ISSN 1392-1207. MECHANIKA. Vol. 30, No. 3, 2024: 294–299

The Thermal Error Characteristic Analysis and Thermal Error Modeling Compensation of Aspheric Grinder

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https://doi.org/10.5755/j02.mech.34090

1. Introduction

With the rapid development of modern Industry, the demand for high-precision NC machine tools in the corresponding manufacturing fields has increased greatly [1-5], the key factor affecting the steady improvement of numerical control accuracy is the deformation of the main structural components of CNC machine tools under the action of thermal temperature field and alternating load, and the thermal deformation is the most significant. The experiment shows that the thermal deformation error accounts for 35% - 65%, which is the most important factor to affect the numerical control accuracy [6, 7], it has become a hot topic to be solved urgently in manufacturing industry [8-11]. Aiming at the cause of thermal deformation error of aspherical grinder, the basic form of thermal error of aspherical grinder was analyzed, and the thermal error modeling scheme was put forward based on multiple linear regression modeling algorithm, a complete thermal error compensation system is constructed to realize the axial thermal error compensation of the grinding wheel of aspherical grinder.

2. Structure analysis of aspheric grinder

The study object structure diagram is shown in Fig. 1. From Fig. 1, it can be analyzed that the aspherical grinder has two working spindles, which are the grinding wheel shaft and the workpiece shaft, and has three linear feed shafts, they are feed axis x, feed Axis y and rotary axis respectively. The Feed Shaft X is installed on the body of the machine tool, and the feed shaft Y is installed above the feed Shaft X to form a cross motion slide table. The rotary shaft is installed directly above the cross motion slide table and moves with the movement of the cross motion slide table, the grinding wheel shaft is arranged above the rotating shaft, and the workpiece shaft is arranged on the other side of the bed body.

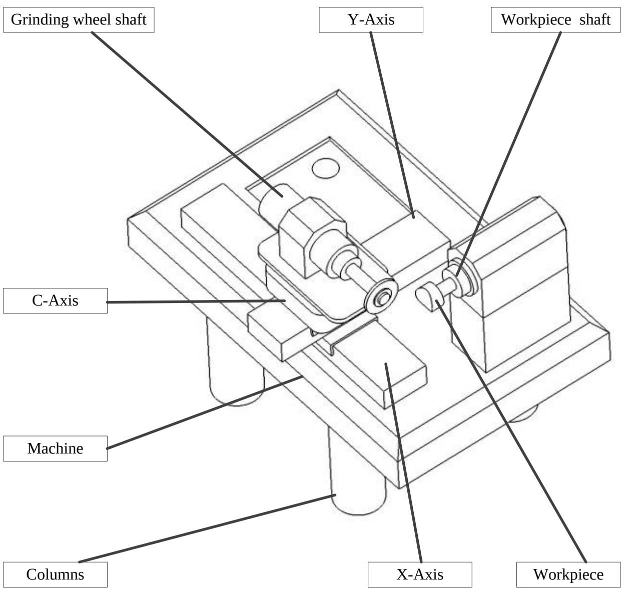

Fig. 1 Structure diagram of aspheric CNC grinder

3. Thermal error analysis of aspheric grinder

The internal and external heat sources of NC machine tools are the main reasons that lead to the thermal deformation error caused by the change of temperature field [12-14]. Due to the differences in material, shape, structure and size of the main structural parts of NC machine tool, the equilibrium temperature field of NC machine tool is broken after being heated, resulting in different degrees of thermal expansion and cold contraction of the main structural parts, thus the thermal deformation, and then make CNC machine parts processing error. For any NC machine tool, the existing linear axis and working spindle will produce thermal error because the equilibrium state of the temperature field of the machine tool is broken, including: grinding wheel axis, workpiece axis, feed axis x, feed axis y and rotation axis will produce thermal error, specifically as follows:

1. 5 items thermal error of grinding wheel shaft:

Thermal drift error 3 items: $\delta_x(s, T)$, $\delta_y(s, T)$ and $\delta_z(s, T)$, 2 items center line rotation angle tilt error: $\varepsilon_x(s, T)$, $\varepsilon_z(s, T)$.

