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Adaptability of Burgers Rheological Model and its Improved Model to the Creep Properties of Glue Laminated Bamboo

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1. Introduction

Bamboo plays a very important role in the world's forest resources and is known as the second forest. At present, there are more than 1200 species worldwide [1]. They are mainly distributed in tropical and subtropical regions of Asia, Africa and South America, and few bamboo species are distributed in temperate and chill zones. Recently, the global forest area has fallen sharply, but the area of bamboo forest increases by about 3% per year [2]. Bamboo, which has the strong growth ability and excellent material, is a very good engineering material. Making full use of bamboo can reduce the dependence on timber resources and is beneficial to the protection of soil and water [3]. In recent years, the development and utilization of bamboo engineering materials has greatly broadened the use of bamboo. Bamboo industry is related to construction, building materials, home and other fields.

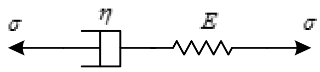
In today's trend of sustainable development, there is a renewed interest to use bamboo for modern building and bridge structures [4]. As an anisotropic material with mechanical properties which vary in the longitudinal, radial and transverse directions, the raw bamboo is generally not directly applied to the engineering. After a series of mechanical and chemical processing, the raw bamboo is made of the plate-like material with the adhesive or bamboo self-binding force in the condition of certain temperature and pressure. The engineered bamboo mainly includes the glue laminated bamboo (GLB) and reconstituted bamboo (RB). GLB is a new kind of bamboo man-made board. It is a kind of bamboo engineering material which is made up of a piece of bamboo or a piece of bamboo by anti-corrosion, drying, gluing and other processes. The structural beams of GLB not only have good strength properties, but also have good ductility [5].

Many scholars have studied the mechanical properties of GLB, including tension, compression, shear and bending properties [6-9]. Gottron et al. [10] studied the creep properties of the raw bamboo. But few scholars studied the rheological properties of the glued laminated bamboo. The present work investigates the bending creep properties and builds the rheological model of GLB.

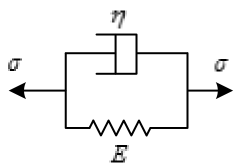
2. Theoretical background

2.1. Burgers rheological model

The simplest way to simulate the combined viscous and elastic behavior of a material is the use of mechanical analogies that include viscous elements (dashpots) and elastic elements (springs). The simplest rheological models are the Maxwell and Kelvin-Voigt models. The Maxwell model, shown in Fig. 1, a, is a combination of a dashpot and spring in series. And the Kelvin-Voigt model, shown in Fig. 1, b, is combination of a dashpot and spring in parallel. The Burgers model, shown in Fig. 2 is a combination of Maxwell and Kelvin-Voigt models in series.



a



b

Fig. 1 The simple rheological models: a - The Maxwell rheological model, b - The Kelvin rheological model

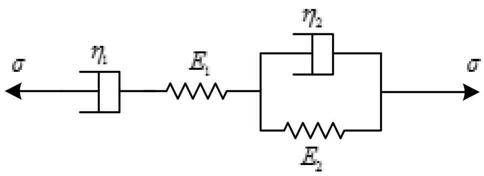


Fig. 2 The Burgers rheological model

Although the Burgers rheological model (BRM) does not represent the structure of the glue laminated bamboo, it has the advantage of being described by a differential equation, whose response to an applied stress can be easily solved analytically, giving a relatively good description of the behavior of the glue laminated bamboo. The total strain is given by Eq. (1):

$$\varepsilon(t)=\frac{\sigma}{E_1}+\frac{\sigma}{E_2}\left[1-exp\left(-t\cdot\frac{E_2}{\eta_2}\right)\right]+\frac{\sigma}{\eta_1}\cdot t\,, \tag{1}$$

where σ is the applied stress, t is the time, E_1 is the instantaneous elastic modulus of the spring and η_1 is the viscosity of the dashpot of the Maxwell element, E_2 is the elastic modulus of the spring and η_2 is the viscosity of the dashpot of the Kelvin-Voigt element. The ratio η_2 / E_2, the relaxation time τ of the element, is a measure of the time required for the extension of the spring to its equilibrium length while retarded by the dashpot; thus, the summation represents the retarded response of the material (decreasing creep rate). The viscous element of the Maxwell element largely contributes to the region of steady state creep.

In order to best fit the experimental data using a nonlinear least-squares optimization algorithm, some authors have preferred to use a phenomenological function of time, as follows:

$$y(t)=A+B\left(1-e^{-ut}\right)+Ct\,, \tag{2}$$

where A, B, C and u are empirically determined parameters and $y(t)$ is the deflection of the bending deformation. Eq. (2) is very simple and has fewer fitting parameters than the Burgers model, but it presents the disadvantage of not being able to predict other material properties.

