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Kinematic Analysis of the Cyclo-Variator Transmission Mechanism

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1. Introduction

Mechanical variable speed transmissions are used to provide a variable speed output from a constant speed power source. The variator that will be presented in this paper is based on multiple disks. The aim of this paper is to analyse the driver link system of a Beier variator and determine how the length of the links change the variation of the output transmission [1].

The analytical expressions for direct and inverse kinematics were used with MATLAB to analyse the behaviour of the driver link system. As validation of the results obtained with geometrical analysis, we built the same system using Solidworks and compared the result for direct kinematics between the two software. The analysis resulted in the determination of a motion law for the input that approximates the linear variation of the transmission ratio. Furthermore, the determined motion law was compared with the ideal motion law.

2. State of the art

The Beier transmission is composed of two pairs of multiples disks: one pair is fixed, the second one is mobile. By varying the distance between the disk packages, the effective radius of the mobile package is modifying, thus changing the transmission ratio. To increase the transmitted torque to the central transmitting disk packages, around the central package can be added between 3 and 6 mobile packages and as result a more considerable torque can be transmitted. The strategy of using multiple disks, gives the opportunity to increase the transmission torque, but not to increase the overall dimensions of the assembly. Multiple friction contacts reduce the contact pressure and the intensity of frictional forces. The contact surfaces can be lubricated because the decrease of the friction coefficient is being compensated by the increased number of contacts [2].

3. Analytic analysis of the driver link system of for multiple disks transmission

The structure analysed by us, represents a link mechanism for modifying the distance between the disks packages to obtain a variable transmission. The central fixed package is mounted in the joint *x*. The mobile part of the transmission is composed from three pairs of disks that are positioned in the centres of the joints c_1, c_2 and c_3. The variator mechanism contains one slider-crank mechanism and three four-bar mechanisms. The input motion is transmitted to the entire mechanism through the common revolute joint b_1.

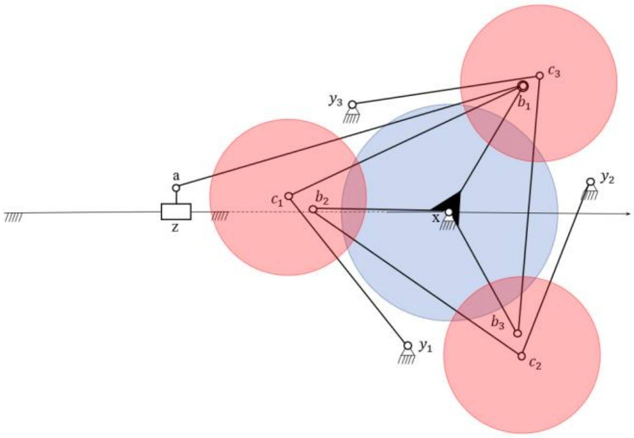

Fig. 1 Representation of the driver link system

All three four-bar mechanism share the revolute joint *x*, so we can simplify the analysis to the slider-crank mechanism and the first four-bar mechanism. For the other mechanisms, the modified distance between the disks will be the same [3-5].

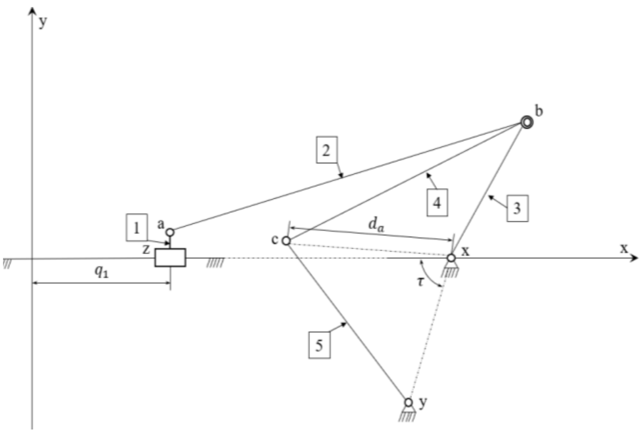

Fig. 2 Representation of the slider-crank and four-bar mechanism

The slider-crank is composed from the elements *1*, *2* and *3*, while the four-bar mechanism is composed from elements *3*, *4* and *5*.

