



different surface materials is explored. These materials include steel, non crystalline silica, bronze, silicon and silicon carbonized. The tilting angle and the width of this bearing are respectively  $\theta$  and  $l$ . The thickness of the intermediate continuum fluid film is generally termed as  $h$ , and its values on the entrance and exit of the bearing are respectively termed as  $h_i$  and  $h_o$ . The surface separation on the exit of the bearing is  $h_{tot,o}$ .

### 3. Analysis

Here, the used multiscale lubrication analysis is the same with that presented in ref. [16]. For conciseness, this content is not repeated. Considering the surface roughness and the surface elastic deformation, the surface separation on the  $J^{\text{th}}$  discretized point shown in Fig. 1 is formulated as:

$$h_{tot,J} = h_{oo} + x_J \tan \theta + \frac{R_z}{2} \sin(\omega x_J + \varphi) - \frac{2}{\pi E_v} \int_0^l p(s) \ln(x_J - s)^2 ds, \quad (1)$$

where  $h_{oo}$ ,  $\omega$  and  $\varphi$  are respectively constant,  $R_z$  is the maximum height of the surface roughness,  $E_v$  is the compound Young's modulus of elasticity of two bearing surfaces [16].

For the numerical analysis, the  $(N+1)$  discretized points are evenly distributed in the whole hydrodynamic area of the bearing, as shown in Fig. 1.

### 4. Calculation

Here, the numerical solution procedure is the same with that presented in [16]. The weak, medium and strong fluid-bearing surface interactions have been respectively considered. The parameter values for characterizing these interactions have been shown in [16]. In all the calculations, it was taken that:  $l = 600 \mu\text{m}$ ,  $\omega = (2.094\text{E}-6)$  rad/m,  $\varphi = \pi$ . The values of the other operational parameters are the same with those shown in [16]. For different bearing materials, the Young's modulus of elasticity ( $E$ ) and the Poisson's ratio ( $\mu$ ) are respectively shown in Table 1.

Table 1  
Young's modulus of elasticity and Poisson's ratio for different bearing materials

Materials	$E$ , GPa	$\mu$	$E_v$ , GPa
Non crystalline silica	73	0.17	75
Bronze	108	0.32	120
Silicon	190	0.1	192
Steel	193	0.3	209
Silicon carbonized	450	0.25	480

## 5. Results

### 5.1. Comparison of the surface separation profiles respectively for elastic steel surfaces and rigid surfaces

Fig. 2, a – c show the comparisons between the surface separation profiles respectively for elastic steel surfaces and rigid surfaces for different fluid-bearing sur-

face interactions when  $R_z = 4 \text{ nm}$ ,  $w = 4000 \text{ N/m}$  and  $u = 4\text{E}-4 \text{ m/s}$ . Owing to the surface elastic deformation, the original surface roughness is flattened. In the case of the rough surface, for a given load and sliding speed, the fluid-bearing surface interaction appears to have a negligible influence on the shape of the surface separation distribution, but lifts the surface separations in the whole hydrodynamic area if the interaction strength between the fluid and the bearing surface is increased. Whenever the bearing surfaces are rigid or elastically deformed, the surface separations in the whole hydrodynamic area are significantly increased with the increase of the interaction strength between the fluid and the bearing surface. These results are the same with those for the smooth bearing surfaces as shown in [16].

### 5.2. Comparison of the film pressure profiles respectively for elastic steel surfaces and rigid surfaces

Fig. 3, a – c show the film pressure distributions respectively for elastic steel surfaces and rigid surfaces for different fluid-bearing surface interactions when  $R_z = 4 \text{ nm}$ ,  $w = 4000 \text{ N/m}$  and  $u = 4\text{E}-4 \text{ m/s}$ . These pressure profiles well correspond to the surface separation distributions in Fig. 2, a – c. For rigid surfaces, we can notice a bit film pressure ripples owing to the surface roughness. But no apparent film pressure ripples are observed for the rough elastic bearing surfaces.

