Using of Grasshopper Optimization Algorithm Approach for Optimal Weight Design Problem of the Spur Gear

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1. Introduction

Depending on the designer's needs, the design of the machine elements involves a complex optimization process that takes into account specific goals such as weight, strength, corrosion, wear, deflection, etc. The optimization of the entire system is more complex and dependent on different parameters than the optimization of a single mechanical component. For this reason, element-based optimization applications yield more effective results. Therefore, many heuristic optimization approaches designed with inspiration from nature have been used for this purpose in mechanical systems [1-3].

In mechanical systems, gears are at the head of the machine elements that enable movement to be transferred from one place to another. Gears are used extensively in all areas of industry for motion and mechanical power transmission. Due to their complex geometry and contact structures, they have a large number of design parameters. One of the most important criteria affecting gear performance is the minimization of weight without losing strength. Many studies have been conducted in the literature using analytical and heuristic optimization methods recently. Yokota et al. used the genetic algorithm method in their design problem [4]. After this study, which is one of the first studies on this subject, Savsani et al. obtained better results with Particle Swarm Optimization (PSO) and Simulated Annealing Optimization (SA) methods [5]. The Grey Wolf Optimization (GWO) method inspired by nature and designed by Mirjalali et al. [6] was applied by Dörterler et al. [7] for the same problem. In their study, weight optimization of the spur gear prepared parametrically was performed with variables and constraints defined by Yokota et al. [8]. When the literature was reviewed, it was seen that the most ideal results were obtained with this algorithm.

Gear optimization is a highly researched field in engineering design. Evolutionary algorithms play a key role in gear optimization, particularly through the use of intelligent techniques such as GA, PSO, ABC, and ACO [7]. In the literature, it has stated that the gears obtained with the optimization approach work quieter [10]. On the other hand, many studies have also been done to reduce gear volume and weights. For example, Marjanovic et. al. [11] has proposed new software to find minimum volume of the gearbox. With the developed software, it is aimed to minimize the existing gearbox. Thompson et al. employed the quasi-Newton minimization method to optimize 2-3 stage gears for minimum volume and surface fatigue cycle [12]. In other study, Zeyveli et al. the minimum volume problems of gearboxes have been applied for two stage gears [13]. Barbieri et al. [14] have been investigated static and dynamic trans-

mission error. They have used micro geometric modification obtained by genetic algorithm for noise reduction. In their research, Rai et al. [15] incorporated additional design constraints, including scoring, alongside bending strength, contact strength, contact ratio, and interference. The results were compared with those obtained using traditional design methods to identify a more effective solution and optimization approach.

This study addressed the gear optimization problem for minimum weight using the Grasshopper Optimization Algorithm (GOA). As a recently introduced meta-heuristic optimization technique, GOA has demonstrated strong performance in engineering applications. It was applied to the design of a spur gear for minimum weight for the first time. The results of this study indicate that GOA can produce lighter gear designs compared to previous approaches.

2. Grasshopper Optimization Algorithm

GOA is an algorithm that was inspired by the behaviour of grasshoppers in nature [6]. It mathematically models and imitates the movement of grasshopper swarms for the solution of optimization problems. Despite generally appearing individually in nature, grasshoppers can create swarms at a continental scale. Millions of them jump and eat almost all vegetation in their path when they are nymph. In this larval phase, the movement of grasshoppers is slow. However, in their adulthood period, they form swarms in the air and travel over large distances. The search process of food source takes place in nature-inspired algorithms in two ways. The process in which the search agents move abruptly is called "exploration". Whereas, in "exploitation", they tend to move locally.

2.1. GOA algorithm

The swarming behaviour of grasshoppers can mathematically be modeled as in Eq. (1) [9]:

$$X_i = S_i + G_i + A_i, \tag{1}$$

where X_i is the location of the *i-th* grasshopper, S_i is the social interaction, G_i is the gravity force on the *i-th* grasshopper, and A_i is the wind advection. The social interaction is defined as:

$$S_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} s\left(d_{ij}\right) d_{ij}, \tag{2}$$

where d_{ij} is the distance between the *i-th* and the *j-th* grasshopper, d_{ij} is the unit vector from the *i-th* grasshopper

to the *j-th* grasshopper, and *s* is a function to denote the strength of social forces and calculated as follows:

$$s(r) = fe^{\frac{-r}{l}} - e^{-r}, \qquad (3)$$

where *f* represents the intensity of attraction and *l* is the attractive length scale. Fig. 1 shows the effect of *s* on the social interaction of grasshoppers. The social behaviour changes with the parameters *f* and *l*.

