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Modeling and Characterization of a Flow in a Porous Medium by a Coupled Approach Between the Navier-Stokes and Darcy Equations

Khelifa HAMI*, Abdelkrim TALHI**

**Laboratory of Environmental and Energy Systems, Institute of Science and Technology, University Center Ali KAFI Tindouf, Tindouf 37000, Algeria, E–mail: hamikhelifanada@gmail.com*

***Laboratory of Environmental and Energy Systems, Institute of Science and Technology, University Center Ali KAFI Tindouf, Tindouf 37000, Algeria, E–mail: karim.talhi@gmail.com*

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Nomenclature

u_{in} – infiltration rate, m. s-^{1}; u – velocity along the x-axis, m. s-^{1}; v – velocity along the y-axis, m. s-^{1}; h – height, m; e – thickness, m; k – coefficient of the permeability, m^2. s-^{1}; ϕ – porosity of the medium, %; ρ – density, kg. m^{-3}; μ – dynamic viscosity, kg. m^{-1}.s^{-1}.

1. Introduction

Porous media are materials that are distinguished by the coexistence of a solid matrix with a network of channels, the pores, where one or more fluids can coexist [1]. A distinction is generally made between granular materials, resulting from the stacking of grains of matter forming a nonconvex solid matrix (we also speak of "unconsolidated" media), and materials whose solid matrix is continuous (then called "consolidated").

There are several levels of description of flows in porous media, the size and number of which may vary depending on the materials and applications considered [2-3]. However, we generally admit at least three scales of description characterized by the following properties:

The pore scale (also referred to as the microscopic scale in these works): this is the smallest scale of the porous medium (from micrometer to millimeter) in the sense that it explicitly takes into account the geometry (complex) of the solid skeleton and the multiphase character of the flows: phases and interfaces are identifiable. Despite the restricted size of these levels of description, the use of continuum mechanics remains valid for most of the materials considered.

The Darcy scale (also referred to as the macroscopic scale in these works): This is the usual scale for describing the porous medium (from millimeters to ten centimeters). For most engineering problems, a description at the pore scale is of little interest, either because the applications are more oriented towards a global description of the properties of the porous medium. Or because describing the resolution of the problem from the microscopic scale is difficult to implement due to the complex geometry of this scale and the resources necessary for solving it pore by pore. Therefore, a resolution of the problem at the Darcy scale involves macroscopic state variables: at each point of the domain, the physical quantities result from an average over a minimum volume of the porous material, chosen so that its properties are representative of the environment. For multiphase flows, the interfaces are not visible and the occupation of the pore space is described in terms of mass or volume fraction.

Large scales: This is the scale of the environment studied as a whole (from ten centimeters to kilometers). The study of these environments generally calls for a combined approach of geostatistical data and numerical simulations of the averaged equations of the Darcy scale over areas of constant properties.

The transition from a description of the microscopic scale to higher scales is therefore done by assuming that the physical properties of the material remain unchanged once the unit of description of the system is large enough. Thereby, the microscopic dynamics in a (VER) around different points in the material will show strong similarities. This hypothesis, then justifies the use of an averaged description of the physics of the Darcy scale, assuming that the elementary volume of the domain is in fact a REV, and therefore that the underlying microscopic physics varies little.

Note that despite this assumption, we observe in practice a dependence of the physical properties on the size of the (VER) for highly heterogeneous porous media, especially towards large scales (Fig. 1). This is due to the structuring of porous media which, depending on geological events (deposits by layers of sediment, earthquakes, etc.), will lead to an arrangement of pore networks that differ from one area to another. At the medium scale, depending on the heterogeneity of the material, we can consider the existence of different REVs. Therefore, areas with different dynamics, which will have to be taken into account. Under these conditions, a statistical description of REV may be preferred. On the first scales of description, a disparity of the physical properties is present, according to the considered zones of the material. However, observed on a sufficiently large volume, these properties vary little from one point to another [3]. The same invariance is observed for larger arbitrary volumes. However, for heterogeneous media, at large scales, variations can again appear on large scales due to a difference in microstructure.