The third element and the revolute joint *b* are shared between the two mechanisms, thus creating the bond between the slider-crank and four-bar mechanism [6].

The transmission ratio of the disk packs can be identified by determining the distance d_a [5]:

$$i = \frac{R}{r}, \tag{1}$$

$$r = d_a - R, \tag{2}$$

where: *i* is total transmission ratio; *R* is radius of the central fixed disk pack; *r* is engaged radius of the mobile disk pack; d_a is distance between the centres of the two disk packs.

To analyse the mechanism, we assigned the following input parameters: l_{AB}, l_{BX}, l_{CY}, l_{BC}, l_{AZ}, l_{XY}, l_1 are link lengths; τ is angle between the axis passing through the fixed revolute joints of the four-bar mechanism and the translational axis; q_1 is the generalized coordinate of the prismatic joint.

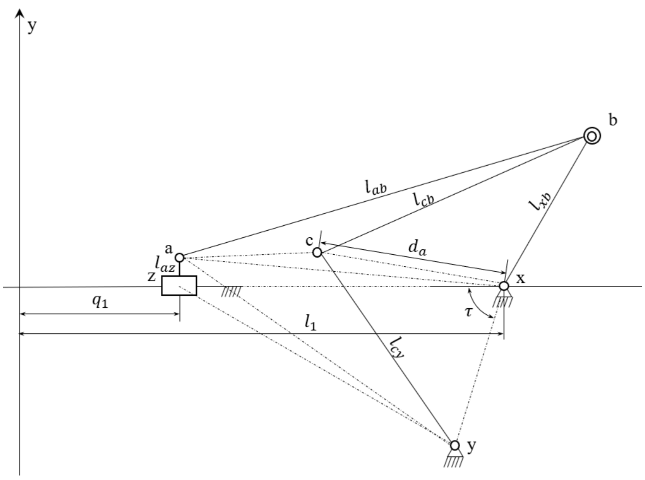

Fig. 3 Representation of the notations assigned to fully analyse the driver link system

To solve the equation that describes the behaviour of the interest parameter d_a, in terms of the input parameters and the generalized coordinate q_1, the following approach was used.

First, the distance between the centres of the revolute joints *a* and *x* is computed as following:

$$l_{AX} = \sqrt{l_{AZ}^2 + (l_1 - q_1)^2}. \tag{3}$$

Having the distance l_{AX} computed, the angles between the horizontal axis, the line l_{AX}, and the link 3 can be computed as following:

$$\varphi_{ZXA} = acos\left(\frac{l_{AX}^2 + (l_1 - q_1)^2 - l_{AZ}^2}{2l_{AX}(l_1 - q_1)}\right), \tag{4}$$

$$\varphi_{BXA} = acos\left(\frac{l_{AX}^2 + l_{BX}^2 - l_{AB}^2}{2l_{AX}l_{BX}}\right). \tag{5}$$

Having the angles φ_{ZXA} and φ_{BXA} determined, the angle between the link 3 and the axis passing through the centres of the fixed revolute joint of the four-bar mechanism is computed as following:

$$\psi = \pi - \varphi_{ZXA} - \varphi_{BXA} - \tau. \tag{6}$$

By knowing the angle ψ, the distance between the centres of the revolute joints *y* and *b* can be computed as following:

$$l_{BY} = \sqrt{l_{BX}^2 + l_{XY}^2 - 2l_{BX}l_{XY}cos(\pi - \psi)}. \tag{7}$$

Having the value of l_{BY} computed, the angles between the link 3, the axis passing through the revolute joint *b* and *y*, and the link 4 are computed as following:

$$\varphi_{CBY} = acos\left(\frac{l_{BC}^2 + l_{BY}^2 - l_{CY}^2}{2l_{BC}l_{BY}}\right), \tag{8}$$

$$\varphi_{XBY} = acos\left(\frac{l_{BX}^2 + l_{BY}^2 - l_{XY}^2}{2l_{BX}l_{BY}}\right). \tag{9}$$

By knowing the angles from (8) and (9), the angle between the links 3 and 4 is:

$$\varphi_{CBX} = \varphi_{CBY} + \varphi_{XBY}. \tag{10}$$

By computing the angle between the links 3 and 4, the interest parameter l_{CX}, representing the distance between the centres of the disk packs, is determined as following:

$$l_{CX} = d_a = \sqrt{l_{BX}^2 + l_{CB}^2 - 2l_{BX}l_{CB}cos(\varphi_{CBX})}. \tag{11}$$