In this illustration, these parameters are taken 1.5 and 0.5, respectively. There is a repulsion from the beginning until the point 2.079. This point is called the comfort zone as there is neither attraction nor repulsion. After the comfort zone, attraction starts and then it gradually decreases from the point 4.
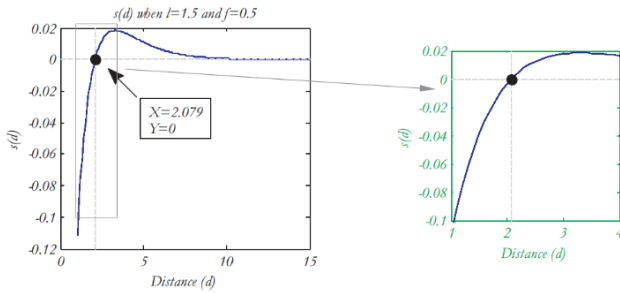

Fig. 1 Function *s* [6]

The gravity force is calculated as follows:

$$G_i = -g e_g , \qquad (4)$$

here *g* is the gravitational constant and e_g is the unit vector towards the center of earth. The wind advection in Eq. (1) is computed as follows:

$$A_i = u e_w , \qquad (5)$$

where *u* represents the constant drift and e_w is the unity vector in the direction of wind. Substituting *S*, *G*, and *A*, Eq. (1) can be rearranged as follows:

$$X_i = \sum_{\substack{j=1 \\ j \neq i}}^{N} s\left(\left|x_j - x_i\right|\right) \frac{x_j - x_i}{d_{ij}} - g e_g + u e_w . \qquad (6)$$

In this model, the grasshoppers rapidly reach their comfort zone and the swarm does not converge to a specified point. Instead, a modified equation is proposed to solve the optimization problem:

$$X_i^d = c\left(\sum_{\substack{j=1 \\ j \neq i}}^{N} c \frac{ub_d - lb_d}{2} s\left(\left|x_j^d - x_i^d\right|\right) \frac{x_j - x_i}{d_{ij}} \right) + T_d , \qquad (7)$$

where ub_d and lb_d are the upper bound and the lower bound in the D_{th} dimension, respectively. T_d denotes the value of the D_{th} dimension in the target grasshopper (best solution found so far), and *c* is a decreasing coefficient to squeeze

the comfort zone, repulsion zone, and attraction zone.

Eq. (7) illustrates that a grasshopper's position is updated based on its current position, the target position, and the positions of all other grasshoppers. To maintain a balance between exploration and exploitation, the coefficient reduces the comfort zone in proportion to the number of iterations:

$$c = c_{max} - l \frac{c_{max} - c_{min}}{L} , \qquad (8)$$

where c_{max} is the maximum value, c_{min} is the minimum value, and their values are defined as 1 and 0.00001, respectively. In the equation, *l* represents the current iteration, and *L* is the maximum number of iterations.

The GOA algorithm's pseudocode is depicted in Algorithm 1. Initially, the GOA generates a random population and calculate solutions using a fitness function. The search agents update their positions and the best solution obtained so far is updated. Also, in each iteration, the coefficient *c* is updated and the distances between grasshoppers are normalized in the range of [1,4]. The position update is repeated until the ending criterion is reached. The position and fitness of the best target are selected as the best approximation for the global optimum solution.

Algorithm 1 Grasshopper Optimization Algorithm
Initialize the swarm $X_i(i = 1, 2, …, n)$
Initialize c_{max}, c_{min} and maximum number of iterations
Calculate the fitness of each search agent
T = the best search agent
while (*l* < *Max number of iterations*)
      Update *c* using Eq. (8)
      for each search agent
          Normalize the distances between grasshopper in [1,4]
          Update the position of the current search agent by the Eq. (7)
          Bring the current search agent back if it goes outside the boundaries
      end for
      update *T* if there is a better solution
      l = *l*+1
end while
return *T*.

2.2. Evaluation function

Various methods have been developed to manage constraints in optimization problems, with penalty functions being the most commonly used approach. In this study, we applied an adaptive penalty method introduced by Yokota et al. [4].

2.3. Penalty function

Penalty function methods transform the constrained optimization problem into an equivalent unconstrained problem. As a general rule, the objective function is added by a term that yields a high cost value for violation of the constraints. There are two different types of methods:

i) Additive Form:

In this method, the penalty function is added to the objective function to define the fitness value:

$$eval(X) = \begin{cases} f(X) & if\ X \in F \\ f(X) + p(X) & otherwise \end{cases}, \qquad (9)$$

where $p(X)$ represent the penalty term and $eval(X)$ is the overall objective function that serves as an evaluation function [16]. If no violation occurs, $p(X)$ becomes zero, and positive otherwise.

ii) Multiplicative Form:

In this form, $p(X)$ is greater than one for a violation case and equal to one, otherwise.