2. 5 items of shaft thermal error:

3 items of thermal drift error: $\delta_x(v, T)$, $\delta_y(v, T)$ and $\delta_z(v, T)$, 2 items center line rotation angle tilt error: $\varepsilon_x(v, T)$, $\varepsilon_z(v, T)$.

3. 3 items x thermal drift error of feed shaft:

$\delta_x(x, T)$, $\delta_y(x, T)$, $\delta_z(x, T)$.

4. 3 items feed Axis y thermal drift error:

$\delta_x(x, T)$, $\delta_y(x, T)$, $\delta_z(x, T)$.

5. 3 items of thermal drift error:

$\delta_x(c, T)$, $\delta_y(c, T)$, $\delta_z(c, T)$.

Therefore, there are 19 thermal errors in the aspherical grinder. As shown in Table 1, the axial thermal drift error $\delta_x(s, T)$ of the grinding wheel shaft is the most significant.

Table 1

Thermal Error Elements

No.	Axis	Thermal error
1	Grinding wheel shaft	$\delta_x(s,T)$
		$\delta_y(s,T)$
		$\delta_z(s,T)$
		$\varepsilon_x(s,T)$
		$\varepsilon_z(s,T)$
2	Workpiece shaft	$\delta_x(v,T)$
		$\delta_y(v,T)$
		$\delta_z(v,T)$
		$\varepsilon_x(v,T)$
		$\varepsilon_z(v,T)$
3	Feed shaft X	$\delta_x(x,T)$
		$\delta_y(x,T)$
		$\delta_z(x,T)$
4	Feed shaft Y	$\delta_x(y,T)$
		$\delta_y(y,T)$
		$\delta_z(y,T)$
5	Axis of rotation	$\delta_x(c,T)$
		$\delta_y(c,T)$
		$\delta_z(c,T)$

4. Multiple linear regression modeling scheme

The establishment of thermal error compensation model of NC machine tool is the key to realize thermal error compensation, the thermal error modeling of NC machine tool can be realized by the method of multiple linear regression modeling with multi-independent variable input and single-dependent variable output [15-17].

Based on the expansion and contraction characteristics of axial thermal deformation of grinding wheel in the actual testing process of aspherical grinder, multiple measuring points of machine tool temperature are established as independent variable input by using multiple linear regression modeling method, the axial thermal deformation elongation of the grinding wheel shaft is dependent on the output of the variable, which can realize the axial thermal error modeling of the grinding wheel shaft of the aspheric grinder.

Suppose there are M independent variables x_1, x_2, x_3,…, x_M have a linear relationship with a dependent variable y, and the temperature rise data of N groups of temperature measuring points and the axial thermal deformation error of grinding wheel can be obtained by actual detection, and the relationship as shown in Eq. (1) can be established.

$$\begin{cases} y_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{21} + \cdots + \beta_M x_{M1} + \varepsilon_1 \\ y_2 = \beta_0 + \beta_1 x_{12} + \beta_2 x_{22} + \cdots + \beta_M x_{M2} + \varepsilon_2 \\ \vdots \\ y_N = \beta_0 + \beta_1 x_{1N} + \beta_2 x_{2N} + \cdots + \beta_M x_{MN} + \varepsilon_N \end{cases} \quad (1)$$

The Eq. (1) is represented by a matrix, and it can be written into an expression (2).

$$Y = X\beta + \varepsilon . \quad (2)$$

In the Eq. (2), the expression for the dependent variable matrix Y is shown in the Eq. (3), the expression for the independent variable matrix X is shown in the Eq. (4), and the expression for the regression coefficient matrix β is shown in the Eq. (5), the expression of the random variable matrix ε is shown in the formula (6). The regression coefficient matrix β and the random variable matrix ε are the desired variables.