Therefore, the function describing the distance between the centres of the disk packs (forward kinematic problem) in terms of the input parameters and the generalized coordinate q_1 is:

$$l_{CX}(q_1) = \sqrt{l_{BC}^2 + l_{BX}^2 - 2l_{BC}l_{BX}cos(A + B)}. \tag{12}$$

Where:

$$A = acos\left(\frac{l_{BX}^2 - l_{XY}^2 + C^2}{2l_{BX}C}\right), \tag{13}$$

$$B = acos\left(\frac{l_{BC}^2 - l_{CY}^2 + C^2}{2l_{BC}C}\right), \tag{14}$$

$$C = \sqrt{l_{BX}^2 + l_{XY}^2 - 2l_{BX}l_{XY}cos(\tau + D + E)}, \tag{15}$$

$$D = acos\left(\frac{l_{AZ}^2 - l_{AB}^2 + l_{BX}^2 + (l_1 - q_1)^2}{2l_{BX}\sqrt{l_{AZ}^2 + (l_1 - q_1)^2}}\right), \tag{16}$$

$$E = acos\left(\frac{l_1 - q_1}{\sqrt{l_{AZ}^2 + (l_1 - q_1)^2}}\right). \tag{17}$$

Having the function $l_{CX}(q_1)$, the function that describes the behaviour of the transmission ratio in terms of the input parameters and the generalized coordinate q_1 is:

$$i(q_1) = \frac{R}{l_{CX}(q_1) - R}. \tag{18}$$

For the inverse kinematic problem (IKP) the distance between the centres of the central and the peripherical disk packs is known by imposing the transmission ratio, and the generalized coordinate q_1 is computed based on the constructive parameters and the imposed transmission ratio *i*.

First, the distance between the central and peripheral disk packs is expressed as following:

$$l_{CX} = \frac{R(1+i)}{i}. \tag{19}$$

By determining the length l_{CX}, the angles between the link 3, the axis passing through the centres of the disk packs and the support axis of the two fixed revolute joints of the four-bar mechanism are determined as following:

$$\varphi_{BXC} = acos\left(\frac{l_{BX}^2 + l_{CX}^2 - l_{BC}^2}{2 l_{BX} l_{CX}}\right), \tag{20}$$

$$\varphi_{XYC} = acos\left(\frac{l_{XY}^2 + l_{CX}^2 - l_{CY}^2}{2 l_{XY} l_{CX}}\right). \tag{21}$$

Having the angles φ_{BXC} and φ_{XYC} determined, the angle between the link 3 and the support axis of the four-bar mechanism is:

$$\varphi_{BXY} = \varphi_{BXC} + \varphi_{YXC}. \tag{22}$$

By determining the angle between the link 3 and the support axis of the four-bar mechanism, the angle between the support axis of the prismatic joint and the link 3 is determined as following:

$$\varphi_{ZXB} = \varphi_{BXY} - \tau. \tag{23}$$

Having the angle of the crank φ_{ZXB} determined the height of the revolute joint *b* from the support axis of the prismatic joint is determined by:

$$h_b = l_{BX} sin(\varphi_{ZXB}). \tag{24}$$

By determining the height of the revolute joint h_b, the angle between the horizontal axis passing through the revolute joint *a* and the link 2 is determined as following*:*

$$\varphi_{BAH} = asin\left(\frac{h_b - l_{AZ}}{l_{AB}}\right). \tag{25}$$

Having the angle φ_{BAH} determined, the distance between the prismatic joint and the center of the central disk pack is determined by:

$$l_{ZX} = l_{AB} cos(\varphi_{BAH}) + l_{BX} cos(\varphi_{ZXB}). \tag{26}$$

By determining the value of l_{ZX}, the equation that determines the generalized coordinate q_1 is:

$$q_1 = l_1 - l_{ZX}. \tag{27}$$

Therefore, the function describing the generalized coordinate q_1 of the driver link system (inverse kinematic problem) in terms of the input parameters and the distance between the centres of the disk packs l_{CX} is:

$$q_1(l_{CX}) = l_1 - \sqrt{l_{AB}^2 + (l_{AZ} - A)^2} - B. \tag{28}$$

Where:

$$A = l_{BX} sin(C + D - \tau), \tag{29}$$

$$B = l_{BX} cos(C + D - \tau), \tag{30}$$

$$C = acos\left(\frac{l_{CX}^2 + l_{BX}^2 - l_{BC}^2}{2 l_{CX} l_{BX}}\right), \tag{31}$$

$$D = acos\left(\frac{l_{CX}^2 + l_{XY}^2 - l_{CY}^2}{2 l_{CX} l_{XY}}\right). \tag{32}$$

Having the function q_1 (l_{CX}), the function that describes the behaviour of the generalized coordinate q_1 in terms of the transmission ratio is obtained by substituting (19) in (28-32)

4. Analysis of the driver link system using MathWorks MATLAB and Dassault Systems SOLIDWORKS

MATLAB was used to create a function for the analysis that describes the dependency of the distance between the centres of the central and peripherical disk packs and the transmission ratio based on the input parameters and the generalized coordinate q_1. The determination of the distance between the centres of the two disk packages was implemented using the equations presented in previous chapter. First, the mechanism was defined using the following length of links and the following parameters of the fixed revolute joints: l_{AB} = 130 mm; l_{BX} = 38 mm; l_{CY} = 74 mm; l_{BC} = 60 mm; l_{XY} = 70 mm; l_{AZ} = 8 mm; l_1 = 160 mm; τ = 60°; R = 32,5 mm.

The first analysis made was a visualisation of the behaviour of the length l_{CX} according to a linear variation of the generalised coordinate q_1 from 0 to 50 mm representing a constant velocity motion profile.

From the previous figure it can be observed that the behaviour of the function $l_{CX}(q_1)$ is very close to the behavior of a linear function (Fig. 4. Red / thin line).

To validate the previous results, Dassault Systems SOLIDWORKS was used to construct the driver link system according to the same geometrical parameters as the MATLAB model. After constructing the model, the same approach of iteration through the stroke of the prismatic joint was implemented and the following results for the measurement of the distance between the centres of the central and peripherical disk packs were achieved.