$$eval(X) = \begin{cases} f(X) & if\ X \in F \\ f(X)\,p(X) & otherwise \end{cases}. \qquad (10)$$

3. Optimization of the Design Parameters of a Spur Gear for Minimum Weight

The problem of reducing the weight of single-stage spur gears with heuristic optimization methods was first handled by Yokota et al [4]. The spur gear geometry subject to the problem is given in Fig. 2.
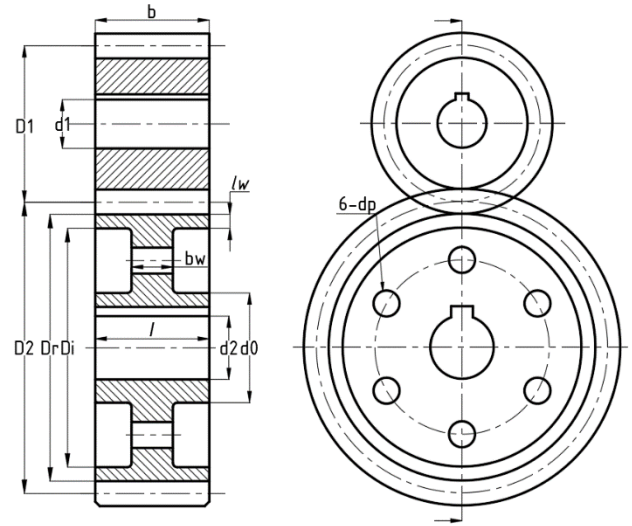

Fig. 2 Parametric spur gear technical drawing [7]

In the problem designed as per the minimum weight target, five nonlinear design variables have been optimized. These are the torsional strength of the shafts, the bending strength and the geometric dimensions of the gears in the system. The design vector defined by Yokota et al. is shown in Eq. (11), and the objective function is shown in Eq. (12).

$$MinF(b, d_1, d_2, Z, m) = W, \qquad (11)$$

$$W = F_{(x)} = \frac{\pi}{4}\frac{\rho}{1000}\left\{bm^2 Z_1^2\left(1+i^2\right) - \left(D_i^2 - d_0^2\right)(l - b_w) - nd_p^2 b_w - \left(d_1^2 + d_2^2\right)b\right\}. \qquad (12)$$

Table 1

Constraints, design parameters and formulations

Constraints		Design variables	
$G1(x) = Fs \geq b_1 \quad G2(x) = \dfrac{F_s}{F_p} \geq b_2 \quad G3(x) = d_1^3 \geq b_3$		$20 \leq b \leq 32 \qquad 10 \leq d_1 \leq 30$	
$G4(x) = d_2^3 \geq b_4 \quad G5(x) = ((1+i)mZ_i)/2 \leq b_5$		$30 \leq d_2 \leq 40 \qquad 18 \leq Z_1 \leq 25$	
		$m = (2.75, 3, 3.5, 4)$	
Formulations			
$D_r = m(iZ_1 - 2.5)$	$l_w = 2.5m$	$b_w = 3.5m$	$d_0 = d_2 + 25$
$D_i = D_r - 2l_w$	$d_p = 0.25\,(D_i - d_0)$	$D_1 = mZ_1$	$D_2 = imZ_1$
$N_2 = N_1/i$	$Z_2 = Z_1 D_2/D_1$	$v = \pi D_1 N_1/60000$	$b_1 = 1000P/v$
$F_p = 2K_v K_w D_1 b Z_2/(Z_1 + Z_2)$	$b_3 = 4.97 \times 10^6 P/(N_1\tau)$	$b_4 = 4.97 \times 10^6 P/(N_2\tau)$	$F_s = \pi K_v K_w \sigma bmy$
Constant values			
$i = 4$	$\rho = 8\,\text{mg/m}^3$	$n = 6$	$K_v = 0.389$
$\sigma = 294.3\,\text{MPa}$	$y = 0.102$	$N_1 = 1500\,\text{rpm}$	$K_w = 0.8$
$b_2 = 0.193$			

Here, b is face width, mm; d_1 and d_2 are diameters of pinion and gear shaft, mm respectively. D_i denotes inside diameter of rim, mm; d_0, d_p and D_r are outside diameters of boss, drilled hole diameter, and dedendum circle diameter, mm; respectively. $F_{(x)}$ represents the objective function; m and i are module and gear transmission ratio of a gear pear, mm. Z_1 and Z_2 show the number of teeth on pinion and gear. b_w and l_w are thickness of web, mm and rim, mm; n is the number of drilled holes and ρ shows the density of gear material, mg/m³. l is length of boss, mm and equal to b. P is the power to be transmitted, kW; τ and σ denote shaft shear strength and gear material strength in MPa. K_v, K_w and y are velocity factor, load factor and Lewis tooth form factor, respectively. N_1 and N_2 are speed of pinion and gear shaft in rpm; v is pitch line velocity, m/s.