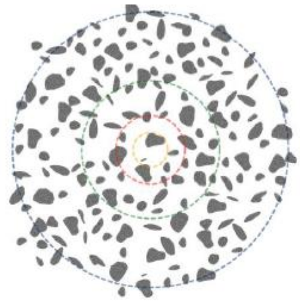

Fig. 1 Representative elementary volume (REV) of a porous medium

The Navier-Stokes equations are not directly applicable to the porous material, as it is not known what happens microscopically in the pores with respect to pressures and velocities. It is therefore necessary to find by other means a macroscopic law valid on the scale of the VER (representative elementary volume) for a porous material, connecting pressure, speed, and external forces. We will see that such a law was found experimentally by Darcy.

This problem has been studied for a long time since Beavers and Joseph [4] study Navier-Stokes flows above a porous block and already speak in their introduction of an extensive analytical literature. They show the existence of a sliding speed at the surface of the porous medium and propose a boundary condition. Subsequently, Saffman [5] mathematically justified this boundary condition and showed that Darcy's speed term could be neglected. Therefore, he gave the condition known as the Beavers-Joseph-Saffman condition. Certain numerical studies concerning the coupling of the equations of Stokes and Darcy take into account this boundary condition, while others do not consider it.

Many authors have coupled the Navier-Stokes equations for the fluid zone and the Darcy equation for the porous zone. Correa and Loula [6] propose a coupling between a Navier-Stokes finite element formulation and a mixed Darcy formulation using Taylor-Hood elements.

J. M. Urquiza et al. [7] couple a mixed finite element formulation of Navier-Stokes with a formulation of the Darcy equation taken in the form of a Poisson equation.

G. Pacquaut et al. [8] express the weak formulation of each of the equations by including the Beavers-Joseph-Saffman condition. A surface integral in taking into account the boundary conditions being common to each of the formulations, they were able to inject the Darcy equation directly into the Navier-Stokes formulation, which allowed the coupling. In principle, this ultimately amounts to writing the weak formulation of Brinkman's equation.

G. N. Gatica et al. [9] also carry out a (Navier-Stokes) -Darcy coupling by comparing several types of elements to obtain a stable formulation. This time again, the solved formulation ultimately returns a weak formulation of the Brinkman equation. Masud [10] does the same type of study, this time clearly solving the Brinkman equation, which he writes under its strong formulation, however calling it the (Navier-Stokes) -Darcy equation.

H. Tan and K. M. Pillai [11] also propose a finite element formulation of Brinkman's equations. However, they propose a modified Brinkman equation to take into account a stress jump at the fluid-pore interface.

The coupling between the Darcy and Navier-Stokes equations has been widely studied previously using the finite volume method in domain immersion, due to numerous applications involving porous materials and at the interface between the porous medium [12-13]. The domain immersion technique used was subsequently improved [14-15].

The establishment of numerical models allowing calculations to be carried out at the microscopic scale thanks to a coupling between the Navier-Stokes equations and Darcy's law. The particularity of the reinforcements considered in this type of work is the presence of several scales of porosity. On the other hand, between the pores [16]. Therefore, we have a domain containing both two matrices, one solid and the other fluid.

2. Method

The characterization of flows in porous media has been detailed. First of all, the levels of description of this class of materials have been reviewed and explained. The geometric characterization of porous media has made it possible to give a definition of the representative elementary volume, essential to distinguish each of the scales at which it is possible to describe flows. Then, the direct application of the mechanics of continuums at the microscopic scale made it possible to find the equations of continuity and conservation of the momentum that the fluid had to respect. By applying an averaging operator, these microscopic equations can be formulated at the Darcy scale, where several flow models coexist due to the simplifying assumptions made on the system. Finally, the study of the relative saturation-permeability relationships has highlighted some existing models in the literature, as for the capillary pressure within the medium [17-18]. From a practical point of view, the flow characterization methods that require the fewest assumptions about the system under study result from scaling operations. However, their complete resolution requires the resolution of a macroscopic model coupled with a microscopic model. A good knowledge of the microstructure is therefore essential to validate the relevance of these approaches.

