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Design optimization of square aluminium damage columns with crashworthiness criteria

H.R. Allahbakhsh*, J. Saemi, M. Hourali*****

**Islamic Azad University, Shahrood Branch, Mechanical department, Shahrood, Iran, E-mail: allahbakhshy@gmail.com*
***Islamic Azad University, Shahrood Branch, Mechanical department, Shahrood, Iran, E-mail: jafarsemi@gmail.com*
****Payame Noor University, Shahrood Branch, Iran, E-mail: hourali@yahoo.com*

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1. Introduction

Thin walled structure are used for crashworthiness design [1, 2]. As a major class of energy-absorbing component, the sectional trusses or frames made of aluminum and its alloys are gaining growing popularity in a range of engineering designs mainly due to its low cost and high weight-stiffness efficiency. Besides, the aluminum materials can be produced to almost any shape by using the extrusion process. For the reasons of these design and manufacturing benefits, more and more new aluminum structural members with increasing complexity of sectional configurations are being introduced to further enhancing the structural integrity and crashworthiness. The importance of fracture in these analyses has been increasingly recognized.

In designing such columns, maximizing their energy-absorption capability should always be a major objective. As presented in previous researches, there are two approaches to enhance the performance of the multicell thin-walled columns: either using advanced materials with high mechanical properties [3, 4] or designing optimized wall thickness and cross-sectional dimensions for such columns that can provide the best crash performances [5].

In the latter, the response surface method (RSM) gains extensive popularity as various computational crashing simulation techniques are established, and its applications in crashworthiness design have been substantially explored by a number of researchers, e.g. Lee et al. [6], Avalle et al. [7], Chiandussi et al. [8], Kim [9], Jansson et al. [10], Lee et al. [11], Shariati et al [12], Forsberg and Nilsson [13, 14]. It is noted that in these earlier studies, exhaustive attention has been paid to such simpler and more conventional thin walled sectional structures as squared or circular tubes [6] and their tapered variations [7, 8].

In this paper, the numerical crushing responses of multicell thin-walled aluminum columns are investigated considering the damage evolution. The numerical crash analyze of tubes was performed using the Abaqus finite element software and was validated by comparing against solution published in literature. To seek for the optimal crashworthiness design a set of designs are selected from the design space using the factorial design, which have different thickness column and side length.

2. Damage criteria

In this study, finite element (FE) models of circular tubes were developed using the nonlinear FE code Abaqus. Metal sheets and thin-walled extrusions made of aluminum alloys may fail due to one or a combination of the following failure mechanisms: ductile failure due to nucleation, growth, and coalescence of voids; shear failure due to fracture within shear bands and failure due to necking instabilities [15]. If the model consists of shell elements, a criterion for the last failure mechanism is necessary because the size of the localized neck is of the order of the sheet thickness and, hence, cannot be resolved with shell elements of dimensions one order of magnitude larger than the thickness.

Abaqus/Explicit offers a number of damage initiation criteria to model the onset of necking instabilities in sheet metals. These include the Forming Limit Diagram (FLD), Forming Limit Stress Diagram (FLSD), Müschenborn-Sonne Forming Limit Diagram (MSFLD), and Marciniak-Kuczynski (M-K) criteria. The first three criteria utilize the experimentally measured forming limit curves in the appropriate strain or stress spaces. The last criterion introduces virtual thickness imperfections in the sheet metal and analyzes the deformation in the imperfection zone to determine the onset of the instability.

The strain-based FLD criterion is limited to applications where the strain path is linear. On the other hand, the stress-based FLSD criterion is relatively insensitive to changes in the strain path. However, this apparent independence of the stress-based limit curve due to the strain path may simply reflect the small sensitivity of the yield stress to changes in the plastic deformation. The M-K criterion can capture the effects of nonlinear strain paths accurately; however, it is computationally expensive, especially if large numbers of imperfection orientations are introduced. It has been verified that the results obtained using the MSFLD criterion are similar to those obtained using the M-K criterion but with a much reduced computational expense. Therefore, in this paper we choose the MSFLD damage initiation criterion for necking instability. For specifying the MSFLD damage initiation criterion, the forming limit curve of the material is required. In Abaqus this criterion can be specified by converting the forming limit curve from the space of major versus minor strains to the space of equivalent plastic strain versus ratio of principal strain rates. Abaqus also allows direct specification of the forming limit curve for the MSFLD criterion. All models in this study are made of aluminum alloy (E = 70 GPa, ν = 0.3, and ρ = 2700 kg/m^3). We use the forming limit curve based on the experimental work of Hooputra [16].

