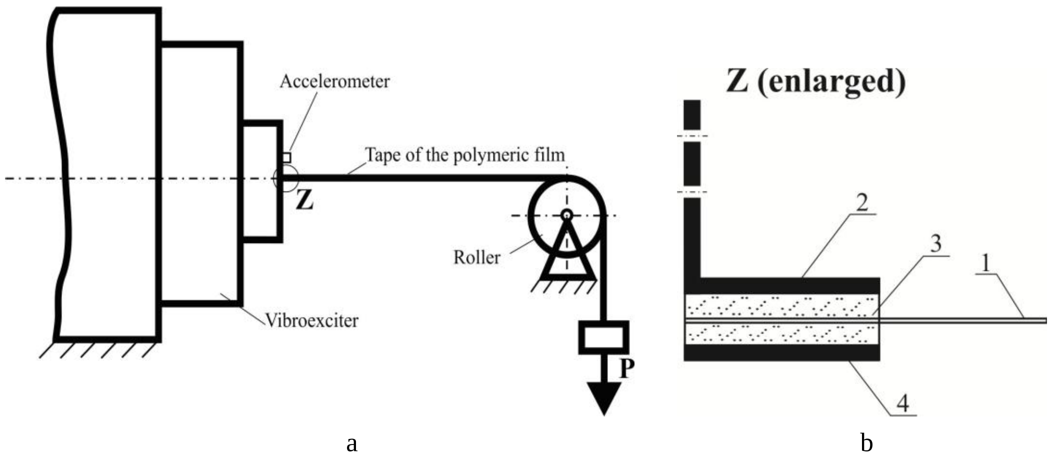

Fig. 5 Experimental setup for investigation of the film: a) general view: *P* denotes the loading force; b) detailed view of the fastening of the polymeric film: *1* – the polymeric film, *2* – aluminium angle, *3* – foamed acrylic two-sided sticky tape Tesa 52017 (thickness 0.38 mm, adhesion 25.67 kg/m), *4* – keeping strip

Experimental setup shown in Fig. 5 was designed and produced. In this experimental setup one end of the polymeric film 1 (Fig. 5, b) was fastened (glued) between the aluminum elbow 2 and the keeping strip 4. This end was fastened to the vibroexciter. To the other end of the polymeric film which was also glued to the aluminum elbow the weight (load) *P* was attached. The position of this load enables to apply symmetric as well as non-symmetric loadings. In order to achieve minimum distortion in the distribution of stresses in the film in the whole tested area of the film several methods of fastening of the polymeric film to the keeping aluminum elbows were investigated. It was determined experimentally that the smallest effect to the non-uniformity of stresses in the film has the special type of fastening when the film is glued to the aluminum angle by using a two-sided sticking tape. When fastening the polymeric film by presses, screwing by screws, gluing with glues or fastening in some other ways the undesirable non-uniformity of stresses through the whole width of the keeping elements was observed (the width is up to 200 mm). Because of this non-uniformity of stresses the results of investigations become inaccurate. Thus for this purpose foamed acrylic two-sided sticky tape Tesa 52017 (thickness 0.38 mm, adhesion 25.67 kg/m) was used.

As shown in the structural diagram of the setup for experimental investigations (Fig. 5, a) the accelerometer KD – 32 is fastened to the operating element of the vibroexciter – to the vibrating membrane. Thus the mass of the accelerometer has no effect to the loading of the film and at the same time to the characteristics of vibrations of the film.

Frequencies of the experimentally determined eigenmodes are given in Fig. 6 and Table 1. Thus in the process of experimental analysis the resonance of the accelerometer and of the investigated multilayered film could not take place.

