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Multiaxial Fatigue Criterion Using Total Strain Energy Parameters Associated with Cumulative Damage Model

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1. Introduction

The invulnerability and enduringness of structures in service has become as urgent a necessity as it was in the past owing to the sudden failure and damage of complex systems such as for example but not limited aircraft, automobiles, nuclear power plants, and tanks under pressure loads can lead a great deal of damage, significant financial losses and also cause environmental and ecosystem damage. As several of these structures are subjected to repeated complex loadings as multiaxial fatigue, the study of these type loadings becomes one of the fundamental issues in designing of these structures. In general, the loads applied are frequently complex, which corresponds to non-proportional principal stresses or that abruptly changed direction in a cycle of loading. It is highly difficult to specify the fatigue behaviour of structures and materials beneath such loadings. In fact, damage of a material is defined as a modification of its physical and mechanical properties, that is to say the degradation which accompanies a solicitation either monotonous or variable over time (loading responsible for the majority of failure of mechanical systems. Thereupon, the geometric complexity and/or different load gathering for a lot of structures and components often result in a multiaxial stress state. Predicting the fatigue life of these structures and components become a great defy for researchers and designers. Multiaxial fatigue has been the subject of intensive research for further than half a century. However, no theory of multiaxial fatigue is universally accepted. Markedly, fatigue test campaigns with a high number of cycles under multiaxial loads were born very early, but only became widespread in the mid-1930s

Fig. 1 schematically summarizes the different types of stress, the experimental devices used have first solicited shafts in combined torsion-bending, a principle which is still used today [1, 2], then tubular specimens in tension with internal pressure [3] and/or torsion and more recently cruciform specimens due to with two or four cylinders [4].

Bibliography lists more than 45 multiaxial fatigue criteria. In the early 1950s, a usually function denoted by E is used to developed a significant number of these criteria; this function establishes a parameter, which is a function of

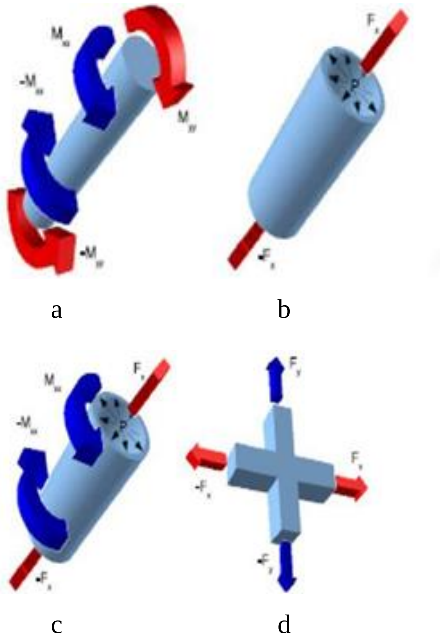


Fig. 1 Types of loading used in multiaxial fatigue: a – torsion-bending, b – tension-internal pressure, c – tension-torsion-internal pressure and d – biaxial tension

the type of solicitation and fatigue properties of studied materials. The service life N for a multiaxial loading state $[\sigma_{ij}(t)]$ is estimated when the fatigue function of the criterion E equal to unity (E=1) [5].

All the multiaxial fatigue criteria are divided into three distinct approaches which differ in their concept. The most used of these criteria is the first approach is called empirical criteria, where the first studies relating to the formulation of multiaxial fatigue criteria were purely empirical. The second approach is called comprehensive approach. It brings together, among other things, the criteria involving invariant of the stress tensor or its deviator. The third approach, critical plane type, where the fatigue analysis using the notion of critical plane is very effective, because the concept of the critical plane is based on the mode of failure or the mechanism of initiation of cracks.

Plastic strain energy concept has been applied to match multiaxial fatigue strength, particularly in the low-cycle domain [6-9]. Likewise, there are also been a number of researches [10-11], in which the elastic strain energy is combined to the plastic strain energy to proceed with the multiaxial fatigue of the polycyclic. The criterion developed by Froustey and Lasserre is a specific energy approach allowing taking into account the little influence of the phase shift in bending and torsion combined to the resistance to polycyclic fatigue [12]. Moreover, Palin luc [13] proposes a criterion on the basis of the works of Froustey and Lasserre, the author defines two quantities which he compares to locate the multiaxial cycle compared to the limit of fatigue of the material.

Ellyin assumed as a starting point that the damage to a material is the result of plastic strain energy over a loading cycle. The author proposed this criterion in the first place for the low-cycle domain, and then this author made an extension to fatigue with a bulky number of cycles. Ellyin proposes to link the strain energy density to the lifetime of metals in multiaxial fatigue [14]. Other works take up this criterion in its initial form [15-18], so these works are based on the assumption that the plastic deformation is nil for fatigue stresses with a high number of cycles [19, 20]. As well as, Garud [21] proposes a multiaxial fatigue criterion in the low-cycle domain. The author assumes in his proposal that the plastic strain energy is the most influencing damage parameter on the initiation and propagation of fatigue loading cracking. Contrary to what Garud assumes, Glinka considers that the two energy parameters (elastic and plastic) respectively associated with the normal and shear strains on the critical plane are the main reasons for the fatigue crack [22].