3. Response surface method

Response surface methodology (RSM) is a method for understanding the correlation between multiple input variables and one output variable. In this approach, an approximation $\tilde{y}(x)$ to the response of the aluminium columns is assumed a series of the basic functions in a form of

$$\tilde{y}(x)=\sum_{j=1}^{N}a_j\varphi_j(x) \tag{1}$$

where N represents number of basis function $\varphi_i(x)$, $x\in R^n$ A typical class of basis functions is the polynomials, for instances, whose full quartic form is given as

$$\left.\begin{array}{ll}\tilde{y}=a_0+a_1x_1+a_2x_2+\cdots+ & \text{linear terms}\\ +a_{12}x_1x_2+a_{13}x_1x_3+\ \cdots + & \text{interaction terms}\\ +a_{11}x_1^{\ 2}+a_{22}x_2^{\ 2}+\cdots + & \text{quadratic terms}\\ \vdots & \\ +a_{111}x_1^{\ 3}+a_{222}x_2^{\ 3}+\cdots+ & \text{cubic terms}\\ \vdots & \\ +a_{1111}x_1^{\ 4}+a_{2222}x_2^{\ 4}+\cdots & \text{quartic terms}\end{array}\right\} \tag{2}$$

To determine the regression coefficient $a=(a_1,a_2,...,a_N)$ in Eq. (2), a large number of FE analyses $y^{(i)}(i=1,2,...,M)$ are needed $(M\rangle N)$. The method of least-square can be used to determine the regression coefficient vector ***a*** by minimizing the errors between the FE analysis ***y*** and the response function $\tilde{y}$. The least squares function can be expressed as

$$E(a)=\sum_{i=1}^{M}\varepsilon_i^2=\sum_{i=1}^{M}\left[y^{(i)}-\sum_{j=1}^{N}a_j\varphi_j\left(x^{(i)}\right)\right]^2 \tag{3}$$

The regression coefficient vector $a=(a_1,a_2,...,a_N)$ can be evaluate by $\frac{\partial E(a)}{\partial x}$, which is

$$a=\left(\Phi^T\Phi\right)^{-1}\left(\Phi^T y\right) \tag{4}$$

where matrix Φ denotes the values of basis functions evaluated at these M sampling points, which is

$$\Phi=\begin{bmatrix}\varphi_1\left(x^{(1)}\right) & \cdots & \varphi_N\left(x^{(1)}\right)\\ \vdots & \ddots & \vdots\\ \varphi_1\left(x^{(M)}\right) & \cdots & \varphi_N\left(x^{(M)}\right)\end{bmatrix} \tag{5}$$

By substituting Eq. (5) into Eq. (1), the RS model can be fully defined. The numerical errors in the RS model can be measured using several criteria. The relative error (*RE*) between the response surface established and the FEA solution $y(x)$ is

$$RE=\frac{\tilde{y}(x)-y(x)}{y(x)} \tag{6}$$

The sum of squares of the residuals (*SSE*) and the total sum of squares (*SST*) are two important properties in evaluating the model's accuracy

$$SSE=\sum_{i=1}^{M}\left(y_i-\tilde{y}_i\right)^2 \tag{7}$$

$$SST=\sum_{i=1}^{M}\left(y_i-\overline{y}_i\right)^2 \tag{8}$$

where $\overline{y}_i$ is the mean value of FEA result y_i.

The typical statistical parameters used for evaluating the model fitness are the F statistic, coefficient of multiple determination R^2, adjusted $R^2{}_{adj}$ statistic and root mean squared error (*RMSE*), respectively, as

$$R^2=1-SSE/SST \tag{9}$$

$$F=\frac{\left(SST-SSE\right)/P}{SSE/\left(M-P-1\right)} \tag{10}$$

$$R_{adj}^2=1-\left(1-R^2\right)\frac{M-1}{M-P-1} \tag{11}$$

$$RMSE=\sqrt{\frac{SSE}{M-P-1}} \tag{12}$$

where P is the number of nonconstant terms in the *RS* model. It should be pointed out that, however, these measures may not be completely independent each other and there may be some interconnections. In general the larger the values of R^2 and $R^2{}_{adj}$ and the smaller the value of *RMSE*, is better fitness [17].