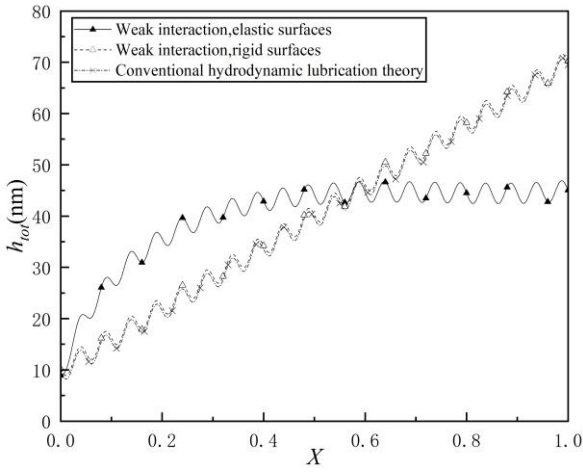
Fig. 4, a – c show the influences of the surface roughness on the film pressure distribution respectively for elastic steel surfaces and rigid surfaces for different fluid-bearing surface interactions when  $h_{tot,o} = 18 \text{ nm}$  and  $u = 4\text{E}-4 \text{ m/s}$ . Whenever the bearing surfaces are rigid or elastic, the film pressures are overall increased with the increase of the surface roughness. However, this effect is more significant for rigid surfaces than for elastic surfaces. The effect of the surface roughness on the film pressure appears to be strongly dependent on the fluid-bearing surface interaction whenever the bearing surfaces are rigid or elastically deformed. However, for elastic steel surfaces such a dependence is weaker than for rigid surfaces. We also notice that when the value of  $R_z$  is increased to  $6 \text{ nm}$ , there is a bit film pressure ripples especially for the weak interaction even when the rough bearing surfaces are elastically deformed. However, for the strong fluid-bearing surface interaction the film pressure ripples are largely reduced because of the generated much higher film pressures and the resulting much greater surface elastic deformations as compared to the case of the weak fluid-bearing surface interaction.

### 5.3. Variations of the carried load of the bearing with the surface roughness respectively for elastic steel surfaces and rigid surfaces

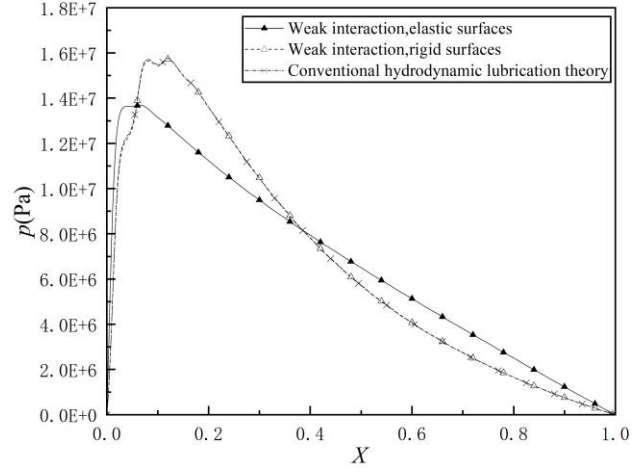
Fig. 5 shows that the carried load of the bearing is significantly increased with the increase of the surface roughness  $R_z$  especially for the strong fluid-bearing surface interaction whenever the bearing surfaces are rigid or elastically deformed. However, the increase slope of the bearing load with the surface roughness appears smaller for the elastic steel surfaces than for rigid surfaces. elastically deformed. However, the increase slope of the bearing load with the surface roughness appears smaller for the elastic

steel surfaces than for rigid surfaces. Also, for the same operating condition, the carried load of the bearing for the elastic steel surfaces is significantly smaller than that for rigid surfaces. These phenomena are particularly apparent

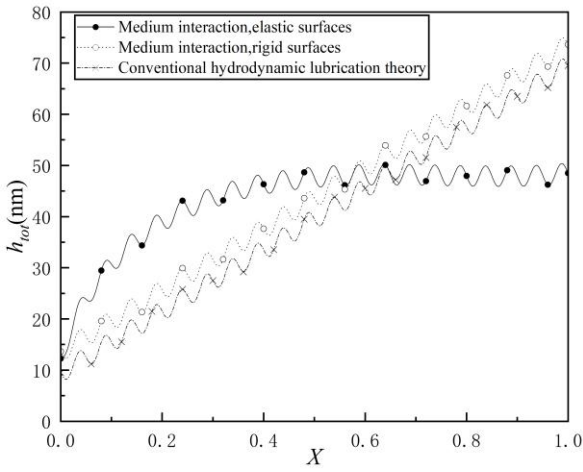
for the strong fluid-bearing surface interaction. Whenever the bearing surfaces are rigid or elastically deformed, the carried load of the bearing is significantly increased with the increase of the interaction strength between the fluid