Hence, in 1999, Macha proposed a multiaxial fatigue criterion valid in the polycyclic field, this criterion is founded on the shear strain energy parameter calculated on a critical plane, and it assumes that this energy is the cause of fatigue cracking [23]. Jing et al [24] proposed a strain energy density model established on the concept of critical plane to estimate the life of metals subjected to multiaxial fatigue loading, principally for nonproportional loadings. Their method is based on the normal and shear strain energy densities on maximum principal strain range plane. Means of the probability density functions of the fatigue error advance a correlative analysis of the capableness of the aforementioned methods to estimate the fatigue life in notched bars under proportional bending-torsion [24]. Furthermore, in author’s previous study [25-26] a cumulative strain energy density, as well named fatigue toughness, was applied to estimate the fatigue life of notched members under multiaxial loading.

2. Proposed criterion

Strain energy variation is extensively used in plasticity approach and is also suggested as a model for uniaxial fatigue analysis [5, 27]. The pertinence of this model for the explanation of fatigue processes seems to be promising, particularly in materials subjected to random thermomechanical loading. The model does not contain a strain energy density division into the elastic and plastic parts, as in the event of the parameters suggested by Smith-Watson-Topper (SWT) [28], Hoffman and Seeger [29], Bergman and Seeger [30]. The strain energy is determined from the following equation:

$$w = \frac{1}{2}\sigma\varepsilon . \tag{1}$$

As a function of time, this strain energy is expressed by:

$$w(t) = \frac{1}{2}\sigma(t)\varepsilon(t) . \tag{2}$$

Under maximum stresses and strains and, respectively, the strain energy is written:

$$w_a = \frac{1}{2}\sigma_a\varepsilon_a . \tag{3}$$

The Manson-Coffin-Basquin equation allows us to calculate the strain as follow:

$$\varepsilon_a = \varepsilon_a^e + \varepsilon_a^p = \frac{\sigma_f^{'}}{E}\left(2N_f\right)^b + \varepsilon_f^{'}\left(2N_f\right)^c . \tag{4}$$

Substituting Eq. (4) in Eq. (3) we obtain:

$$w_a = \frac{1}{2}\sigma_a\left[\frac{\sigma_f^{'}}{E}\left(2N_f\right)^b + \varepsilon_f^{'}\left(2N_f\right)^c\right]. \tag{5}$$

One can express Basquin’s law as follows:

$$\sigma_a = \sigma_f^e\left(2N_f\right)^b , \tag{6}$$

$$w_a = \frac{1}{2}\left[\frac{\left(\sigma_f^{'}\right)^2}{E}\left(2N_f\right)^{2b} + \sigma_f^{'}\varepsilon_f^e\left(2N_f\right)^{b+c}\right]. \tag{7}$$

2.1. Low cycle fatigue case

The low-cycle fatigue dovetail to high stresses, greater than elastic limit. Hence, momentous plastic deformation causes frequently a fracture. The nonlinear comportment of materials under cyclic uniaxial loading (hysteresis loop) can be expressed by the Ramberg-Osgood equation [31].

$$\varepsilon_t = \varepsilon_e + \varepsilon_p = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{\frac{1}{n'}} . \tag{8}$$

Here n' and K are the material constants of the cyclic consolidation curve and if Eq. (8) is considered, the following equation determines these constants [32, 33]:

$$\left.\begin{aligned} n &= \frac{b}{c} \\ K &= \frac{\sigma}{(\varepsilon)} \end{aligned}\right\} . \tag{9}$$

The authors in literature use various methods to calculate the strain energy due to a post elastic limit stress. The criterion proposed by Molski and Glinka can be cited [34-36]:

$$W_t = W_e + W_p = \frac{\sigma^2}{2E} + \frac{2\sigma}{n'+1}\left(\frac{\sigma}{K}\right)^{\frac{1}{n'}}. \quad (10)$$

2.2. Polycyclic fatigue case

Under lower loads (fatigue at a high number of cycles), the plastic part of strain energy is neglected and Eq. (7) becomes:

$$w_a = \frac{1}{2}\left[\frac{\left(\sigma_f^{'}\right)^2}{E}\left(2N_f\right)\right]. \quad (11)$$

By setting:

$k = \dfrac{\sigma_f{}'^2}{2E}$ and $c' = 2b$, the Eq. (11) becomes:

$$w = k(2N). \quad (12)$$

2.3. Case of multiaxial fatigue

The proposed model is generalized in the case of a multiaxial stress state. It is established on the analysis of the stresses and the corresponding deformations in a critical plane, taking into account their signs. This model is developed on the basis of the authors' considerations [37-39]. The origin of fatigue cracking is the part of strain energy density opposite to the normal stress $\sigma_n(t)$ over the normal strain $\varepsilon(t)$, i.e. $W_n(t)$ and the work of stress $\tau(t)$ on the shear strain $\varepsilon_{ns}(t) = \frac{1}{2}\gamma_{ns}$ in the direction $\vec{S}$ on the plane with the normal $\vec{\eta}$, i.e. $W_{ns}(t)$ (Fig. 2), [40].