Darcy's law describes the movement of fluid through the interstices of a saturated porous medium, which is mainly driven by a pressure gradient, and wherein momentum transfer due to shear stresses in the fluid is negligible the Darcy's Law interface calculates the pressure, the velocity field is then determined by the pressure gradient, fluid viscosity and permeability.

The porous medium studied in this work presents a homogeneous and isotropic material fed by a source of the ifiltration velocity (Fig. 2). The walls at the top, bottom and the left side wall approximately the source are considered impermeable. The influence of several physical parameters are taken into account to simulate this type of material.

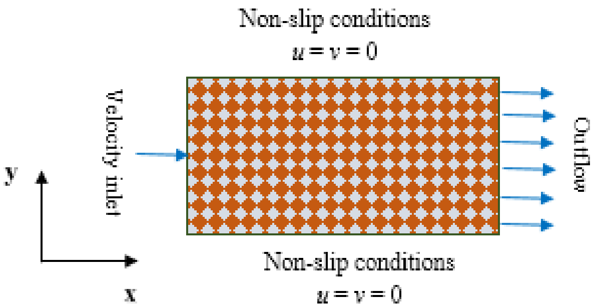

Fig. 2 Physical model

2.1. Mathematical model

Darcy's law was established under specific conditions, which limit its validity. The main underlying assumptions are:

- Homogeneous, isotropic and stable solid matrix;
- Homogeneous, isothermal and incompressible fluid;
- Negligible kinetic energy;
- Permanent flow regime;
- Laminar flow;

- The spatial variations in density (compressibility, heterogeneity) and viscosity (temperature) of the liquid phase are low enough for their effect to be generally neglected.

Equation of continuity:

$$\frac{\partial u}{\partial x}=\frac{\partial v}{\partial y}. \qquad (1)$$

Porous materials are modeled by adding momentum source terms of the fluid flow equation described as Eqs. (2) and (3). Darcy's law is considered as the source term in these two equations. $\frac{\mu_f}{k}u$ and $\frac{\mu_f}{k}v$ are the source terms for the directions (X, Y) in momentum Eqs. (2), and (3).

Equation of momentum along the X-axis:

$$\rho_f\left(\frac{u}{\phi^2}\frac{\partial u}{\partial x}+\frac{v}{\phi^2}\frac{\partial u}{\partial y}\right)=-\frac{\partial p}{\partial x}+\frac{\mu_f}{\phi}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)-\frac{\mu_f}{k}u. \qquad (2)$$

Equation of momentum along the Y-axis:

$$\rho_f\left(\frac{u}{\phi^2}\frac{\partial v}{\partial x}+\frac{v}{\phi^2}\frac{\partial v}{\partial y}\right)=-\frac{\partial p}{\partial y}+\frac{\mu_f}{\phi}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)-\frac{\mu_f}{k}v. \qquad (3)$$

At the level of the top, bottom and left side wall (non-slip conditions) $u = v = 0$; source level (input velocity) $u_{in} = 10^{-4}$, m/s; at the level of the right side wall (outflow) $\varphi = u\cdot(h\cdot e)$, m^3/s.

The algorithm for the coupled Navier-Stokes, and Darcy equations resolution (Fig. 3), summarizes the steps in solving this study.

- τ – represent the step of the iterative calculation;
- $\tau = 0$ – initialization of the calculation;
- $(\tau+1)$ –next step of the iterative calculation;
- The convergence test chosen for this simulation is $R < 10^{-4}$ (weighted residue).