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_N \end{pmatrix}, \quad (3)$$

$$X = \begin{pmatrix} 1 & x_{11} & x_{21} & \cdots & x_{M1} \\ 1 & x_{12} & x_{22} & \cdots & x_{M2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1N} & x_{2N} & \cdots & x_{MN} \end{pmatrix}, \quad (4)$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \vdots \\ \beta_M \end{pmatrix}, \quad (5)$$

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_N \end{pmatrix}. \quad (6)$$

The least squares principle of applied mathematics theory is used to analyze the regression coefficient matrix β and the random variable matrix ε. Suppose, b_0, b_1, b_2,…, b_M is the least square estimate of the regression coefficient matrix β, then the multivariate linear regression model can be expressed by an Eq. (7).

$$y = b_0 + b_1 x_1 + \cdots + b_M x_M . \quad (7)$$

According to the principle of least squares, b_0, b_1, b_2,…, b_M should satisfy the requirement that the sum of squares of the dependent variables output y_t residuals be minimized, i.e. that the solution given by the expression (8) is the minimum.

$$Q = \sum_{t=1}^{N} (y_t - \hat{y}_t)^2 = \\ = \sum_{t=1}^{N} (y_t - b_0 - b_1 x_{t1} - \cdots - b_M x_{tM})^2 . \quad (8)$$

For the non-negative quadratic power (8), there must be a minimum solution. By using the extremum theorem of differential calculus to solve the Eq. (8), we can get b_0, b_1, b_2, …, b_M, the solution is shown in Eqs. (9).

$$\begin{cases} \dfrac{\partial Q}{\partial b_0} = -2\sum_{t=1}^{N} (y_t - b_0 - b_1 x_{t1} - \cdots - b_M x_{tM}) = 0 \\ \dfrac{\partial Q}{\partial b_i} = -2\sum_{t=1}^{N} (y_t - b_0 - b_1 x_{t1} - \cdots - b_M x_{tM}) = 0 \end{cases} \quad (9)$$

By transformation of Eqs. (9), formula (10) can be obtained.

$$y_t = \mu_0 + \beta_1 x_{t1} + \beta_2 x_{t2} + \cdots + \beta_M x_{tM} \quad (10)$$

The multivariate linear regression equation of the corresponding formula (10) is shown in the formula (11).

$$\hat{y} = \mu_0 + b_1 x_1 + b_2 x_2 + \cdots + b_M x_M \tag{11}$$

In the multivariate linear regression Eq. (11), b_0, b_1, b_2, …, b_M is a parameter for solving the thermal error model, which can be obtained by differential calculation of Eqs. (9). The random variable matrix ε can be set to 0, x_1, x_2, x_3, …, x_M is an independent variable temperature measurement point temperature rise data, through the temperature sensor can be detected, dependent variables, for the thermal error model output, that is, grinding wheel axial thermal error value.

5. Grinding wheel axial thermal error modeling

Establishing the mapping relationship between thermal error and temperature field is the key problem of thermal error compensation for NC machine tools. For aspheric surface grinder, the establishment of the key temperature measurement points and the axial thermal error model of grinding wheel axis can effectively improve the machining accuracy of the grinder. Three key temperature measurement points are selected as the final model independent variable inputs, which are defined as T_1, T_2 and T_3. In the process of modeling, the axial thermal error data of grinding wheel axis was chosen as dependent variable Y, and 60 sets of temperature data were used as the base of regression coefficient matrix β and random variable matrix ε to solve the thermal error model.

In order to establish the accurate model of axial thermal error of grinding wheel shaft, the key temperature measuring points T_1, T_2, T_3 and the axial thermal error measuring data of grinding wheel shaft were substituted into equation group (1), and the Eq. (12) was obtained, the locations of key temperature measuring points T_1, T_2 and T_3 are shown in Fig. 2.

$$\begin{cases} y_1 = \beta_0 + \beta_1 x_{11} + \beta_2 x_{21} + \beta_3 x_{31} \\ y_2 = \beta_0 + \beta_1 x_{12} + \beta_2 x_{22} + \beta_3 x_{32} \\ \qquad \vdots \\ y_{60} = \beta_0 + \beta_1 x_{1.60} + \beta_2 x_{2.60} + \beta_3 x_{3.60} \end{cases} . \tag{12}$$