4. Problem description

The crashworthiness of the aluminum columns is expressed in terms of specific energy absorption (*SEA*). The *SEA* is defined as

$$SEA=\frac{Total\ energy\ absorotion\ E_{total}}{Total\ structural\ weight} \tag{13}$$

Two factors have to be study during this design. At first, based on the human safety issues, the peak load P_m that occurs during the crash should not be greater than a certain criteria, which is an important issue in crashworthiness. Also, the two design variables of the optimized aluminum columns, its side's length and thickness (Fig. 1), only vary between their upper and lower bounds. Thus, this optimization problem is formulated as

$$\begin{cases} & Maximaize: y=SEA(x)\\ So\ that & Max\ \ PL(x)\le 70\\ & x^L\le x\le x^U\end{cases} \tag{14}$$

where $PL(x)$ is the response polynomial function of peak crushing force. $x=(x_1,x_2,\ldots,x_k)$ indicate the vector of k design variables of the aluminum columns.

$x^L = \left(x_1^L, x_2^L, ..., x_k^L\right)$ and $x^U = \left(x_1^U, x_2^U, ..., x_k^U\right)$ are the lower and upper bounds of the design variables, respectively.

5. FE models and crashworthiness analysis

FE models are created for aluminum columns and they are used for the crashworthiness analyses. For the two continuous variables (*a, t*) the factorial design method was adopted in design of experiments (DOE). FEA results of *SEA* and the maximum crushing force P_m are acquired from the analyses and will later be used for constructing corresponding RS models. The structures considered in this study include the two square thin-walled columns. The side length *a* of the cross-sections and the thickness *t* of the thin wall are chosen as design variables, and the constraints of these two design parameters are given as $40 \le a \le 60$, $1 \le t \le 3$ millimetre. The effects of these parameters on the following response of the aluminium column evaluate for crashing. In this work, the lengths *L* of the aluminium column structures are a constant of 200 mm. The square thin walled configurations with the S1 and S2 sections as shown in Fig. 1, respectively.

Columns were nominated as follows: 40-40-1-S1. The numbers following show the side length and the thickness of S1 column are 40 mm and 1 mm, respectively.

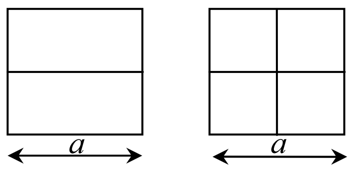


Fig. 1 Cross-sections of square thin walled columns. a) S1, b) S2

For validation of FEA, deformation mode and load-deformation curve are of interest. Fig. 2 shows the comparison of from the present simulations with experimental and theoretical results [6].

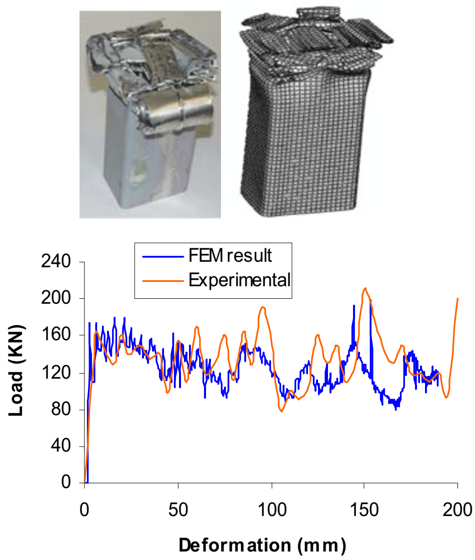


Fig. 2 Comparison of the experimental and numerical results

Figs. 3-5 shows the deformation modes and load-deformation curves for square cross-section columns.

It can be seen that in Fig. 3 the peak crushing force and the energy absorbed for S2 is more than S1. Also Fig. 4 shows that with increasing *t* the peak crushing force and the energy absorbed decrease. Fig. 5 indicate that with increasing side length the peak force increases but the load deformation will not change considerably.