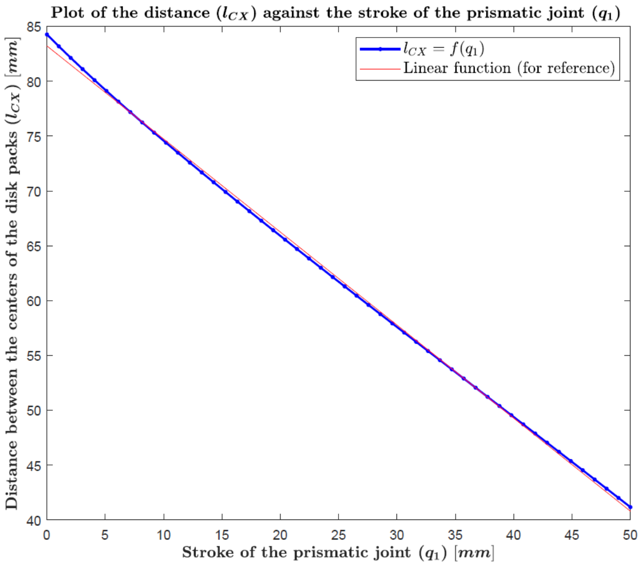

Fig. 4 Representation of the distance l_{CX} against q_1

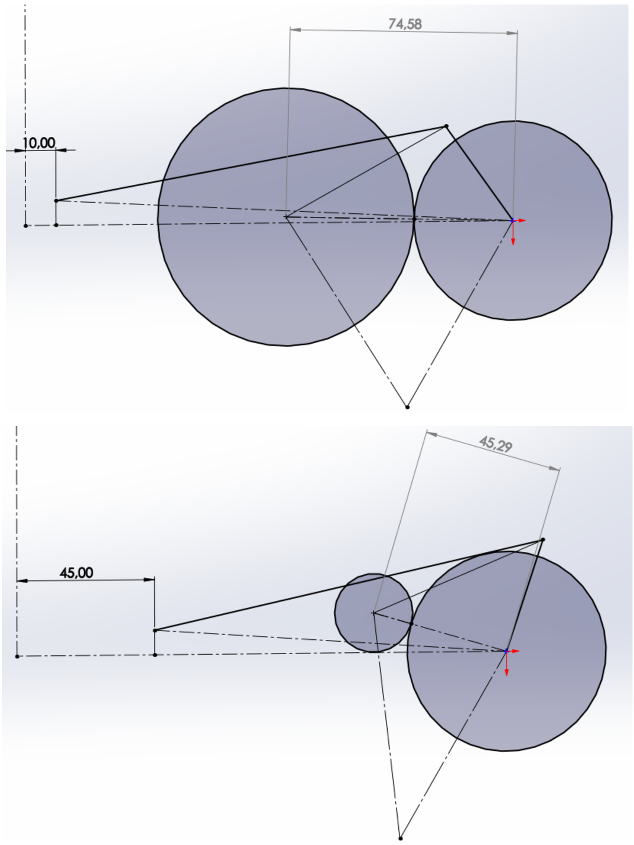

Fig. 5 Representations of the model analysed with SOLIDWORKS

From the comparison of the results obtained by using MATLAB and SOLIDWORKS for the analysis, it was concluded that the function implemented in MATLAB is correct by giving the consistent and low valued errors (maximal absolute error of $4.748 \cdot 10^{-6}$) between the two software packages.

Table 1

Results achieved with MATLAB and SOLIDWORKS

Stroke q_1, mm	Distance l_{CX}, mm	
	MATLAB	SOLIDWORKS
0	84.2611575036225	84.26115750
5	79.2125100483902	79.2125091
10	74.5758676887008	74.57587123
15	70.1683825394567	70.16838342
20	65.8999283080192	65.89993089
25	61.7173197770893	61.71732023
30	57.5858132158235	57.58581311
35	53.4812519383046	53.481251
40	49.3860036622092	49.38600212
45	45.2860558922457	45.28605565
50	41.1673221581297	41.16732255

After determining the behaviour of the distance between the centres of the central and peripheric disk packs it was proceeded into visualizing the transmission ratio dependence to the stroke of the prismatic joint.

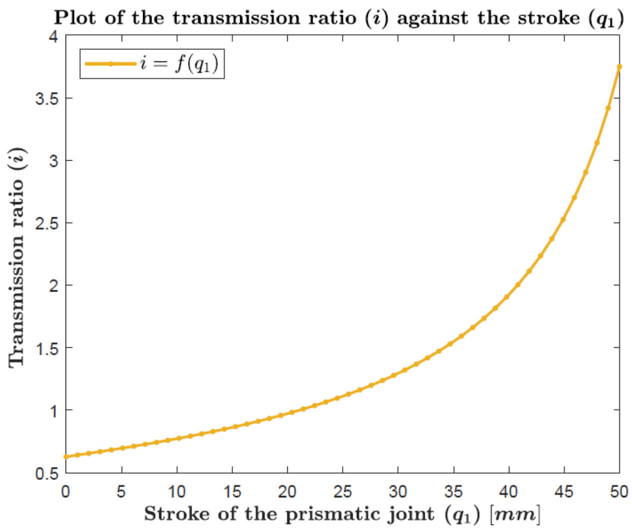

Fig. 6 Representation of the transmission ratio against the stroke of the prismatic joint

By utilising the "Curve Fitting Tool" from MATLAB, a regression can be computed for the values of the transmission ratio according to the given values of the stroke q_1.

Table 2

Curve fitting tool results from MATLAB

Regression type	Exponential
Number of terms	2
Equation	$f(x) = A \cdot e^{B \cdot x} + C \cdot e^{D \cdot x}$
Results	
A	0,6151
B	0,02233
C	0,001054
D	0,1494
Goodness of fit	
SSE:	0,003786
R-square:	0,9999
Adjusted R-square:	0,9999
RMSE:	0,009073

By using the “Curve Fitting Tool” it was observed that for a linear input, the mechanism has its best fit regression as an exponential for its output, and for an optimal usage of this mechanical variable speed transmission is expected to behave in a linear way by modifying the stroke of the slider-crank mechanism.