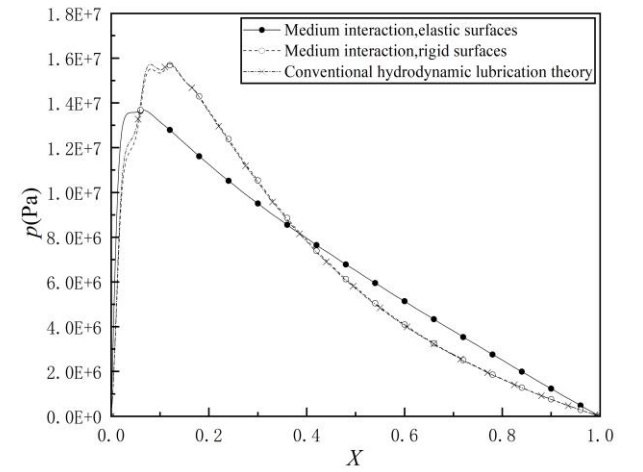
a



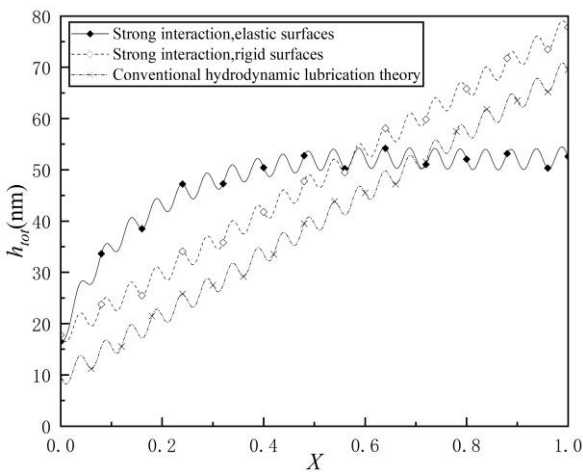
a



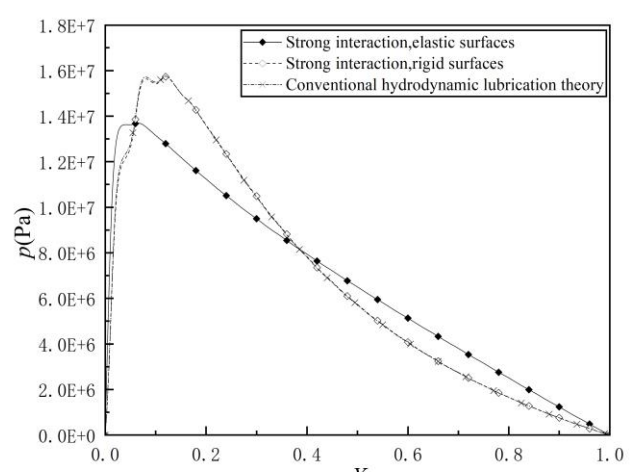
b



b



c

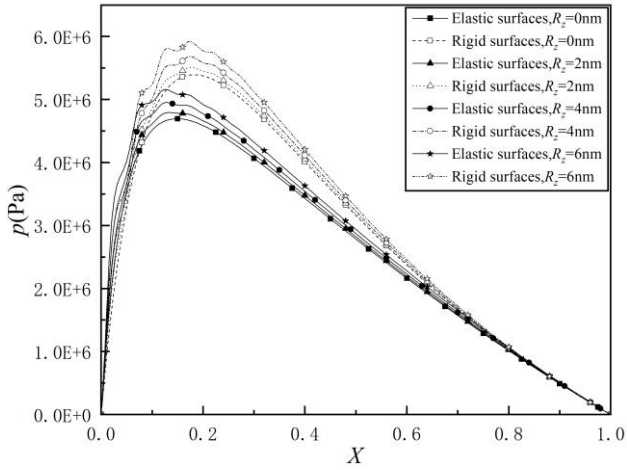


c

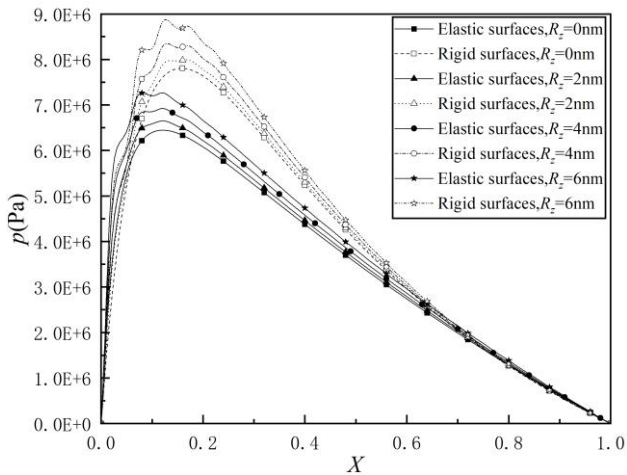
Fig. 2 Comparison between the surface separation profiles respectively for elastic steel surfaces and rigid surfaces for different fluid-bearing surface interactions when  $R_z = 4$  nm,  $w = 4000$  N/m and  $u = 4E-4$  m/s: a – for the weak interaction, b – for the medium interaction, c – for the strong interaction

Fig. 3 Comparison of the film pressure profiles respectively for elastic steel surfaces and rigid surfaces for different fluid-bearing surface interactions when  $R_z = 4$  nm,  $w = 4000$  N/m and  $u = 4E-4$  m/s: a – for the weak interaction, b – for the medium interaction, c – for the strong interaction

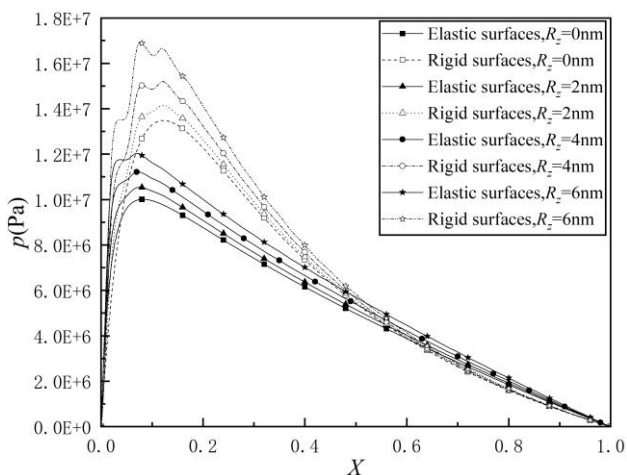
and the bearing surface especially for greater surface roughness. However, this effect is more stronger for rigid surfaces.



a



b



c

Fig. 4 Influences of the surface roughness on the film pressure distribution respectively for elastic steel surfaces and rigid surfaces for different fluid-bearing surface interactions when  $h_{tot,o} = 18$  nm and  $u = 4E-4$  m/s: a – for the weak interaction, b – for the medium interaction, c – for the strong interaction