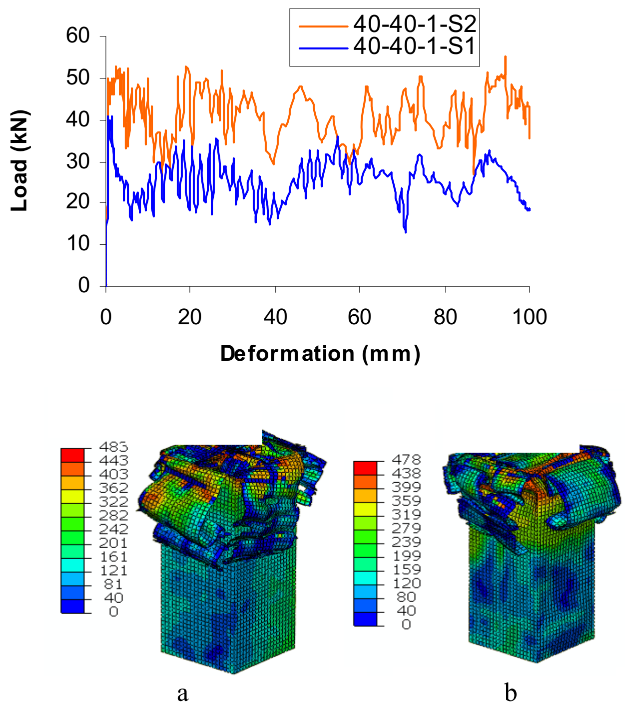


Fig. 3 Plots of load-deformation the shell deformations and the von Mises stress (MPa) for a) 40-40-1-S2 and b) 40-40-1-S1

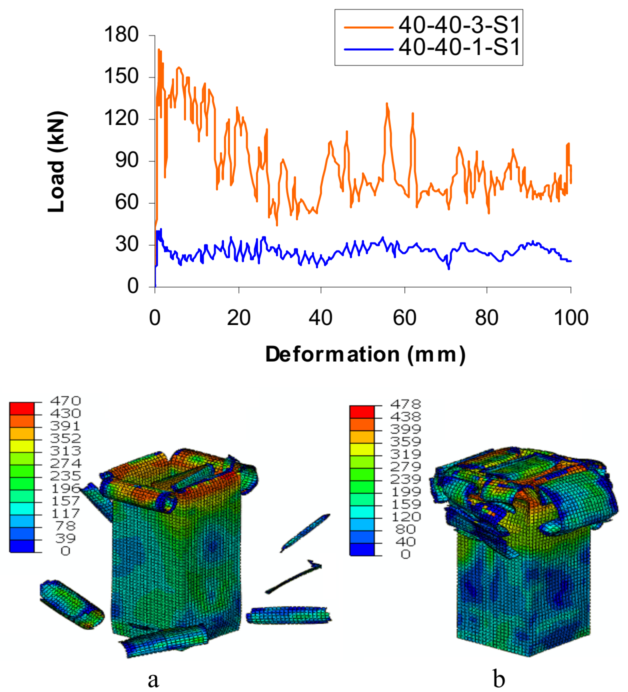
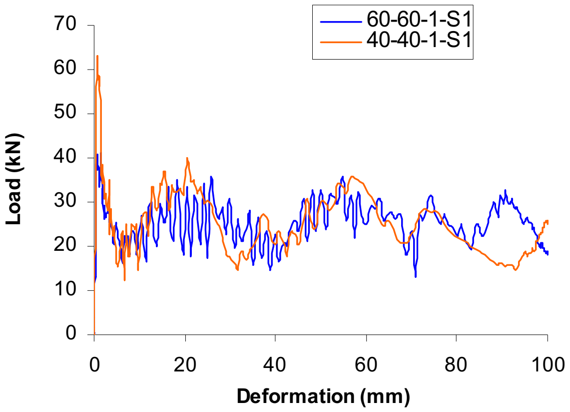
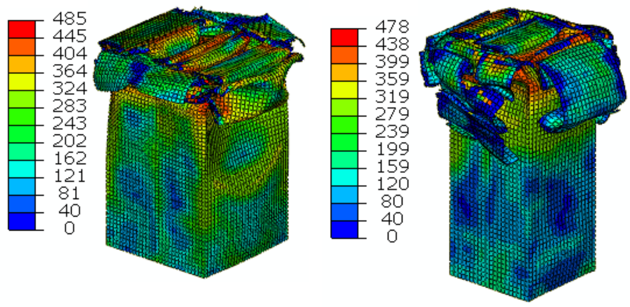


Fig. 4 Plots of load-deformation the shell deformations and the von Mises stress (MPa) for a) 40-40-3-S1 and b) 40-40-1-S1



a



a b

Fig. 5 Plots of load-deformation the shell deformations and the von Mises stress (MPa) for a) 60-60-1-S1 and b) 40-40-1-S1