The exponential regression that fits the best the current output curve has the following two term form solution:

$$i(q_1) = A \cdot e^{B \cdot q_1} + C \cdot e^{D \cdot q_1}. \tag{33}$$

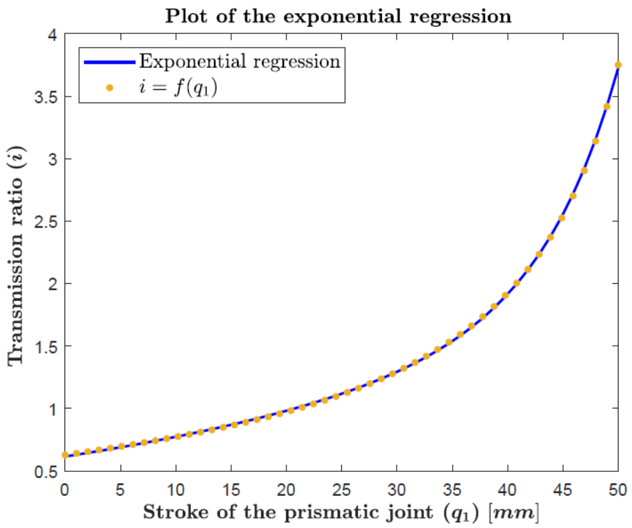

Fig. 8 Representation of exponential regression determined to fit the variation for the transmission ratio

The previous results can be explained by the fact that the parameter has an almost linear behaviour and the fact that the equation representing the transmission ratio in terms of l_{CX} has the form from (18), which represents a rational function and with the following arrangement of terms, the function can be expressed as the hyperbolic function $f(x) = \dfrac{1}{x}$:

$$i(q_1) = \frac{1}{\dfrac{l_{CX}(q_1)}{R} - 1}. \tag{34}$$

To try the linearization of the behaviour of the transmission ratio, based on the stroke of the prismatic joint, different common polynomial or power series motion profiles were tried as laws for the input and the best fit achieved by us was determined by using the following input motion law (Fig. 9) and (35).

$$q_1(t) = v\sqrt{t \cdot (c - t)}, \tag{35}$$

where: t is time, s; c is imposed time constant, s; v is minimal speed for non-zero time, mm/s.

The equation that approximates the behaviour of the variation for the transmission ratio is determined by the equation that describes the linear regression which fits the best the variation of the transmission ratio. The “Curve Fitting Tool” was used to determine the linear regression and the following results were obtained:

$$i(t) = A \cdot q_1(t) + B, \tag{36}$$

where: A=0,06508 and B=0,6508.

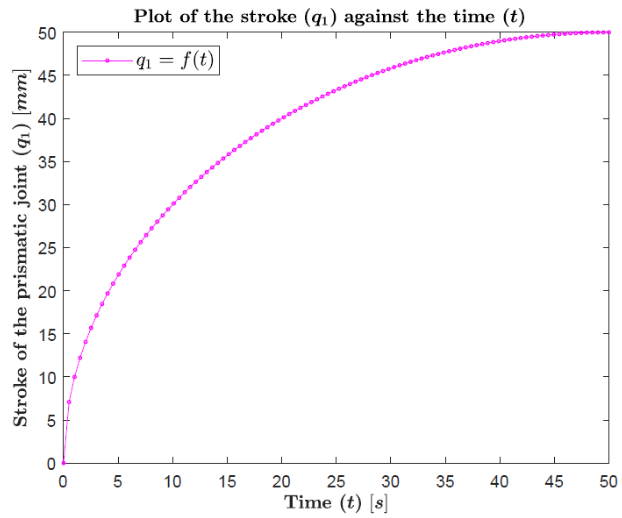

Fig. 9 Representation of the motion law for the stroke of the prismatic joint

The result of the variation of the transmission ratio, based on the motion law of the prismatic joint, is represented in the following figure. A linear regression was applied for the transmission ratio variation to determine an approximation of the dependence between the motion of the prismatic joint and variation of the transmission ratio.

Furthermore, the R-squared factor of the linear regression is 0.9947, indicating a satisfactory approximation for the relationship between the input and output behaviour.