The model in the multiaxial case is formulated on the following assumptions.

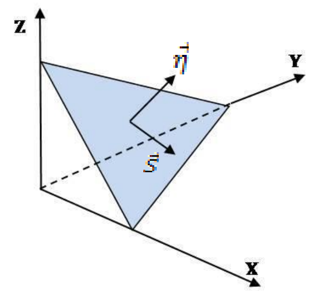


Fig 2 Vector of critical plane in normal $\vec{\eta}$ and tangential/shear direction $\vec{S}$

The $\vec{S}$ direction on the critical plane occurs with the maximum shear mean direction of the strain energy density $W_{ns,max}(t)$.

At the state limit, force in material is defined by the maximum value of the linear association of the energy parameters $W_n(t)$ and $W_{ns}(t)$, where the strain energy subjected to multiaxial loading fulfils the conditions of the next equation:

$$\frac{\max_t\left[\alpha W_{ns}(t) + \beta W_n(t)\right]}{\gamma} = 1, \quad (13)$$

with α, β and γ which are material constants determined from uniaxial fatigue tests. From the Eq. (13), equivalent strain energy density $W_{eq}(t)$ can be obtained as follows:

$$W_{eq}(t) = \alpha W_{ns}(t) + \beta W_n(t), \quad (14)$$

$$\left.\begin{aligned} W_{ns}(t) &= \frac{1}{2}\tau_{ns}(t)\varepsilon_{ns}(t)\,sgn\left[\tau_{ns}(t),\varepsilon_{ns}(t)\right](t) \\ W_n(t) &= \frac{1}{2}\sigma_n(t)\varepsilon_n(t)\,sgn\left[\sigma_n(t),\varepsilon_n(t)\right] \end{aligned}\right\}, \quad (15)$$

$sgn[X,Y]$ is defined by:

$$sgn[X,Y] = \frac{sgn(X) + sgn(Y)}{2} = \begin{cases} 1 & \text{for } sgn(X) = sgn(Y) = 1 \\ 0 & \text{for } sgn(X) = -sgn(Y) \\ -1 & \text{for } sgn(X) = sgn(Y) = -1 \end{cases}. \quad (16)$$

2.4. Damage variable

If one notes, the damage by the variable, one can define a virgin state of the material (the part was never requested by cycles of constraints when $D = 0$) and a broken state when $D = 1$. After n loading cycles, the following expression gives the variable by the expression [5, 27]:

$$D_i = \frac{W_{edi} - W_i}{W_u - W_i}. \quad (17)$$

Here: W_{edi} is the strain energy owing to the damaged stress, W_i is the strain energy owing to applied stress, W_u is the strain energy owing to the ultimate stress.

Fig. 3 shows the algorithm of the damage accumulation and estimation of the life by the proposed method. Additionally, in this study, the Wohler curve of the bending test ($\sigma - N$) was used to determine the same curve with the energy parameters ($W - N$). In other words, to estimate the life of multiaxial fatigue tests (bending-torsion) using this algorithm, knowledge of the Wohler curve in bending is sufficient.

3. Results and discussions

The validation of the proposed method for multiaxial fatigue analysis has been obtained by the use of bending-torsion tests with proportional and nonproportional loading status as experimental data extracted from the literature [41-43]. Furthermore, Figs. 4, 5 and 6 evinces the variation of the total strain energy of 6082-T6 Al alloy, 30NCD16 steel and SM45C steel respectively in terms of the cycle's number and these curves are represented by the Eq. (12). As can be seen in these figures the growth of the