6. Results of design optimization

In this section, the RS models are constructed based on the FEA results. In order to validate the set of design points and the orders of polynomials the different polynomial RS models are constructed, and then evaluated their accuracies using Eqs. (6) - (12). The results of approximations are summarized in Table.1. Since the larger values of R^2 and R^2_{adj} and the smaller values of *RE* and *RMSE* indicate a better fitness of the *RS* models, it is found that compared to other response functions the quartic polynomial functions provide the best approximation on the column's responses and therefore should be used for optimum design. As a result of the least square procedure, the quartic response functions of *SEA* and Max PL foe S1 and S2 are, respectively, given as

$$\begin{aligned} MAX\ \ PL(t,a) &= -220.294 + +4430.691t - 1100.737a - \\ &-13695.754t^2 + 438.994at + 1323.832a^2 + 15319.991t^3 + \\ &+3359.999t^2a - 1654.857ta^2 - 490.239a^3 - 6266.662t^4 - \\ &-2079.999t^3a - 45.714a^2t^2 + 428.799ta^3 + 57.173a^4 \end{aligned} \quad (15)$$

$$\begin{aligned} MAX\ \ PL(t,a) &= -4696.73 + 43631.84t - 2034.47a + \\ &+130097.52t^2 - 1308.51at + 2509.02a^2 + 172200.0t^3 + \\ &+3192.4863t^2a - 470.462ta^2 - 1092.58a^3 - 87600.02t^4 + \\ &+1493.335t^3a - 1756.73a^2t^2 + 539.73ta^3 + 137.17a^4 \end{aligned} \quad (16)$$

$$\begin{aligned} SEA(t,a) &= 245.63 - 348.38t - 347.15a + 35.27t^2 + \\ &+610.855at + 186.725a^2 - 1053.33t^3 + 601.632t^2a - \\ &-532.408ta^2 - 17.599a^3 + 1466.668t^4 - 906.666t^3a + \\ &+231.836a^2t^2 + 59.733ta^3 - 1.92a^4 \end{aligned} \quad (17)$$

$$\begin{aligned} SEA(t,a) &= -4945.41 + 44139.86t - 920.45a - \\ &-132419.27t^2 - 1646.19at + 1196.17a^2 + 176073.39t^3 + \\ &+2171.58t^2a + 654.75ta^2 - 590.45a^3 - 87800.03t^4 - \\ &-133.33t^3a - 737.95a^2t^2 + 21.33ta^3 + 93.119a^4 \end{aligned} \quad (18)$$

The RS of SEA and peak force are shown in Fig. 6 respectively. It can be seen that in Fig. 6 with increasing *t* and decreasing *a*, the SEA increases and with increasing *t* and *a* the peak force increases. The results of approximations are shown in Table 2 for aluminium columns. The optimal results can be acquired using the nonlinear programming (fmincon), which is provided by MATLAB. "fmincon" attempts to find a constrained minimum of a scalar function of several variables starting at an initial estimate [18] . The optimization results are summarized in Table 2.

Table 1
Accuracy of different polynomial RS models for spot welded columns

RS model	R^2	R^2_{adj}	RMSE	RE interval (%)
Quadratic polynomial	0.9989	0.998	0.0099	[-2.1, 2.5]
Cubic polynomial	0.9991	0.998	0.0093	[-0.9 , 1.3]
Quartic polynomial	0.9998	0.999	0.0012	[-0.4 , 0.9]