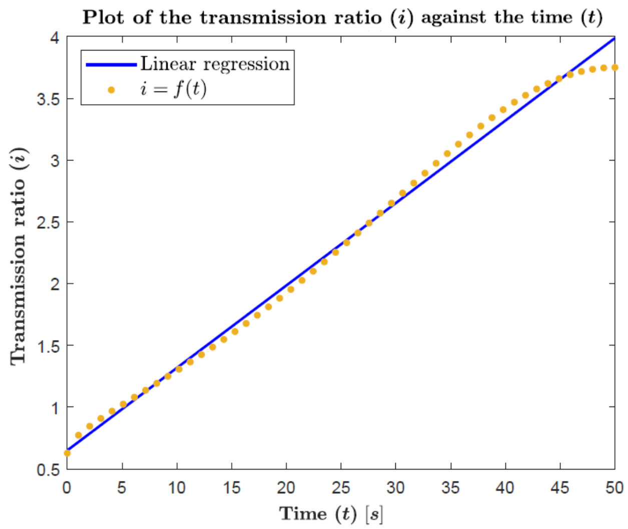

Fig. 10 Representation of the variation over time of the transmission ratio and the linear regression

Furthermore, by using the equations that solves the inverse kinematic problem (IKP), a proper motion law for the stroke of the prismatic joint can be determined to achieve linear variation of the transmission ratio. The linear variation of the transmission ratio over time has the following form:

$$i(t) = A \cdot t + B, \tag{37}$$

where: A=0,06243 and B=0,62788.

The values of A and B were computed by constructing the line, which describes the variation of the transmission ratio, to pass through the minimal and maximal value of the transmission ratio in order to achieve the same range for the generalised coordinate q_1 (stroke of the prismatic joint). By applying the imposed variation profile of the transmission ratio, the required behaviour of generalised coordinate is q_1.

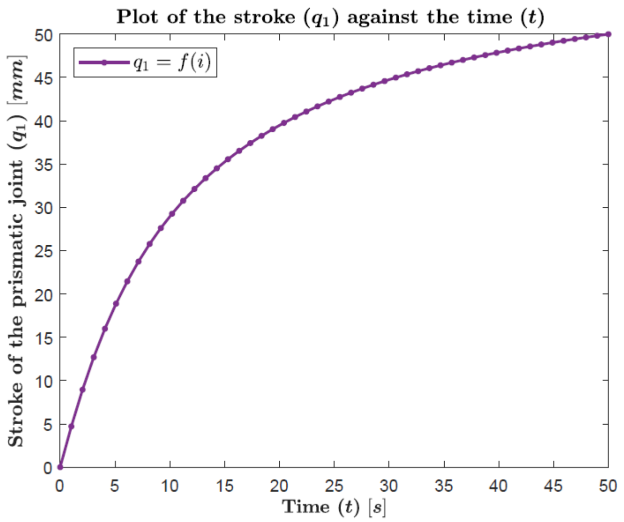

Fig. 11 Representation of the ideal motion law for the input to achieve linear output variation

6. Conclusion

In this paper, the kinematic analysis of a driver link mechanism for multiple disks variator was achieved. The study is based on the analysis of the distance between the central fixed disk packages and the mobile disk packages.

The analytical equations were implemented in MATLAB for visualizing the dependence between the generalized coordinate q_1, described by the position of the prismatic joint of the slider-crank mechanism and the output, which represents the distance between the centre of the central and peripherical disk packs and also the behaviour of the total transmission ratio based on the generalized coordinate q_1. From the representation achieved, it was observed that for an imposed motion law consisting of a constant velocity motion profile, the behaviour of the output has is best characterized by an exponential regression. Also, if the behaviour of the output is desired to vary linearly, then the motion law for the input needs to be computed using the equations that solve the inverse kinematic problem.

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KINEMATIC ANALYSIS OF THE CYCLO-VARIATOR TRANSMISSION MECHANISM

S u m m a r y

This paper presents the working principle of the Beier variator that is composed of three mobile disk packages and one fixed. The actuation system of the moving disk packages consists of a slider-crank mechanism and three four-bar mechanism. The paper presents the geometrical approach for determining the transmission ratio of the mechanism based on the distance between the disk packages. MATLAB was used for determining the motion law for the input in order to obtain a linear output variation.

Keywords: Beier variator, Beier disks, slider-crank mechanism, four-bar mechanism, MATLAB.

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