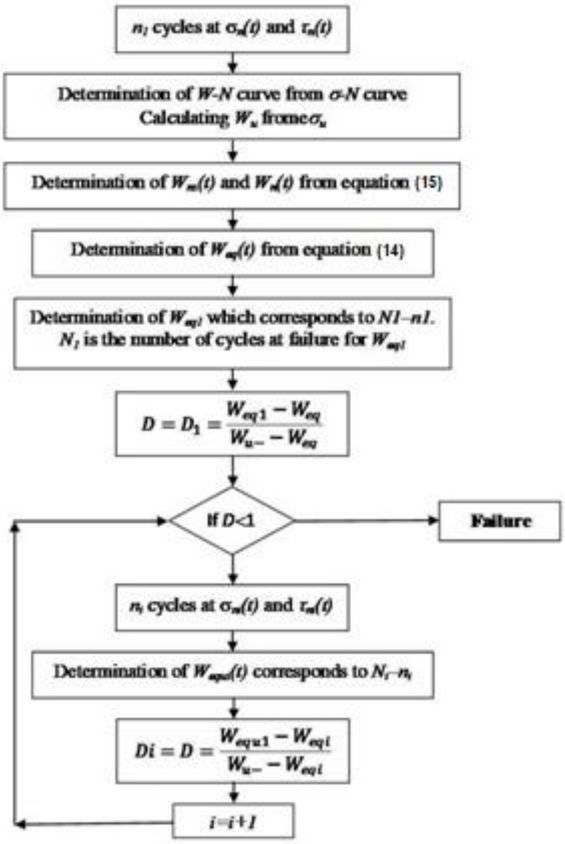


Fig. 3 Algorithm of the damage accumulation and estimation of the life by total strain energy

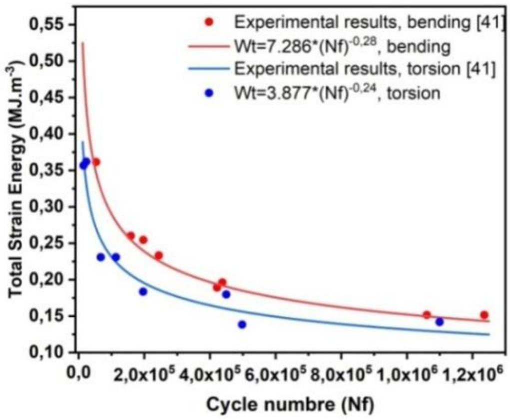


Fig. 4 Cyclic strain energy release rate W_t versus number of cycles for 6082-T6 Al alloy

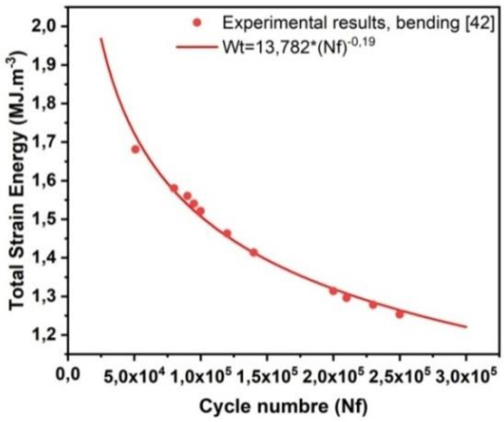


Fig. 5 Cyclic strain energy release rate W_t versus number of cycles for 30NCD16 steel

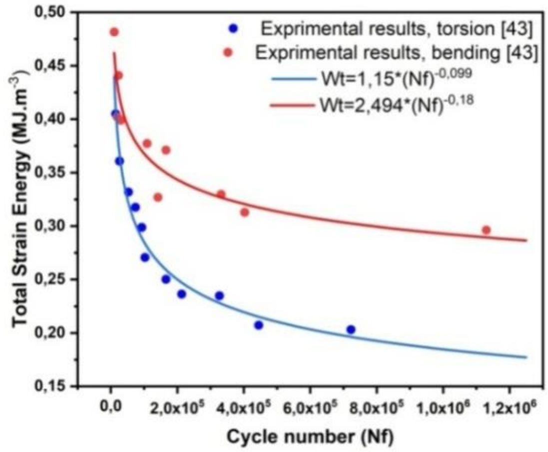


Fig. 6 Cyclic strain energy release rate W_t versus number of cycles for SM45C steel

total strain energy corresponding to the cycle's number is strongly nonlinear.

Three different loading trajectories for multiaxial fatigue, i.e., proportional loading (in phase loading) and non-proportional loading (angles out of phase loading) with and without mean stress as can be seen in Fig. 7 in order to see how well this model can handle complex loads in strain energy terms.

Likewise, the equivalent strain energy density is calculated by Eq. (14) also represented in this figure.

REP % (relative error of prediction) of total lifetime compared to the experimental results is used in this paper for the evaluation of the performances of the model. The following expression defines this error:

$$REP(\%) = \frac{\text{Experimental value-Estimated value}}{\text{Experimental value}} \times 100 .$$

A damage model could be considered as a good precision if the relative error REP of its forecast remains lower than 20% in absolute value [5]. This value of 20% takes into account systematic or accidental errors that may come from the measurements of certain parameters from curves.

A model is called:

- conservative if *REP* (%) > 0, the values of its predictions are lower than the experimental values; the law provides some security
- non-conservative if *REP* (%) < 0, the forecast values are higher than the experimental values; in this case, the model does not guarantee safety.