2.2. Improved Burgers rheological model

The viscosity of the dashpot of the Maxwell element in the Burgers rheological model is constant. In other words, it is the linear viscous body. But in many cases, the viscosity of the material is related to the stress level and loading time. So, we assume that the viscosity coefficient is expressed as follows:

$$\eta_1(\sigma,t)=\eta_0\left(\frac{\sigma}{\sigma_0}\right)^{\beta}\left(\frac{t}{t_0}\right)^{\lambda}, \tag{3}$$

where η_0 is the initial viscosity coefficient of the dashpot of the Maxwell element, σ_0 is the reference stress, t_0 is the reference time, β represents the parameter related to the viscosity coefficient and the stress and λ represents the parameter related to the viscosity coefficient. By substituting formula (3) into formula (1), the improved Burgers rheological model can be obtained:

$$\varepsilon(t)=\frac{\sigma}{E_1}+\frac{\sigma}{E_2}\left[1-exp\left(-\frac{E_2}{\eta_2}t\right)\right]+ \\ +\frac{\sigma}{\eta_0\left(\sigma/\sigma_0\right)^{\beta}\left(t/t_0\right)^{\lambda}}t. \tag{4}$$

The stress is invariable when the creep behavior of material is studied at a certain stress level. Following the form of formula (2) and assuming that is equal to (1), we write Eq. (4) as follows:

$$y(t)=A+B\left(1-e^{-ut}\right)+Ct^{1-\lambda}\,, \tag{5}$$

where A, B, C, u and λ are the parameters of material property, which can be determined by experiments. And $y(t)$ is the deflection.

3. Experimental

3.1. Material

The experimental materials, GLB, are purchased from Taohuajiang bamboo material technology Co. Ltd., Taojiang County, Hunan Province, China. The average density of GLB is 0.63 g/cm^3. The average flexural modulus of elasticity of GLB is 8.3 GPa. The average bending strength is 90.3 MPa. Fig. 3 shows a series of specimens of GLB for creep test. The rectangular dimensions of each specimen were 300 (Length), 30 (Width), 25 (Thickness) mm.

Fig. 3 The specimens of GLB for tests

3.2. Creep tests

Fig. 4 shows the test equipment and loading method. Creep tests were conducted in three-point center concentrated load bending at room temperature. The sample is placed on the supports with the distance 240mm.And the load is placed at the center point (Fig. 5). The specimens were respectively tested under different stress levels 18.06 MPa, 36.12 MPa, 54.18 MPa, 72.24 MPa. The loading duration of each stress level is 15 hours. The displacement is obtained by the dial indicator which is read every 5 minutes within the first hour, every 15 minutes within the first 2-3 hours, every 30 minutes within the 4-5 hours, and every 1 hour within the 6-15 hours.

Fig. 6 shows the creep curves of the glued laminated bamboo under different stress levels. The initial creep stage and steady state creep stage can be observed in the test curve in Fig. 6. But the accelerated creep stage can't be

Fig. 4 The specimens of GLB for tests

observed. In the initial creep stage, the instantaneous elastic deformation increases with the increase of load. And the total amount of creep also increases with load. The creep at low loads increases very slowly. But the creep growth is clearly faster at high loads. However, the damage can't be observed in the short term, which requires a longer trial period.