3. Results and discussions

The idea of this method is to fix the solution to known values on the nodes. Then we build the final solution by interpolating between each node. So, choosing a fine mesh means that we have a good knowledge of the solution we want to obtain. Conversely, for a coarse mesh, we then have a more approximate idea of the solution to be obtained. We do not know how in practice an interpolation is carried out, but we think that this one is done without taking into account the physical solution between two nodes.

We noted the value of the residuals for each of the solutions. We found residuals in the same order of magnitude when we expected to have large residuals for coarse meshes. In conclusion, if we increase the number of nodes, then we decrease the errors due to interpolations between nodes, and the numerical solution is closer to reality. We also noted that the computation time increases with the fineness of the mesh, but this increase is not significant for the meshes chosen in our simulations (Error < 1 %), Fig. 4.

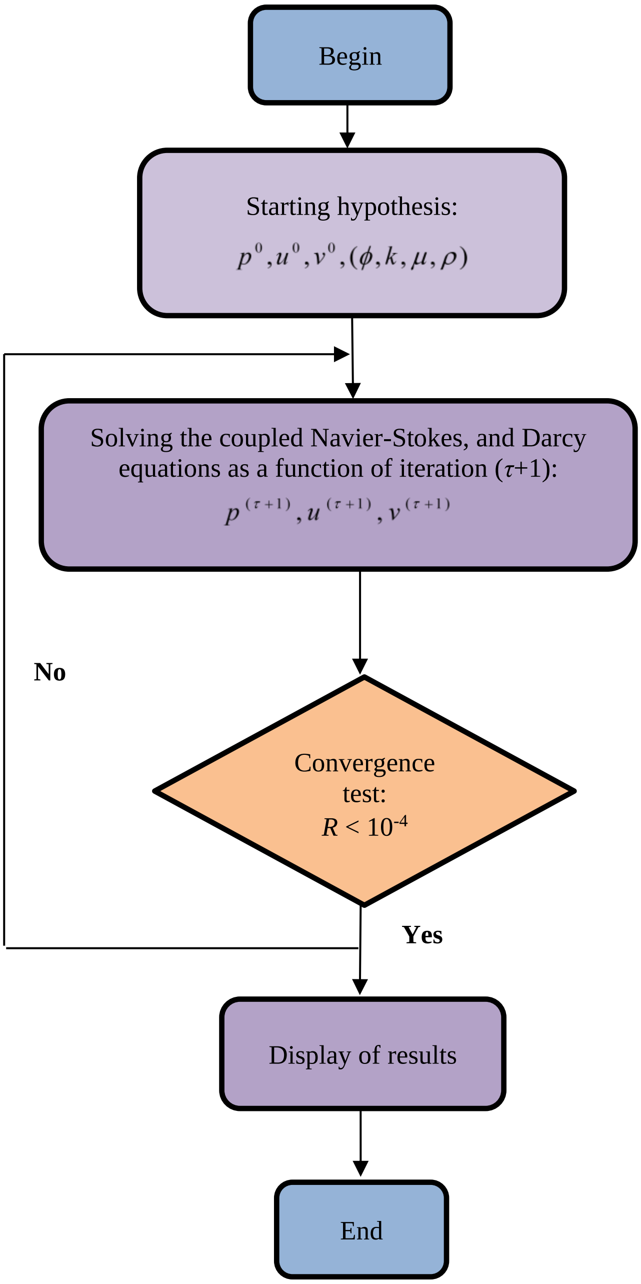

Fig. 3 Algorithm for the coupled Navier-Stokes, and Darcy equations resolution

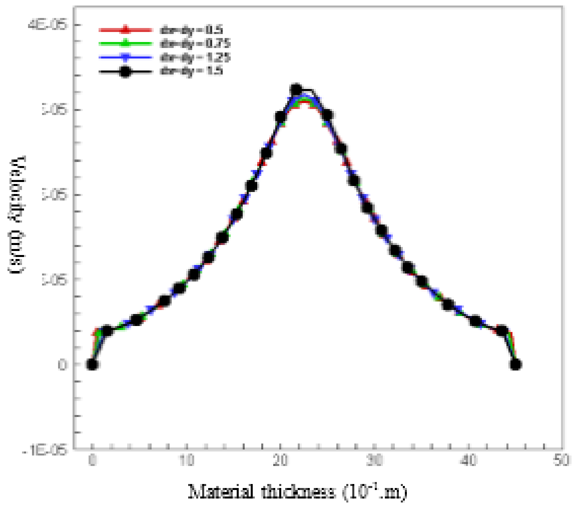

Fig. 4 Mesh test

The evolution of the velocity and the pressure in the vertical plane in the middle of the material studied, are represented in Fig. 5. Comprises two successive phases, the transient phase, which depends strongly on the initial state of the system or the numerical solution is unstable and the steady-state phase, Independent of the initial state of the system (initial conditions), where our numerical solution has become stable. The results obtained confirm a passage between an initial computation times towards an inertial computation time of the system.