The Eq. (12) is represented by a matrix, and can be written as an expression (13)

$$Y = X\beta + \varepsilon . \tag{13}$$

In the Eq. (13), the expression for the dependent variable matrix Y is shown in the expression (14), the expression for the independent variable matrix X is shown in the Eq. (15), and the expression for the regression coefficient matrix β is shown in the Eq. (16), the expression of the random variable matrix ε is shown in the Eq. (17);

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ \vdots \\ y_{60} \end{pmatrix}, \tag{14}$$

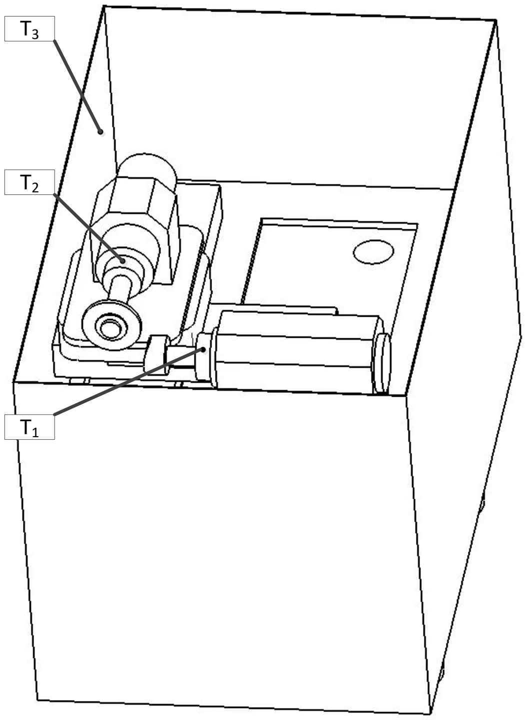

Fig. 2 Layout of key temperature measurement points

$$X = \begin{pmatrix} 1 & x_{11} & x_{21} & x_{31} \\ 1 & x_{12} & x_{22} & x_{32} \\ \vdots & \vdots & \vdots & \vdots \\ 1 & x_{1.60} & x_{2.60} & x_{3.60} \end{pmatrix}, \tag{15}$$

$$\beta = \begin{pmatrix} \beta_0 \\ \beta_1 \\ \beta_2 \\ \beta_3 \end{pmatrix}, \tag{16}$$

$$\varepsilon = \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \vdots \\ \varepsilon_{60} \end{pmatrix}. \tag{17}$$

The least squares principle of applied mathematics theory is used to analyze the regression coefficient matrix β and the random variable matrix ε. Assuming that b_0, b_1, b_2, b_3 are the least squares estimates of the regression coefficient matrix β, the multivariate linear regression model is represented by an algorithm (18).

$$y = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 \tag{18}$$

According to the principle of least squares, b_0, b_1, b_2, b_3 should satisfy the minimum of the sum of squares of y residuals of dependent variables, that is, the solution of equation (19) should be the minimum.

$$Q = \sum_{t=1}^{60} (y - \hat{y})^2 = \\ = \sum_{t=1}^{60} (y - b_0 - b_1 x_{t1} - b_2 x_{t2} - b_3 x_{t3})^2 . \tag{19}$$

The extreme value theorem of differential calculus is applied to solve the Eq. (19), and b_0, b_1, b_2, b_3 can be obtained, and the solution is shown by the Eq. (20)

$$\begin{cases} \dfrac{\partial Q}{\partial b_0} = -2\sum_{t=1}^{60}(y - b_0 - b_1 x_{t1} - b_2 x_{t2} - b_3 x_{t3}) = 0 \\ \dfrac{\partial Q}{\partial b_1} = -2\sum_{t=1}^{60}(y - b_0 - b_1 x_{t1} - b_2 x_{t2} - b_3 x_{t3}) = 0 \\ \dfrac{\partial Q}{\partial b_2} = -2\sum_{t=1}^{60}(y - b_0 - b_1 x_{t1} - b_2 x_{t2} - b_3 x_{t3}) = 0 \\ \dfrac{\partial Q}{\partial b_3} = -2\sum_{t=1}^{60}(y - b_0 - b_1 x_{t1} - b_2 x_{t2} - b_3 x_{t3}) = 0 \end{cases}. \quad (20)$$

The solutions of the parameters β_0, β_1, β_2, β_3 obtained by solving the Eq. (20) are shown in Table 2, so that the axial thermal error model of the grinding wheel is shown as an Eq. (21).