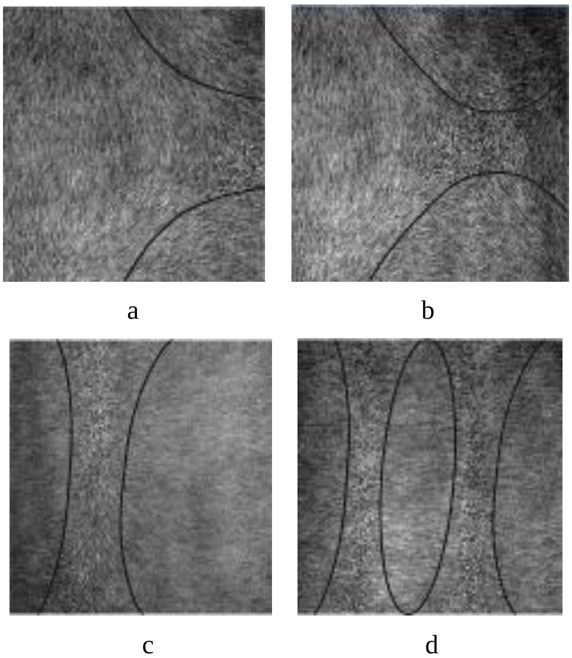

Fig. 6 The experimental mode of the upper surface of the film PET+PAP+LDPE for un-symmetric loading: (a) eigenmode 2: frequency of excitation 185 Hz, amplitude 2.2×10^{-6} m; (b) eigenmode 4: frequency of excitation 208 Hz, amplitude 1.15×10^{-6} m; (c) eigenmode 5: frequency of excitation 221 Hz, amplitude 0.95×10^{-6} m; (d) eigenmode 7: frequency of excitation 259 Hz, amplitude 0.55×10^{-6} m

Table 1

Frequency ranges of vibrations and their amplitude ranges for eigenmodes of multilayered polymeric film

Eigenmode number	Frequency range, Hz	Amplitude range, m
I	168 – 176	$1.2 – 5.9 \times 10^{-6}$
II	178 – 187	$1.2 – 5.4 \times 10^{-6}$
III	189 – 198	$0.9 – 4.8 \times 10^{-6}$
IV	200 – 210	$0.7 – 3.3 \times 10^{-6}$
V	212 – 227	$0.6 – 2.6 \times 10^{-6}$
VI	230 – 248	$0.4 – 1.5 \times 10^{-6}$
VII	250 – 272	$0.2 – 1.0 \times 10^{-6}$

The measurements were carried out at such frequencies generated by the forced vibrator, which coincided with eigenfrequencies of the investigated film. Measurements were carried out when the oscillation frequency was stabilized.

The camera Edmund Optics EO-1312C USB was used. In the experimental tests square samples of the film (0.2×0.2 m) were produced.

5. Experimental results

In Fig. 6 some of the results are indicated: in Fig. 6, a – the eigenmode 2, in Fig. 6, b – the eigenmode 4, in Fig. 6, c – the eigenmode 5 and in Fig. 6, d – the eigenmode 7 of the film PET+PAP+LDPE are shown.

There is correspondence of experimental results with the vibrations of the polymeric film investigated numerically. Figs. 6, (a-d) correspond to Fig. 3. In the numerical images of the first four eigenmodes (see Fig. 3, A-D) horizontal lines dominate, which may be compared with the images obtained in Figs. 6, a and b. Higher eigenmodes (see Fig. 3, E and F) have two vertically located lines, which may be compared with Fig. 6, c. And finally the eigenmode represented in Fig. 3, G has three vertically located lines, which may be compared with Fig. 6, d. Thus one is to compare Fig. 3, B, a with Fig. 6, a, Fig. 3, D, a with Fig. 6, b, Fig. 3, E, a with Fig. 6, c and Fig. 3, G, a with Fig. 6, d.

The performed investigations of polymeric films were in the range of frequencies and amplitudes of vibrations which are presented as the data for Fig. 6 and in the Table 1. The frequency ranges of excitation were from 168 Hz up to 272 Hz.