Tables 1-3 list the estimated life from the proposal compared with the estimates supplied by Mamiya et al. [44] and calculated *REP* (%). These tables group the loading details for every material studied in this investigation cycle's number causing failure. The *REP* (%) are presented in Figs. 8, 9 and 10. Therefore, from these tables and figures one can see that 56% of the results are lower than 20% (absolute value of *REP* (%)). These first observations allowed to say that the proposed model gives good precision.

Fig 11. represent the fatigue life predictions of proposed criterion for the materials studied namely SM45C steel, 30NCD16 and Al6082.

Table 1

Multiaxial loading conditions for bending-torsion tests and lives estimated by the proposed criteria for 6082-T6 Al alloy

Test No	$\sigma_{x,a}$, MPa	$\tau_{xy,a}$, MPa	$\sigma_{x,m}$, MPa	$\tau_{xy,m}$, MPa	ϕ	N_f^{exp} [41]	N_f^{pre} [44]	*REP*,% [44]	N_f^{pre} model	*REP*,% model
1	14	138	1	0	0	14695	18233	-24,08	17134	-16,60
2	18	139	3	0	0	23052	17062	25,98	15653	32,10
3	15	111	1	0	0	67690	103132	-52,36	104596	-54,52
4	16	111	1	0	0	113455	102789	9,40	103854	8,46
5	13	99	3	0	0	196555	250130	-27,26	269899	-37,31
6	24	98	0	0	0	449997	264661	41,19	268985	40,23
7	15	86	2	1	0	497990	783051	-57,24	431540	13,34
8	15	87	1	0	0	1100000	714475	35,05	1243750	-13,07
9	224	4	-1	0	0	52990	30976	41,54	50374	4,94
10	190	5	0	7	0	159000	115190	27,55	155078	2,47
11	188	4	-1	0	0	197275	125737	36,26	176870	10,34
12	180	4	-4	-1	0	244403	178018	27,16	270909	-10,85
13	162	3	0	1	0	421560	414608	1,65	482271	-14,40
14	165	4	-2	1	0	437636	356906	18,45	461253	-5,40
15	145	4	-1	1	0	1060730	1002259	5,51	1090790	-2,83
16	145	4	-1	0	0	1235690	1002259	18,89	1093360	11,52
17	70	118	-3	0	0	71255	41536	41,71	69435	2,55
18	71	117	-1	1	1	78730	43608	44,61	64678	17,85
19	59	100	-1	1	-7	230750	157027	31,95	272670	-18,17
20	61	98	0	0	-18	516985	176164	65,92	320940	37,92
21	53	83	-1	1	-2	1018780	650462	36,15	1219823	-19,73
22	52	82	-2	0	2	1289550	721503	44,05	1523234	-18,12
23	79	129	-1	1	129	20730	19794	4,52	24109	-16,30
24	79	116	-4	0	125	41490	41762	-0,66	64256	-54,87
25	69	110	1	0	126	188882	69364	63,28	107310	43,19
26	68	99	2	0	128	234725	147180	37,30	225650	3,87
27	68	99	2	0	125	368080	147180	60,01	225650	38,70
28	60	94	3	0	126	1016280	240218	76,36	404850	60,16
29	147	106	-2	1	-4	31000	21036	32,14	17956	42,08
30	151	104	-4	0	-3	64090	21139	67,02	18992	70,37
31	163	81	-2	0	-5	124460	43067	65,40	146606	-17,79
32	147	90	1	-1	-8	132215	44370	66,44	78404	40,70
33	146	76	-3	-1	-6	232370	88434	61,94	316267	-36,10
34	118	82	-3	1	-5	315795	145991	53,77	343860	-8,89
35	119	72	1	-1	0	694062	253260	63,51	780490	-12,45
36	188	106	0	0	89	5590	8656	-54,85	7617	-36,26
37	189	106	-5	0	94	27420	8470	69,11	34876	-27,19
38	189	106	1	0	88	34015	8470	75,10	28156	17,22
39	171	99	-4	1	91	44750	16650	62,79	53270	-19,04
40	190	105	-4	0	91	47020	8595	81,72	39644	15,69
41	149	68	0	0	93	114845	116499	-1,44	157715	-37,33
42	151	67	0	0	94	273325	114012	58,29	139658	48,90
43	155	72	1	1	92	445560	80378	81,96	103454	76,78
44	152	47	-1	2	91	456725	252647	44,68	139973	69,35

Table 2

Multiaxial loading conditions for bending-torsion tests and lives estimated by the proposed criteria for SM45 steel

Test No	σ_x, MPa	τ_{xy}, MPa	ϕ	N_f^{exp} [43]	N_f^{pre} [44]	*REP*,% [44]	N_f^{pre} model	*REP*,% model
1	411	0	0	15000	16465	-9,77	15486	-3,24