Table 2

Regression coefficient β

No.	Parameter	Value
1	b_0	-7.793
2	b_1	0.868
3	b_2	-0.690
4	b_3	0.253

$$y = -7.793 + 0.868 \times \Delta T_1 - -0.690 \times \Delta T_2 + 0.253 \times \Delta T_3 . \quad (21)$$

6. Compensation implementation

In order to implement axial thermal error compensation for grinding wheel of aspheric grinder, an adaptive thermal error compensation system is established, as shown in Fig. 3. In the course of compensation, the temperature rise data of key temperature measuring points T_1, T_2 and T_3 are collected and transmitted in real time, and the thermal error compensation system calculates the thermal error by the temperature rise data of key temperature measuring points T_1, T_2 and T_3, therefore, the aspheric surface grinder is driven to carry out axial thermal compensation of the grinding wheel shaft. Thermal error compensation site as shown in Fig. 4.

The data before and after thermal error compensation are fitted and plotted by MATLAB software, as shown in Fig. 5.

According to the analysis of the data fitting graph before and after compensation in Fig. 5, the axial thermal error model of aspherical grinding wheel based on the multiple linear regression theory modeling algorithm can accurately predict the thermal error, the maximum residual error of thermal error after compensation is not more than 0.2 μm, and the residual variance has been reduced to 5.891e-002, which can ensure the precision machining of aspheric grinder, in the start-up stage of aspheric grinder, the thermal error fitting effect is not good, so the thermal error prediction based on the multiple linear regression theory modeling algorithm should make the machine work for a period of time after grinding compensation, is advantageous to the compensation effect.