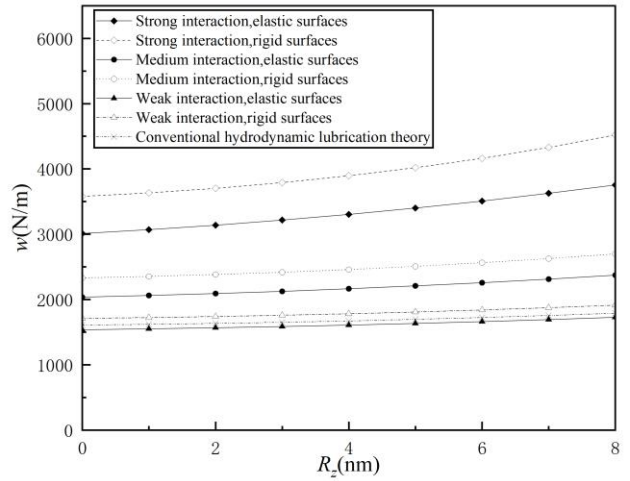


Fig. 5 Variations of the carried load of the bearing with the surface roughness  $R_z$  respectively for elastic steel surfaces and rigid surfaces for different fluid-bearing surface interactions when  $h_{tot,o} = 18$  nm and  $u = 4E-4$  m/s

#### 5.4. Performances of the bearing for different surface materials

Fig. 6 shows that when the operation condition is given, the surface separation profile in the bearing is heavily influenced by the bearing surface material. The material with a large Young's modulus of elasticity such as silicon carbonized gives the surface separation profile close to that for rigid surfaces. While the bearing surfaces made of steel gives the surface separation profile significantly deviating from that for rigid surfaces owing to the pronounced surface elastic deformation. The material with a small Young's modulus of elasticity such as non crystalline silica radically modifies the surface separation profile and moves to the outlet zone the location where the maximum surface separation occurs; In this case, the original surface roughness is reduced to the largest as compared to the cases for the other materials.