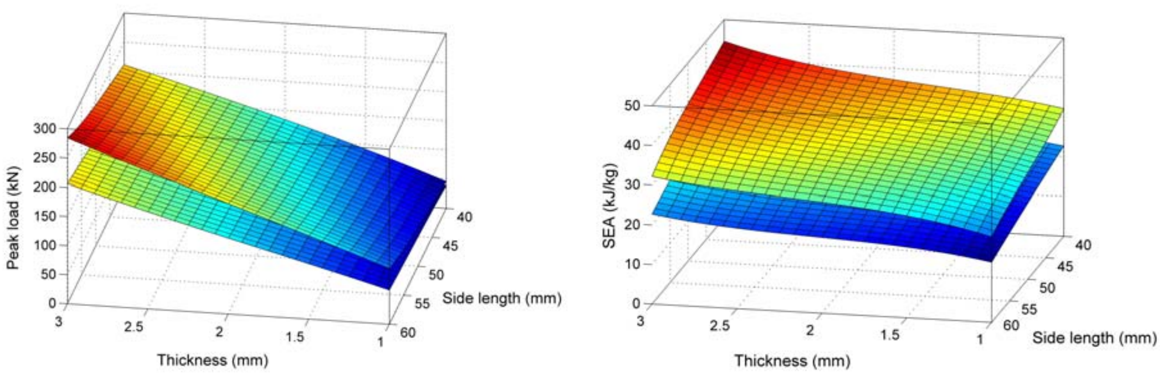


Fig. 6 Response surface of SEA and Peak force for the aluminium columns

Table 2

Optimal square hat section designs

Aluminum column	Optimal design variables, mm	SEA, kJ/kg		Peak force, kN	
		RSM	FEM	RSM	FEM
S1	a=40 , t=1.41	24.63	25.5	70	69.5
S2	a=40 , t=1.25	33.78	33.5	70	70

From Table 2 it can be concluded that for both S1 and S2 columns with square sections, the S2 is the more specific energy the column absorbs when impact occurs. In order to increase the energy-absorption capability, the columns should have minimum side length.

7. Conclusions

This paper presents the crashworthiness design for thin-walled aluminum columns, including the S1 and S2 columns with damage criteria. The optimal S1 and S2 cross-sections are obtained, which provide the best energy-absorption capability during the crashworthiness analyses. During the optimum design the specific energy absorption (SEA) is set as the design objective, which represents the structure's capacity of absorbing the crash energy. The cross-sectional width a and the wall thickness t are selected as two design variables, and the highest crushing force that occurs during the analyses is set as the design constraint. FEA, five-level full factorial design and RSM are employed in this study to formulate the optimum design problems and the optimal designs are finally solved from the derived RS. In this project, Abaqus is used to create the FE model and perform the crashworthiness analyses to provide crash responses of the design samples.

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H.R. Allahbaksh, J. Saemi, M. Hourali

KVADRATINIO SKERSPJUVIO ALIUMINIO KOLONŲ PAŽEIDIMŲ PROJEKTAVIMO OPTIMIZAVIMAS PAGAL ATSPARUMO SMŪGIUI KRITERIJUS

R e z i u m ė

Straipsnyje nagrinėjami aliuminio lydinių deformavimo ir pažeidimo procesai, esant smūginiams krūviams. Skaitinei analizei panaudota ABAQUS programa. Vėliau štampuotų aliuminio dirbinių suirimo procesas buvo ištirtas eksperimentiškai. Galiausiai, ieškant efektyvesnio ir lengvesnio smūgio absorbavimo ir minimalaus smūgio jėgos maksimumo kvadratiniams štampuotiems aliuminio vamzdžiams optimizuoti buvo pritaikyta paviršiaus pasipriešinimo metodologija.

H.R. Allahbakhsh, J. Saemi, M. Hourali

DESIGN OPTIMIZATION OF SQUARE ALUMINIUM DAMAGE COLUMNS WITH CRASHWORTHINESS CRITERIA

S u m m a r y

This paper studied the deformation and damage behaviors of aluminum-alloy under crushing loadings. The numerical analysis is carried out by Abaqus software. subsequently, the collapse behavior of aluminium extrusion damage was experimentally characterized. Finally in order to find more efficient and lighter crush absorber and achieving minimum peak crushing force, response surface methodology (RSM) has been applied for optimizing the square aluminium extrusion tube.

Х.Р. Аллахбаксх, Ю. Саеми, М. Хоурали

ОПТИМИЗАЦИЯ ПРОЕКТИРОВАНИЯ АЛЮМИНИЕВЫХ КОЛОН КВАДРАТНОГО СЕЧЕНИЯ ПО КРИТЕРИЯМ УДАРНОЙ ПРОЧНОСТИ

Р е з ю м е

В статье рассматриваются процессы деформирования и повреждения алюминиевых сплавов при ударном нагружении. Для численного анализа использована программа ABAQUS, после чего процесс разрушения алюминиевых изделий был исследован экспериментально. Наконец для определения более эффективного и облегченного абсорбирования удара и минимального максимума силы удара при оптимизации штампованных алюминиевых труб квадратного сечения была использована методология сопротивления поверхности.

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