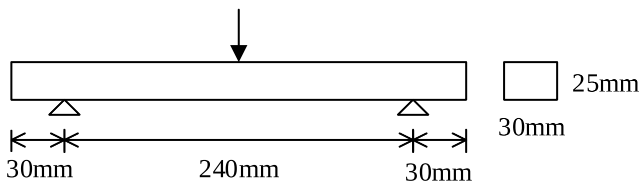


Fig. 5 The loading span and supports

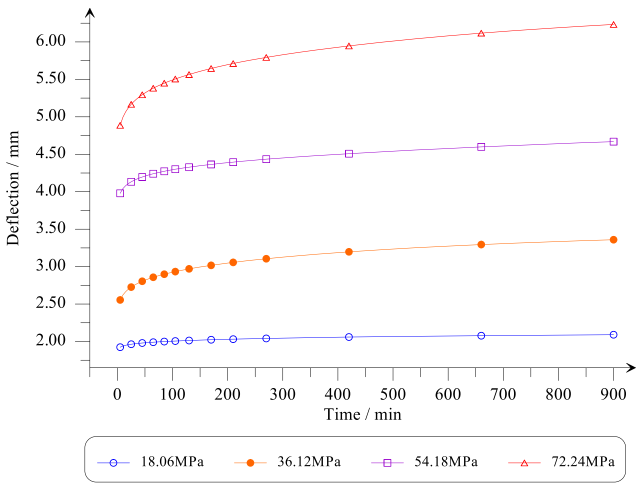


Fig. 6 The creep curves of GLB under different stress levels at room temperature

4. Result analysis and discussion

4.1. Analysis of the Burgers rheological model

According to the Eq. (2), the nonlinear least-squares optimization algorithm is used to fit the creep experimental data of GLB. According to the fitting results, the relevant parameters in Eq. (2) can be obtained under the different stress levels. The parameters of the burgers rheological model are listed in Table 1. RSS stands for the residual sum of squares in Table 1. The standard deviation and correlation of the parameters in Eq. (2) are listed in Table 2. The RSS value is only small when the stress level is 18.06 MPa. The RSS is relatively large under the conditions of other stress levels. The R-squared are all above 0.99. The correlation of parameter *A* is above 0.95, which is the best under four stress levels. The correlation of parameters *u* and *C* is poor, and they are all below 0.9 under four stress levels.

Fig. 7 shows the creep fitting curve of GLB under the different stress levels at room temperature. From the figure, the test data and the fitting curve can match well. According to Fig. 7, the Burgers rheological model can represent the short-term creep of GLB. But the fitting data are less than experimental data in the middle of the curve. And the fitting data are greater than experimental data in the curve tail. The reason is that this part of the deformation is calculated according to the liner viscous deformation of the expression. This shows that the BRM is only suitable for the elastic deformation part of the GLB material.

4.2. Analysis of the improved Burgers rheological model

As described in the previous, the fitting RSS is relatively large. The BRM only adapts to the initial creep stage of GLB material. And BRM is not suitable for the

Table 1

The parameters of the Burgers rheological model of GLB

Stress level	*A*	*B*	*u*	*C*	RSS	R-squared
18.06 MPa	1.93099	0.08798	0.01430	8.58735e-5	5.29173e-4	0.99265
36.12 MPa	2.58004	0.41680	0.01343	4.33848e-4	0.00992	0.99434
54.18 MPa	3.99446	0.32632	0.01677	4.09667e-4	0.00612	0.99486
72.24 MPa	4.92697	0.67641	0.01349	7.50359e-4	0.02613	0.99467

Table 2

The standard deviation and correlation of parameters in BRM of GLB

Stress level	Standard deviation				Correlation			
	A	*B*	*u*	*C*	*A*	*B*	*u*	*C*
18.06 MPa	0.0023	0.0025	9.726e-4	4.116e-6	0.9534	0.9404	0.8851	0.8635
36.12 MPa	0.0096	0.0111	8.521e-4	1.866e-5	0.9508	0.9411	0.8931	0.8755
54.18 MPa	0.0083	0.0084	0.00101	1.261e-5	0.9596	0.9427	0.8637	0.8318
72.24 MPa	0.0155	0.0179	8.546e-4	3.018e-5	0.9509	0.9411	0.8925	0.8747