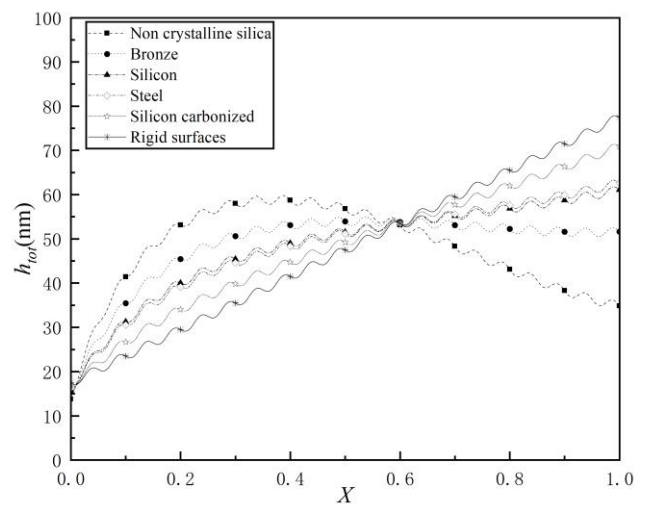


Fig. 6 Comparisons among the surface separation profiles respectively for different bearing surface materials for the medium fluid-bearing surface interaction when  $R_z = 2$  nm,  $w = 2500$  N/m and  $u = 4E-4$  m/s

Fig. 7 shows that when the operating condition is given, the bearing surface materials with different Young's modulus of elasticity give different film pressure distributions owing to the surface elastic deformation. The material of silicon carbonized gives the film pressures very close to those for rigid surfaces. The material of steel gives the film pressure distribution apparently different from that for rigid surfaces. The material of non crystalline silica gives the film pressure distribution the most significantly deviating from that for rigid surfaces. For the material with a small Young's modulus of elasticity, the film pressure ripple is not observed for the given case.

Fig. 8 shows the load ( $w$ ) versus minimum surface separation ( $h_{tot,min}$ ) curves respectively for different bearing surface materials for the medium fluid-bearing surface interaction when  $R_z = 2$  nm and  $u = 4E-4$  m/s. The slopes of these curves represent the film stiffness for different surface materials. It is shown that when the Young's modulus of elasticity of the surface material is greater, the hydrodynamic film stiffness in the bearing is larger. For a

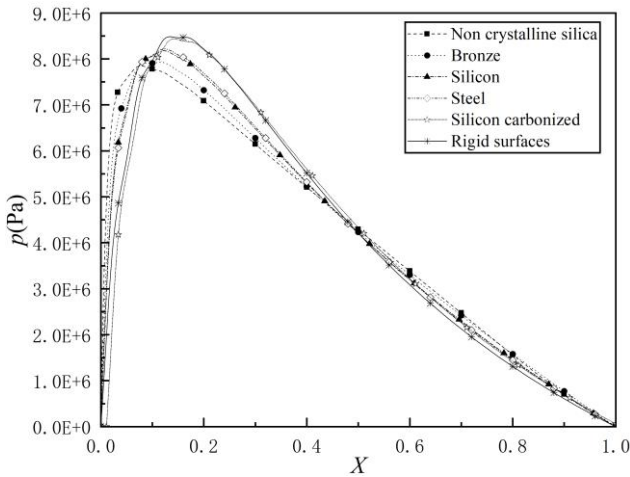


Fig. 7 Comparisons among the film pressure distributions respectively for different bearing surfaces for the medium fluid-bearing surface interaction when  $R_z = 2$  nm,  $w = 2500$  N/m and  $u = 4E-4$  m/s

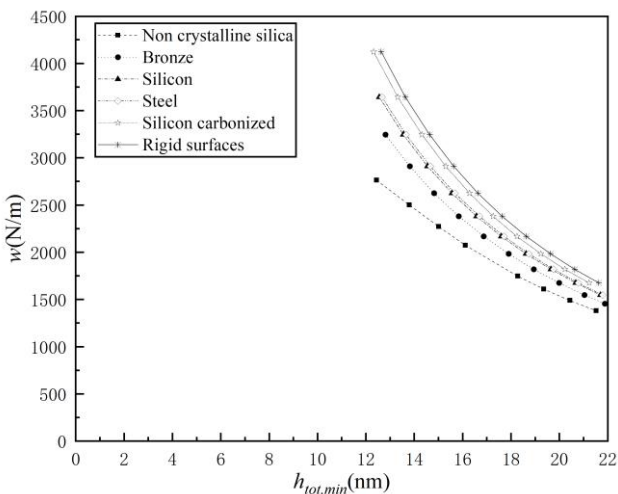


Fig. 8 The load ( $w$ ) versus minimum surface separation ( $h_{tot,min}$ ) curves respectively for different bearing surface materials for the medium fluid-bearing surface interaction when  $R_z = 2$  nm and  $u = 4E-4$  m/s