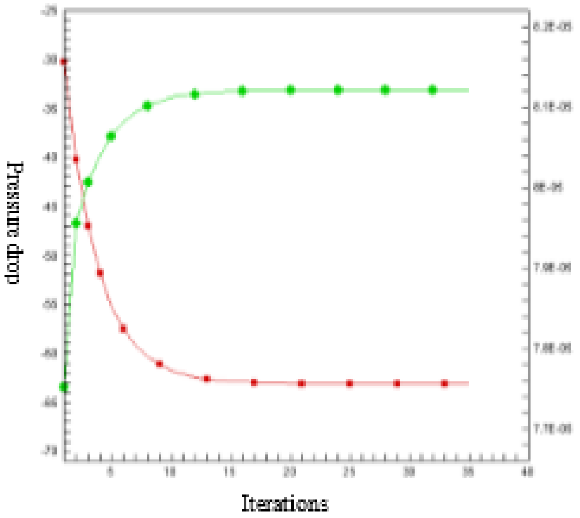

Fig. 5 Convergence test of the numerical solution

The balance between viscous and inertial forces is expressed by a non-dimensional parameter called the Reynolds:

$$R_e = \frac{\rho_f . d . u}{\mu_f}, \tag{4}$$

where: ρ_f is fluid density; d is diameter of the passage through which the fluid moves; u is fluid velocity; μ_f is dynamic viscosity.

We find ourselves theoretically solving the Navier-Stokes equations in the case where $Re << 1$ (i.e. when the inertial terms are negligible). Moreover, the flow in this case is a Darcy-type flow. In the case where the Reynolds number is high, the relationship between the specific discharge and the hydraulic gradient is no longer linear. Therefore, the transition zone of Reynolds numbers in the range 1 to 10 is associated with the upper limit of Darcy's law. The lower limit of Darcy's law is associated with extremely slow flow.

The results presented in Fig. 6, show that the values taken by the pressure drop are independent with respect to the filtration rate, the number of *Re* always remains lower than 1. Thus, the Darcy criterion is systematically validated in this present study.

The resulting velocity field and the velocity vector field presented in Figs. 7 and 8, shows that the velocity is zero near the impermeable walls thanks to the non-slip conditions of the wall, the maximum values are observed at the level of the infiltration zone. The results shown confirm the isotropy and homogeneity of the porous material chosen for this simulation. The flow of this material goes through two successive phases, in the first it is decelerated by the effect of the stresses of the solid matrix of the porous medium and in the second phase, it has become uniform under the effect of the infiltration of these pores.