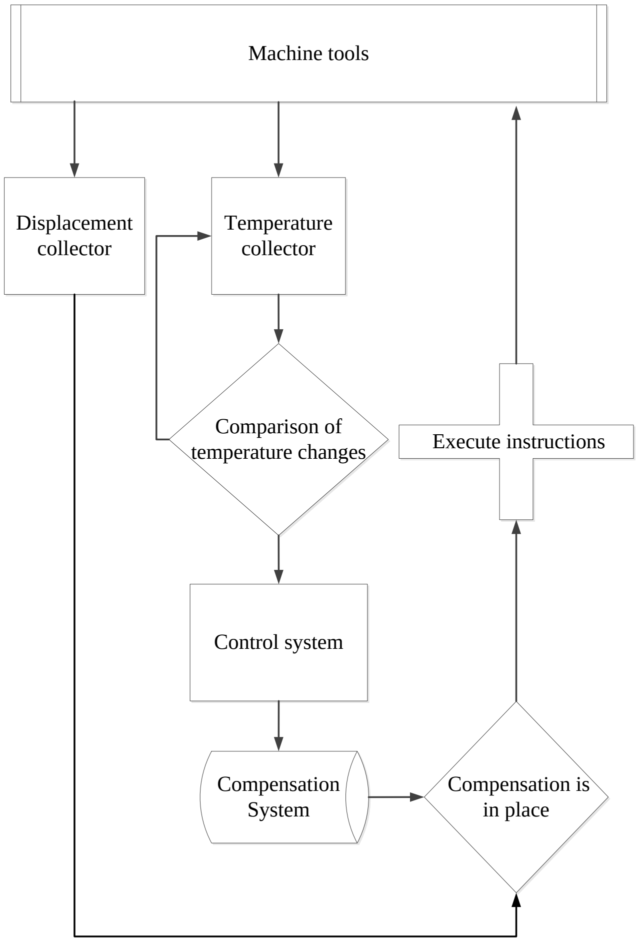

Fig. 3 Thermal Error Compensation System

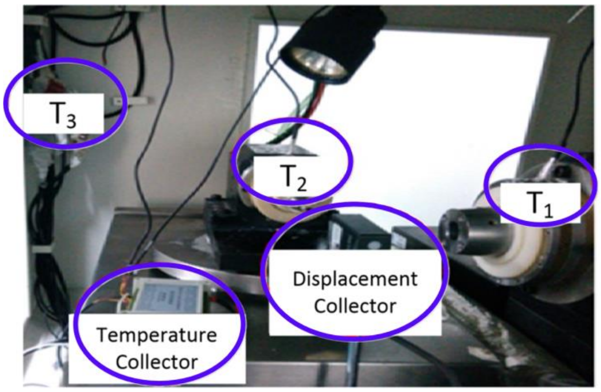

Fig. 4 Thermal error compensation field

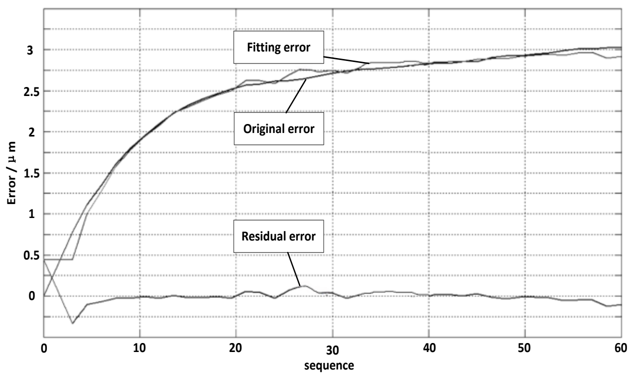

Fig. 5. Thermal error compensation effect diagram

7. Conclusions

1. The structure characteristics of aspheric grinder are discussed, the concrete forms of aspheric grinder thermal error are analyzed, there are 19 kinds of thermal error.

2. Aiming at the existing thermal error elements, a practical scheme of thermal error modeling was designed based on the multivariate linear regression algorithm of statistical theory, and a multivariate linear regression model of axial thermal error of aspheric grinder was established. It lays a foundation for realizing thermal error compensation.

3. A thermal error compensation system is constructed. The experimental results show that the algorithm based on multiple linear regression model and the thermal error compensation system can effectively reduce the axial thermal error of the grinding wheel of aspherical grinder, after compensation, the residual variance is reduced to 5.891e-002, which can meet the precision grinding requirements of aspherical grinder.

Acknowledgments

This work is supported by the General Project of Free Exploration of Science and Technology Department of Jilin Province, No. YDZJ202201ZYTS391.

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THE THERMAL ERROR CHARACTERISTIC ANALYSIS AND THERMAL ERROR MODELING COMPENSATION OF ASPHERIC GRINDER

S u m m a r y

According to the literature analysis, the thermal error of CNC machine tools is the main factor affecting its processing accuracy, accounting for about 35%-65%; This paper discusses the structure assembly of Aspherical Grinder, which is the research carrier of thermal error analysis of NC machine tool, and analyzes the characteristics and concrete forms of thermal error in aspherical grinder Based on the multi-input and single-output characteristics of the multi-linear regression model in the statistical theory, a multi-linear regression method is proposed to realize the thermal error modeling of aspheric grinder, the thermal error model of aspherical grinder is constructed based on matrix calculation, and the thermal error compensation system is designed in practice, the results show that the thermal error model of aspherical grinder based on the multiple linear regression theory can effectively compensate the thermal error of aspherical grinder and calculate the compensation data, the residual error after compensation has been reduced to 5.891e-002, greatly improving the aspheric surface grinder processing accuracy, making it meet the aspheric surface precision grinding processing needs.

Keywords: CNC machine tools, thermal error, multiple linear regression, error modeling, compensation system.

Received April 26, 2023
Accepted June 20, 2004