4. Results and Discussion

In this research, the right weight design problem of a spur gear described by Yokota *et al.* [4] has been solved with two different penalty function methods mentioned above. First, as in the original problem, the evaluation function is defined as follows [4]:

$$eval(X) = f(X)\left[1 - \frac{1}{q}\sum_{i=1}^{q}\Delta_i\right], \tag{13}$$

here, Δ_i is a measure that is the ratio of the value of violation for constraint i over the value of constraint i.

$$\Delta_i = \begin{cases} 0 & G_i \leq b_i, \quad i = 1, 2, \ldots, q \\ \dfrac{G_i - b_i}{b_i} & otherwise \end{cases} \tag{14}$$

In this method, penalty parameters are used in multiplicative form and updated for every new generation. GOA gave better results compared to Yokota's GA algorithm.

GOA produced a feasible solution that all the constraints have been satisfied at the same time with a minimum weight of 3135.51. The convergence curve related to this solution is depicted in Fig. 3.

The design variable space that satisfies the feasible solution is shown in Fig. 4. Here, the colorbar displays the objective function value for design variables b, d_1 and d_2 in their given ranges, and m is set to 2.75.

As a second approach, we adapted a penalty coefficient P in the calculation of evaluation function. In this case, the evaluation function is described with an additive penalty function as follows:

$$eval(X) = f(X) - P\sum_{i=1}^{q}\Delta_i. \tag{15}$$

Within this interval, the penalty term has arised due to the first constraint. For $P = 100$, we obtained an optimal weight of 2929.74. The change in evaluation function value within this interval of P is given in Fig. 5 and comparison of optimization results are presented in Table 2. In the table, GOA-1 represents optimal solution and the results obtained with a penalty function is given as GOA-2. In the simulations, the population size and the number of iterations were selected as 100 and 500, respectively.

Fig. 3 Objective function for feasible solution

Fig. 4 Design variable space for feasible solution

Fig. 5 Change in evaluation function with P

Table 2

Performance comparison of optimization approaches

Design variables	GA [4]	SA [5]	PSO [5]	GWO [7]	**GOA-1**	**GOA-2**
weight, g;	3512.6	3127.71	3127.70	3094.8626	3135.51	2929.74
b, mm;	24	23.7	23.7	23.9031	23.9179	20
d_1, mm;	30	30	30	30	30	30
d_2, mm;	30	36.761	36.763	30	36.7673	36.7665
$Z1$	18	18	18	18	18	18
m	2.75	2.75	2.75	2.75	2.75	2.75

5. Conclusions

In this research, the optimal weight for the spur gear design was identified using the GOA. The algorithm was applied to the case described in the literature considering the previous studies. A large number of tests were performed, and GOA's performance was compared with the results from previous studies. Ultimately, GOA produced lower weight values than its competitors.

GOA provides fast and easy implementation as it requires few parameters. Furthermore, it is equipped with a varying comfort zone coefficient that properly balances exploration and exploitation process. This coefficient helps the grasshoppers not to stuck in their comfort zone and guides them towards the global optimum. These features are important reasons for GOA to achieve better results.

With this research, GOA was employed for the first time to address the minimum weight design problem of spur gears. When compared to the results of GWO, GA, PSO, and SA, GOA demonstrates significant advantages in optimization methods commonly used for gear weight reduction. In optimizing gears for minimum weight, GOA's most notable advantage over previous studies lies in the optimal value and the breadth of the search space.

The reliability and statistical effectiveness of the GOA demonstrate its suitability for optimizing machine components. Moving forward, we aim to enhance the mathematical model of the problem. In this process, various optimization techniques will be applied to the updated model, and the resulting outcomes will be compared both with each other and with the findings from previous studies.

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H. Gökçe

USING OF GRASSHOPPER OPTIMIZATION ALGORITHM APPROACH FOR OPTIMAL WEIGHT DESIGN PROBLEM OF THE SPUR GEAR

S u m m a r y

Gears are undoubtedly the most important parts of motion transmission. One way to increase the motion performance of the system is to reduce the weights of the gears without sacrificing the strength capability. Today, various optimization techniques that rely heavily on analytical and heuristic approaches are used for this problem. In this study, the gear optimization problem in accordance with the minimum weight was solved with Grasshopper Optimization Algorithm (GOA). Introduced in recent years, GOA is a meta-heuristic optimization technique that stands out with its successful performance in engineering applications. This approach was used in a spur gear formation case as per the minimum weight for the first time. Compared to previous studies, the results obtained in this paper show that it is possible to design a lighter gear with GOA.