given load, the material with a smaller Young's modulus of elasticity gives a lower value of the minimum surface separation; Or for a given value of the minimum surface separation, the material with a smaller Young's modulus of elasticity gives a smaller bearing load.

## 6. Conclusions

The combined effects of the adsorbed layer, the surface roughness and the surface elastic deformation are analytically investigated by the multiscale approach in the hydrodynamic inclined fixed pad thrust bearing with low clearance. The coupled bearing surfaces are made of the same material. The moving surface is assumed as perfectly smooth and the stationary surface is imposed with the sinusoidal surface roughness so that the operating condition is steady-state. In the whole hydrodynamic area of this bearing, there is the sandwich film which consists of both the adsorbed layers and the intermediate continuum fluid film. Zhang's closed-form explicit multiscale flow equations were used to respectively describe the flows of the adsorbed layers and the flow of the intermediate continuum fluid. Based on these equations, the numerical calculations were made for the film pressures, the surface separations and the carried loads of the bearing for different surface roughness and different fluid-bearing surface interactions. Based on the calculation results, the conclusions are drawn as follows:

1. Whenever the bearing surfaces are rigid or elastically deformed, the carried load of the bearing is significantly increased with the increase of the surface roughness especially for rigid surfaces and for a strong fluid-bearing surface interaction. However, this increase slope is smaller for elastic surfaces than for rigid surfaces.

2. For the same operating condition, when the Young's modulus of elasticity of the bearing surface material is smaller, the maximum film pressure is lower and the film pressure distribution as well as the surface separation profile in the bearing more significantly deviate from those for rigid surfaces, and the minimum surface separation is smaller.

3. The bearing surface material significantly influences the stiffness of the hydrodynamic film in the bearing. For a given operating condition, smaller the Young's modulus of elasticity of the bearing surface material, smaller the hydrodynamic film stiffness.

4. For the bearing surface material with a large Young's modulus of elasticity such as silicon carbonized, the film pressure and surface separation distributions are close to those for rigid surfaces, and the obvious film pressure ripples can be observed when the maximum height of the surface roughness is only 2nm. However, for the bearing surface with a small Young's modulus of elasticity such as made of non crystalline silica, no film pressure ripples are observed even when the surface roughness is greater, owing to the much greater surface elastic deformation.

5. Due to the surface elastic deformation, the original surface roughness is reduced especially when the film pressures in the bearing are high or the Young's modulus of elasticity of the bearing surface is small.

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MULTISCALE MIXED ELASTOHYDRODYNAMICS IN INCLINED FIXED PAD THRUST BEARING INVOLVING SURFACE ROUGHNESS AND LOW CLEARANCE

S u m m a r y

The film pressures, surface separation profiles and carried loads of the hydrodynamic inclined fixed pad thrust bearing with the sandwich films were calculated by the multiscale approach when the bearing clearance was low and the combined effects of the adsorbed layer, the surface roughness and the surface elastic deformation were incorporated; The sandwich film consists of two physically adsorbed layers respectively on the coupled bearing surfaces and the intermediate continuum fluid film. The results show that the surface roughness intrinsically generates the film pressure ripples, but the rough surface is somewhat flattened and the film pressure ripples are a bit reduced by the surface elastic deformation. For a given sliding speed and a given original bearing clearance, the surface elastic deformation results in the significant reductions of both the maximum film pressure and the carried load of the bearing especially for a strong fluid-bearing surface interaction. In spite of this, the slope of the increase of the bearing load with the magnitude ( $R_z$ ) of the surface roughness is nearly the same for the elastic surfaces with that for the rigid surfaces. The presence of the surface roughness does not qualitatively alter the influence of the surface elastic deformation on the performance of the bearing.

**Keywords:** adsorbed layer, elastic deformation, hydrodynamics, load, pressure, surface roughness, thrust bearing.

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