Test No	σ_x, MPa	τ_{xy}, MPa	ϕ	N_f^{exp} [43]	N_f^{pre} [44]	*REP*,% [44]	N_f^{pre} model	*REP*,% model
2	388	0	0	26100	29773	-14,07	28567	-9,45
3	372	0	0	53000	45912	13,37	44689	15,68
4	364	0	0	74000	57417	22,41	56299	23,92
5	353	0	0	93700	78726	15,98	77991	16,77
6	336	0	0	103000	130796	-26,99	131727	-27,89
7	323	0	0	166000	196272	-18,24	200278	-20,65
8	314	0	0	213000	262479	-23,23	270357	-26,93
9	313	0	0	327000	271233	17,05	279670	14,47
10	294	0	0	445000	516515	-16,07	543734	-22,19
11	291	0	0	723000	573983	20,61	606283	16,14
12	0	278	0	10400	17487	-68,14	6576	36,77
13	0	266	0	23300	30684	-31,69	16036	31,18
14	0	254	0	19500	55256	-183,36	20750	-6,41
15	0	253	0	30000	58105	-93,68	44130	-47,10
16	0	246	0	109000	83079	23,78	77346	29,04
17	0	244	0	166000	92186	44,47	91734	44,74
18	0	230	0	332000	195736	41,04	302661	8,84
19	0	229	0	142000	206912	-45,71	130511	8,09
20	0	224	0	403000	274130	31,98	516264	-28,11
21	0	218	0	1130000	387447	65,71	893480	20,93
22	390	151	0	8500	8080	4,94	8236	3,11
23	349	148	0	24000	20533	14,45	21806	9,14
24	325	153	0	32000	32408	-1,28	35150	-9,84
25	372	93	0	38000	25989	31,61	27661	27,21
26	309	134	0	100000	67916	32,08	83065	16,94
27	265	225	0	12000	12113	-0,94	11405	4,96
28	392	118	90	12700	12025	5,31	12854	-1,21
29	417	78	90	13000	10214	21,43	10133	22,05
30	346	173	90	16000	14247	10,96	17009	-6,31
31	245	216	90	20000	31940	-59,70	26444	-32,22
32	245	211	90	25000	43158	-72,63	34599	-38,40
33	304	186	90	26000	26728	-2,80	33645	-29,40
34	304	152	90	57000	53925	5,39	67369	-18,19
35	314	127	90	100000	68147	31,85	82125	17,88
36	286	143	90	120000	101028	15,81	128838	-7,37
37	167	211	90	290000	231246	20,26	293876	-1,34
38	265	132	90	350000	223940	-9.,77	15486	-3.24

Table 3

Multiaxial loading conditions for bending-torsion tests and lives estimated by the proposed criteria for 30NCD16 steel

Test No	$\sigma_{x,a}$, MPa	$\tau_{xy,a}$, MPa	$\sigma_{x,m}$, MPa	$\tau_{xy,m}$, MPa	ϕ	N_f^{exp} [42]	N_f^{pre} [44]	*REP%* [44]	N_f^{pre} model	*REP%* Model
1	765	0	0	0	0	120000	117002	2,50	121942	-1,62
2	790	0	0	0	0	90000	84511	6,10	86672	3,70
3	795	0	0	0	0	80000	79286	0,89	81038	-1,30
4	780	0	0	0	0	100000	96135	3,87	99246	0,75
5	725	0	0	0	0	200000	201434	-0,72	214966	-7,48
6	708	0	0	0	0	250000	25608	89,76	261856	-4,74

Test No	$\sigma_{x,a}$, MPa	$\tau_{xy,a}$, MPa	$\sigma_{x,m}$, MPa	$\tau_{xy,m}$, MPa	ϕ	N_f^{exp} [42]	N_f^{pre} [44]	*REP%* [44]	N_f^{pre} model	*REP%* Model
7	720	0	0	0	0	210000	216041	-2,88	219585	-4,56
8	752	0	0	0	0	140000	139153	0,61	139085	0,65
9	820	0	0	0	0	51000	57965	-13,66	55557	-8,94
10	785	0	0	0	0	95000	90117	5,14	93855	1,21
11	715	0	0	0	0	230000	231821	-0,79	251469	-9,33
12	660	0	290	0	0	250000	115241	53,90	194885	22,05
13	695	0	290	0	0	120000	73342	38,88	187842	-56,54
14	620	0	450	0	0	140000	90453	35,39	56604	59,57
15	640	0	450	0	0	51000	70122	-37,49	62317	-22,19
16	0	460	290	0	0	120000	171675	-43,06	76288	36,43
17	0	430	450	0	0	250000	298743	-19,50	176288	29,48
18	0	460	450	0	0	120000	171675	-43,06	66460	44,62
19	600	335	0	0	0	80000	117168	-46,46	66460	16,93
20	600	335	0	0	90	100000	117168	-17,17	113787	-13,79
21	548	306	0	0	0	200000	246615	-23,31	130830	34,59
22	500	290	290	0	0	120000	16719	86,07	85539	28,72
23	500	290	290	0	90	210000	16719	92,04	225345	-7,31
24	490	285	450	0	0	95000	90096	5,16	90956	4,26
25	490	285	450	0	90	230000	90096	60,83	225610	1,91