In the Table 1 some of the results of the experimental investigations of the film with the method of speckle photography when the loading of the film was non symmetrical are presented. Intervals of the values of frequencies and of amplitudes of longitudinal vibrations of a tape of film, in which after excitation of resonance vibrations the standing waves (eigenmodes) of transverse vibrations of the tape occur, are given in the second and third columns of the Table 1.

In our previous papers [4, 12] in which vibrations of paper are analyzed for the investigation of the resonant vibrations a special experimental setup was designed. The eigenmodes were determined by using the projection moiré techniques for the tape of paper and for the symmetric load. The several first experimental eigenmodes well correspond to the numerically obtained ones. But the higher eigenmodes do not exhibit such correspondence. Thus similar conclusions previously obtained for the paper on the basis of the current investigation can be considered to be valid for polymeric films.

6. Conclusions

Experimental and numerical investigations of vibrations of a polymeric film were performed. Mutual correspondence of the results is discussed. Non-symmetric loading takes place in a number of engineering problems. Thus the applied numerical model and the designed experimental setup provide the basis for investigation of such problems. Because the results of numerical and experimental investigations have well corresponded.

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E. Kibirkštis, A. Dabkevičius, K. Ragulskis, V. Miliūnas, V. Bivainis, L. Ragulskis

DAUGIASLUOKSNĖS POLIMERINĖS PLĖVELĖS VIRPESIAI ESANT NESIMETRINIAM APKROVIMUI

R e z i u m ė

Apkrautos daugiasluoksnės polimerinės plėvelės virpesiai tirti skaitmeniškai, priimant, kad plėvelė sudaryta iš trijų sluoksnių. Viršutinis ir apatinis sluoksniai yra standūs ir nesideformuoja skersine kryptimi, o vidinis sluoksnis gali deformuotis šia kryptimi. Analizuojamos pirmos savosios formos ir pateikti grafiniai jų tyrimo rezultatai. Apkrautos daugiasluoksnės polimerinės medžiagos PET+POP+ŽTPE (PET+POP+ŽTPE žymi trisluoksnę medžiagą, sudarytą iš PET – PoliEtileno Tereftalato, POP – POPieriaus ir ŽTPE – Žemo Tankio PoliEtileno) eksperimentai buvo atlikti žadinant šią medžiagą priverstiniais virpesiais. Pirmos savosios formos buvo gautos naudojant skaitmeninės raibumų fotografijos metodą ir buvo nustatyta, kad savosios formos kinta keičiant sužadinimo dažnį. Gauti skaitmeninių tyrimų rezultatai palyginti su eksperimentiniais.

Gauti tyrimų rezultatai taikomi nustatant pakavimo medžiagų charakteristikas ir savybes.

E. Kibirkštis, A. Dabkevičius, K. Ragulskis, V. Miliūnas, V. Bivainis, L. Ragulskis

MULTILAYERED POLYMERIC FILM VIBRATIONS UNDER UN-SYMMETRICAL LOADING

S u m m a r y

Vibrations of a loaded multilayered polymeric film have been investigated numerically by assuming that the film consists of three layers. The upper and lower layers are stiff and do not deform in the transverse direction, while the internal layer can get deformed in that direction. The first eigenmodes are analysed and graphical findings of their investigation are presented. Experimental tests on the loaded multilayered polymeric material PET+PAP+LDPE (PET+PAP+LDPE denotes three-layered material made from PET – Poly-Ethylene Terephthalate, PAP – PAPer and LDPE – Low Density Poly-Ethylene) have been performed by exciting this material by forced vibrations. The first eigenmodes have been obtained by using the method of digital speckle photography, and it has been determined that eigenmodes change with the change of the excitation frequency. The obtained results of numerical investigations are compared with the experimental ones.

The obtained study results are applied in determining the characteristics and qualities of packaging materials.

Keywords: multilayered polymeric film, eigenmodes, stresses, vibrations, load, printing quality, speckle photography, experimental setup, digital image, correlation analysis, computerized control.

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