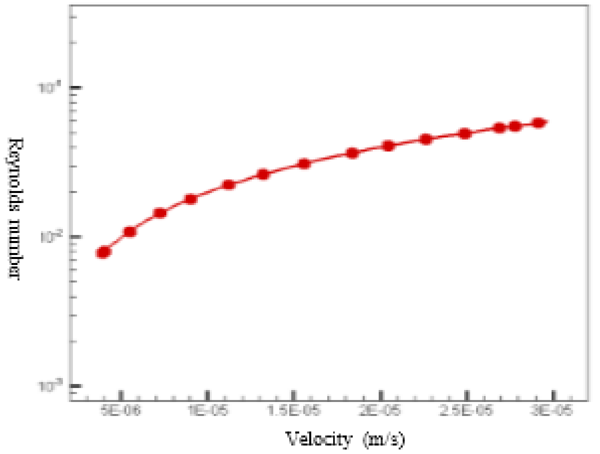

Fig. 6 Validation of the Darcy approach

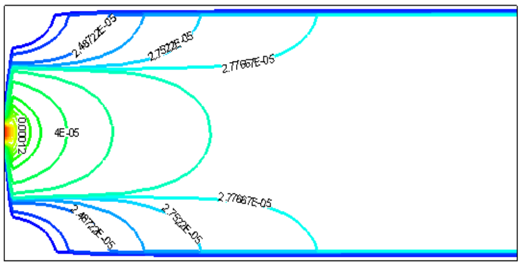

Fig. 7 The resulting infiltration velocity field

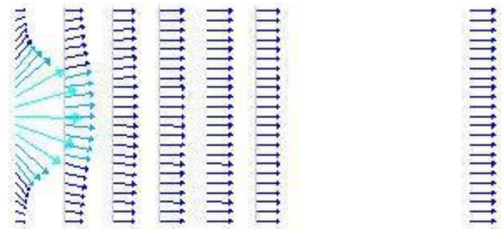

Fig. 8 Infiltration velocity vector field

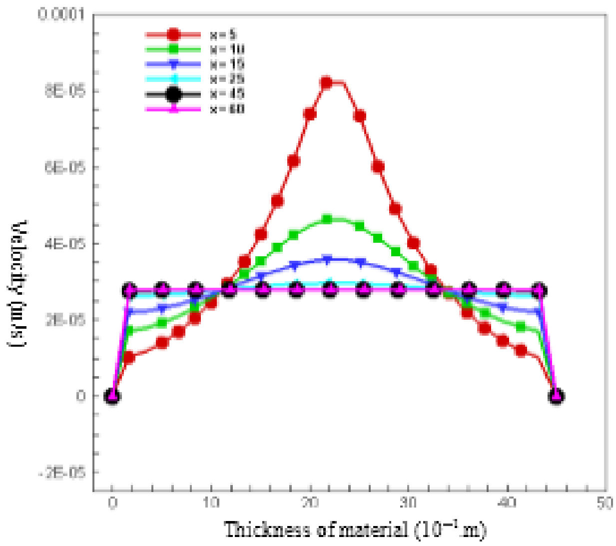

Fig. 9 Velocity profiles in different levels relative to the thickness

The Fig. 9 represent the velocity profiles located between different levels of the thickness of the material relative to the source of infiltration, in all cases; the speed is zero at the level of the impermeable walls thanks to the conduction at the non-slip limits walls. By the effect of the channel, we have a parabolic drop in the speed profile close to the source and another profile with a square shape far from this source of infiltration.

The influence of the infiltration rate of the behavior of the material is illustrated in Fig. 10, we always keep the same shape of the profile presented, but the decelerating flow phase is dominant compared to the uniform flow phase for a maximum flow and vice versa for a minimum flow.