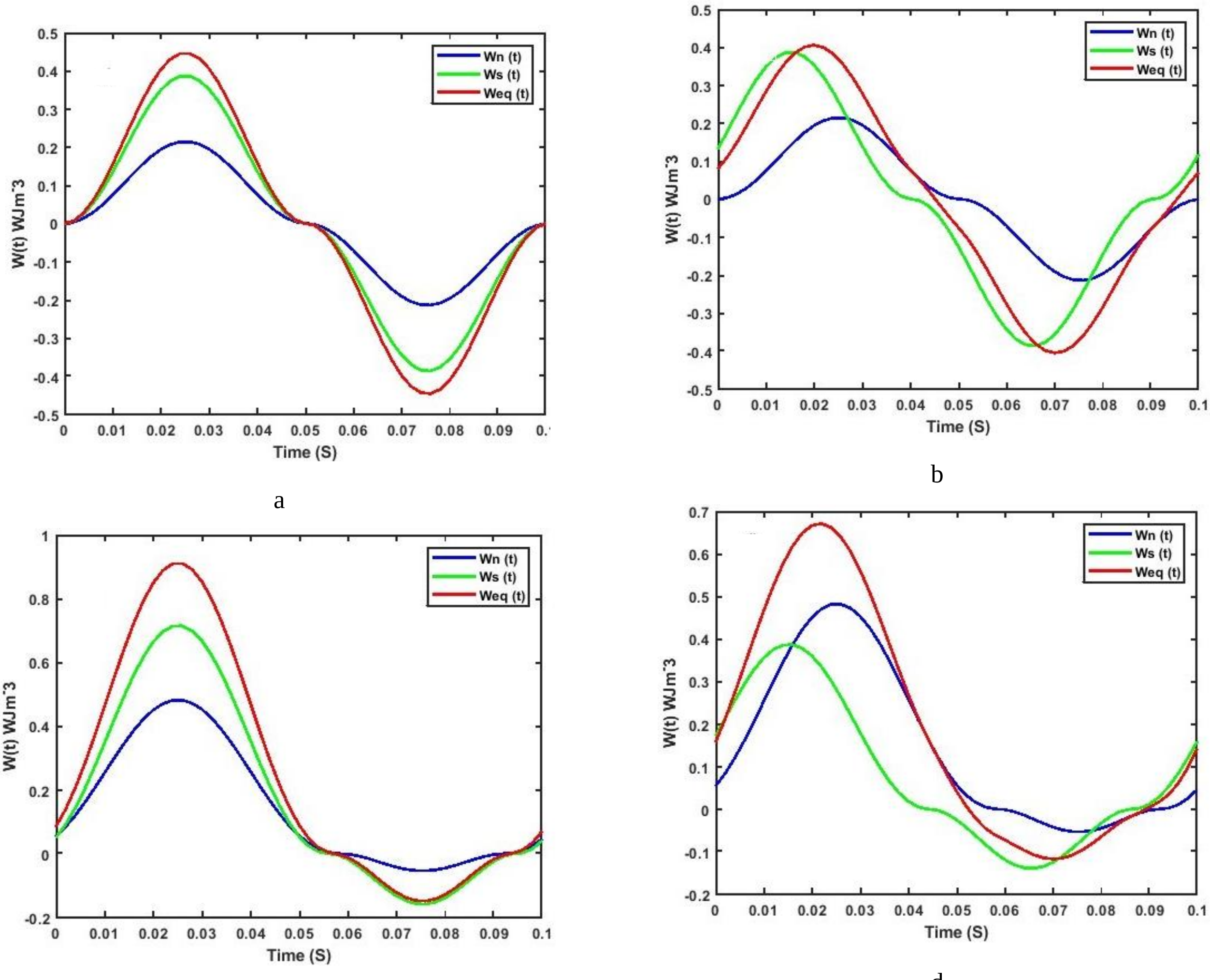


Fig. 7 Different types of cyclic loading: a – proportional multiaxial loading, b – proportional multiaxial loading with mean stress, c – non-proportional multiaxial loading and, d – non-proportional multiaxial loading with mean stress