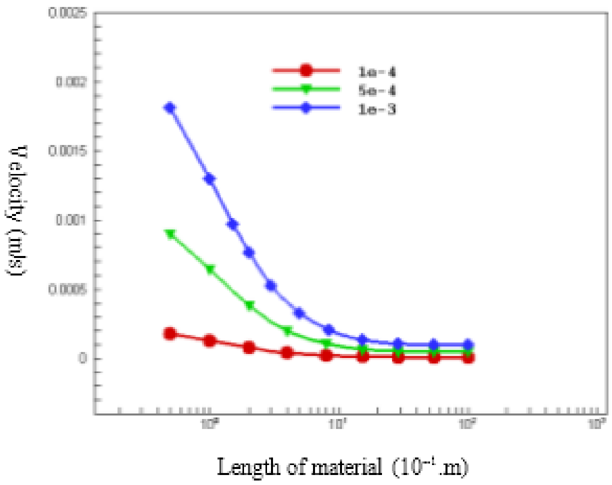

Fig. 10 Effect of infiltration rate

The effect of porosity on the physical behavior of the simulated material with respect to a vertical reference line is shown in Fig. 11. By definition, porosity is also a numerical value defined as the ratio between the volume of the voids and the total volume. In a porous medium (Eq. 5). It is noticed that the speed is worth maximum values if the fluid matrix, which dominates the total volume of the medium compared to the solid matrix, and conversely this speed is worth minimum values in the case where it is the solid matrix, which dominates the totality of the volume of this medium.

$$\phi = \frac{V_{pores}}{V_{pores}}. \quad (5)$$

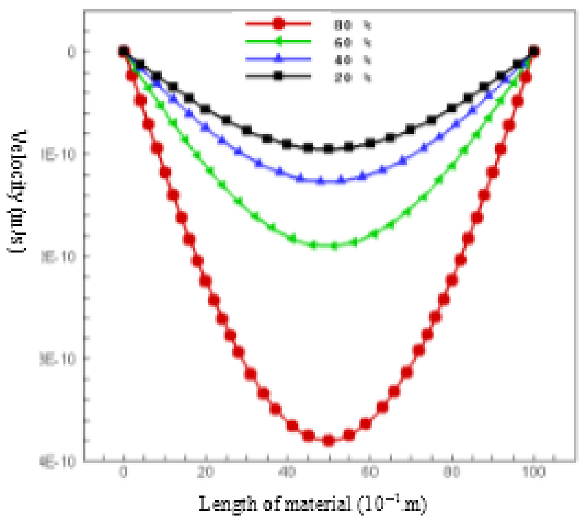

Fig. 11 Effect of porosity

4. Conclusions

Simulations were carried out on the physical behavior of a homogeneous and isotropic porous material by the numerical resolution of a Navier-Stokes approach coupled with Darcy's law in the case where $Re << 1$. The results obtained allowed us to reach the following conclusions: Darcy's approach is systematically validated by a calculation of the Reynolds number ($10^{-2} < R_e < 10^{-1}$). The saturation of the porous material depends on the infiltration rate. The behavior of the material is influenced by several physical and geometric parameters such as the variation of the infiltration rate, the thickness of the material and its porosity. For a porosity of 20 %, and an infiltration rate of 10^{-4} m/s, 5.10^{-4} m/s and 10^{-3} m/s the saturated of the material is 3 times, 5 times, and 8 times of languor with respect to the source of infiltration. For a reference distance in the middle of the material, and for a porosity of 80%, 40%, and 20%, the maximum velocity respectively are 10^{-4} m/s, 5.10^{-6} m/s, and 10^{-7} m/s.

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K. Hami, A. Talhi

MODELING AND CHARACTERIZATION OF A FLOW IN A POROUS MEDIUM BY A COUPLED APPROACH BETWEEN THE NAVIER-STOKES AND DARCY EQUATIONS

S u m m a r y

In this present work, simulations were carried out numerically to characterize a porous material. The basic equations, which govern this problem, are those of the Navier-Stokes equations coupled with the Darcy equation. In order to understand the phenomena at stake, a first test was first completed on the independence of the mesh compared to the numerical solution obtained, the second test is devoted to the validation of Darcy (Re <<1). The characterization of the material is based on physical tests; the first is devoted to the porosity of the material, the second to the thickness, and the trireme to the saturation. The results are presented on the one hand contours for the velocity fields and streamlines, and on the other hand, are illustrated with curves to interpret the physical parameters studied in relation to each other.

Keywords: porous medium, Navier-Stokes, Darcy approach, CFD modelling.

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