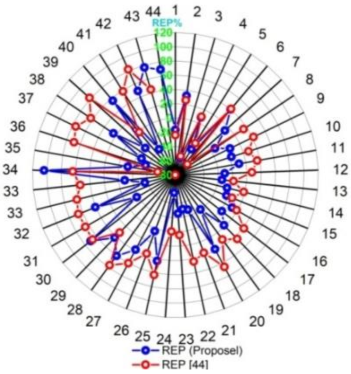


Fig. 8 Comparison of relative prediction errors [41] and the proposed criterion for 6082-T6 Al alloy

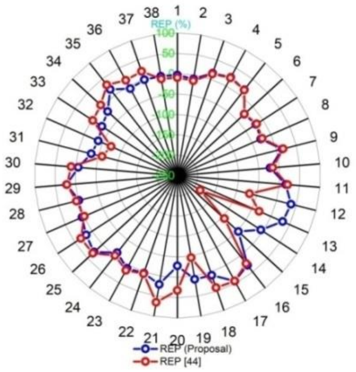


Fig. 9 Comparison of relative prediction errors [41] and the proposed criterion for SM45 steel

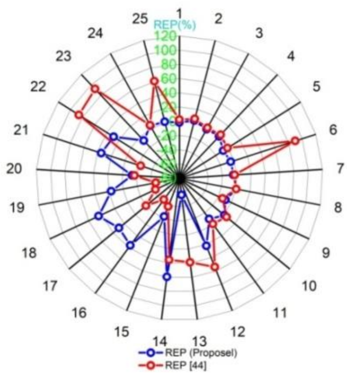
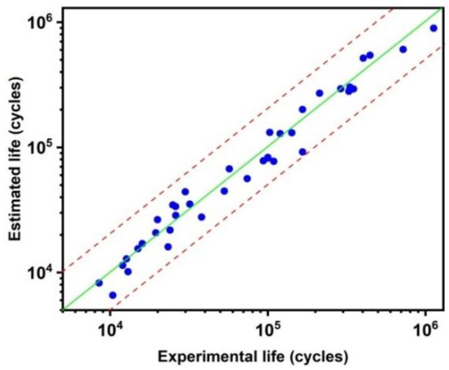
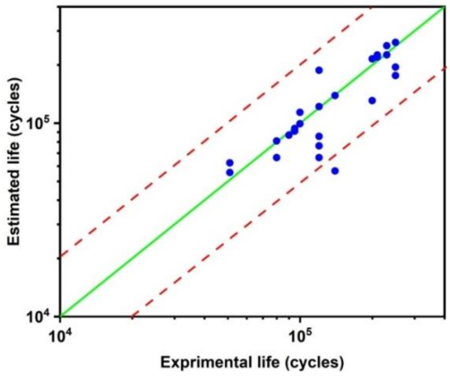


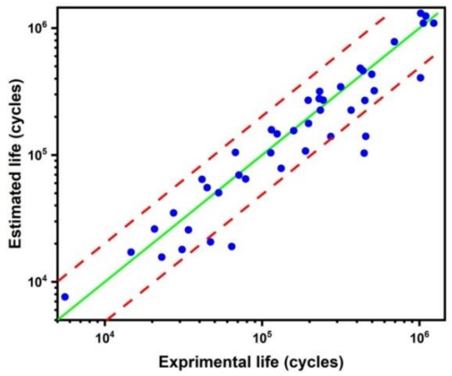
Fig. 10 Comparison of relative prediction errors [41] and the proposed criterion 30NCD16 steel



a



b



c

Fig. 11 Fatigue life predictions of proposed criterion a – SM45C steel, b – 30NCD16, c – Al6082

4. Conclusion

A new multiaxial fatigue life estimation method based on a total energy criterion was proposed in this paper which attempts to use a non-linear damage accumulation approach to make it accomplishable for fatigue life estimation and the state of materials such 6082-T6 Al alloy, SM45 steel and 30NCD16 steel under multiaxial loading conditions for bending-torsion tests. The calibration problem of the model is settled and no more constants to quantified only the Wohler curve parameters. Accordingly, the estimated results by the proposed method are in concordance with experimental data for materials studied in this paper.

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MULTIAXIAL FATIGUE CRITERION USING TOTAL STRAIN ENERGY PARAMETERS ASSOCIATED WITH CUMULATIVE DAMAGE MODEL

S u m m a r y

The knowledge of the causes of damage of materials is a priority for design engineers to avoid sudden failure of equipment in service. One of the most important reasons for the failure of materials is fatigue, this phenomenon can be defined as damage to the metal under repeated stress and lower than the yield stress. The main objective of this study is the development of multiaxial fatigue criterion using total strain energy, then validating this proposal by the results of literature and a comparison with another criterion. The majority of the results obtained by our proposal give conservative results. These predicted results are compared with bending-torsion tests and the agreement is found to be fairly good.

Keywords: damage model, multiaxial fatigue.

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