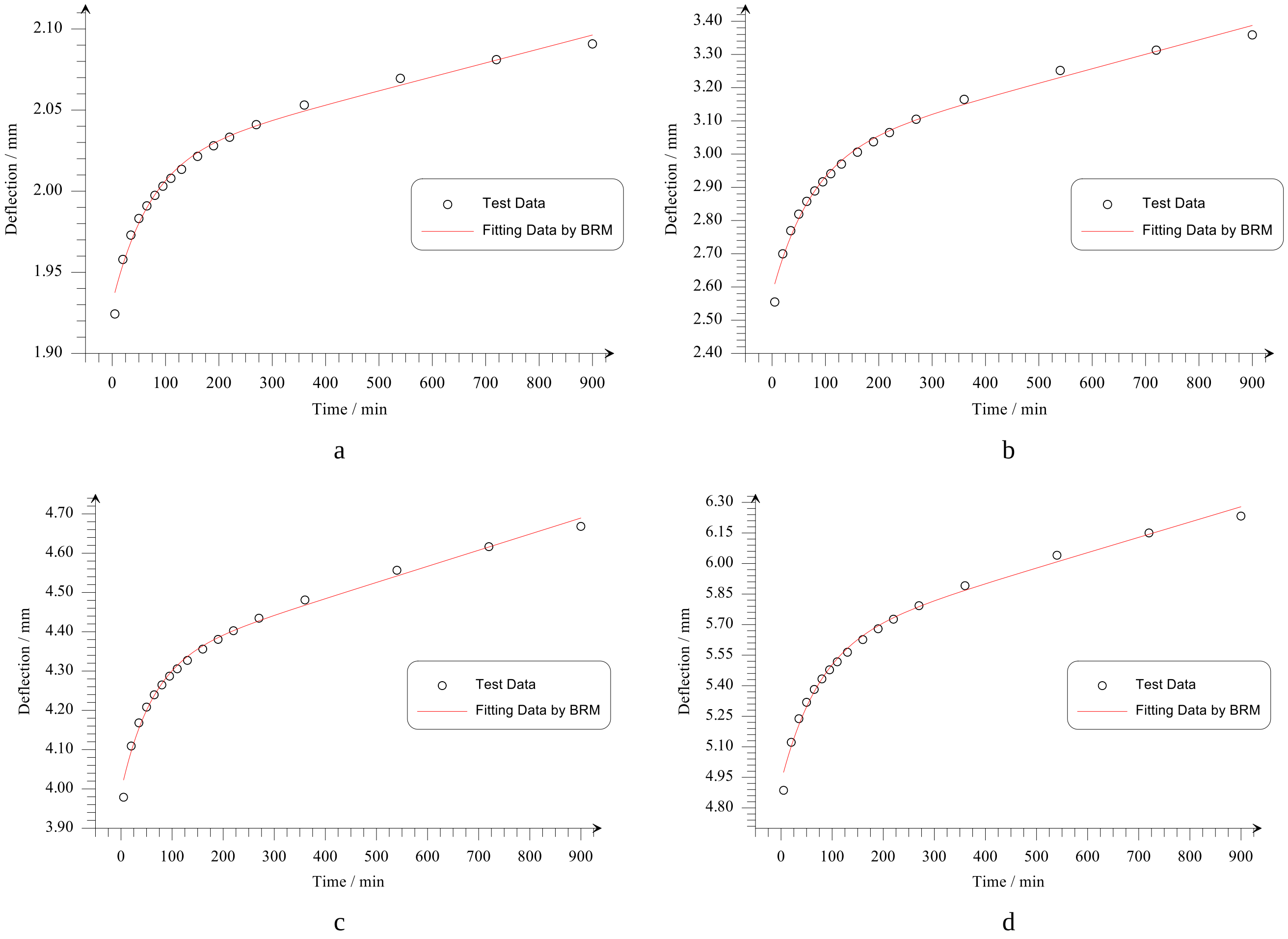


Fig. 7 The creep fitting curve of GLB under different stress levels at room temperature by BRM; a, b, c, and d correspond to stress levels 18.06 MPa, 36.12 MPa, 54.18 MPa, 72.24 MPa respectively

Table 3

The parameters of the improved Burgers rheological model of GLB

Stress level	*A*	*B*	*u*	*C*	*λ*	RSS	R-squared
18.06 MPa	1.57296	0.03213	0.00253	0.31683	0.93639	2.67971e-6	0.99996
36.12 MPa	1.90857	0.09631	0.00292	0.50975	0.85548	1.85892e-5	0.99999
54.18 MPa	3.81406	0.10658	0.04855	0.08749	0.68459	2.33208e-5	0.99998
72.24 MPa	4.00011	0.12650	0.00241	0.67344	0.83121	3.69885e-5	0.99999

Table 4

The standard deviation and correlation of parameters in the Burgers rheological model of GLB

Stress level	Standard deviation					Correlation				
	A	*B*	*u*	*C*	*λ*	*A*	*B*	*u*	*C*	*λ*
18.06 MPa	0.0610	0.0031	7.123e-5	0.0602	0.0104	1.0000	0.9993	0.9749	1.0000	1.0000
36.12 MPa	0.0269	0.0087	1.124e-4	0.0251	0.0052	0.9999	0.9995	0.9898	1.0000	0.9999
54.18 MPa	0.0641	0.5269	1.056e-4	0.0620	0.0059	1.0000	0.9999	0.9999	1.0000	0.9999
72.24 MPa	0.0297	0.0152	1.005e-4	0.0273	0.0047	0.9999	0.9996	0.9894	1.0000	0.9999

steady-state creep and accelerated creep stage of GLB material. So, we use the improved Burgers model formula (5) to fit the data with the nonlinear least-squares optimization algorithm. The parameters of IBRM and the fitting RSS are listed in Table 3. The standard deviation and correlation of the parameters in Eq. (5) are listed in Table 4. It can be seen from Table 3 that the RSS and S-squared of the IBRM method are much smaller than those of the BRM method. The correlation of all parameters of IBRM method is above 0.97. The correlation of parameters *A*, *B*, *C* and λ is closer to or equal to 1 under various stress levels.

Fig. 8 shows the creep fitting curves of experimental data using IBRM at four stress levels. For comparison, the creep fitting curve of BRM is drawn in the figure. According to the Fig. 8, it can be seen that the fitting data of the two models are in good agreement with the experimental data in the initial stage of creep. But the IBRM can better fit the data than the BRM in the middle and the tail of the curve. The reason for this phenomenon is that BRM uses linear viscous body to represent the viscosity of materials. In fact, the viscous deformation of materials is often nonlinear. So, the

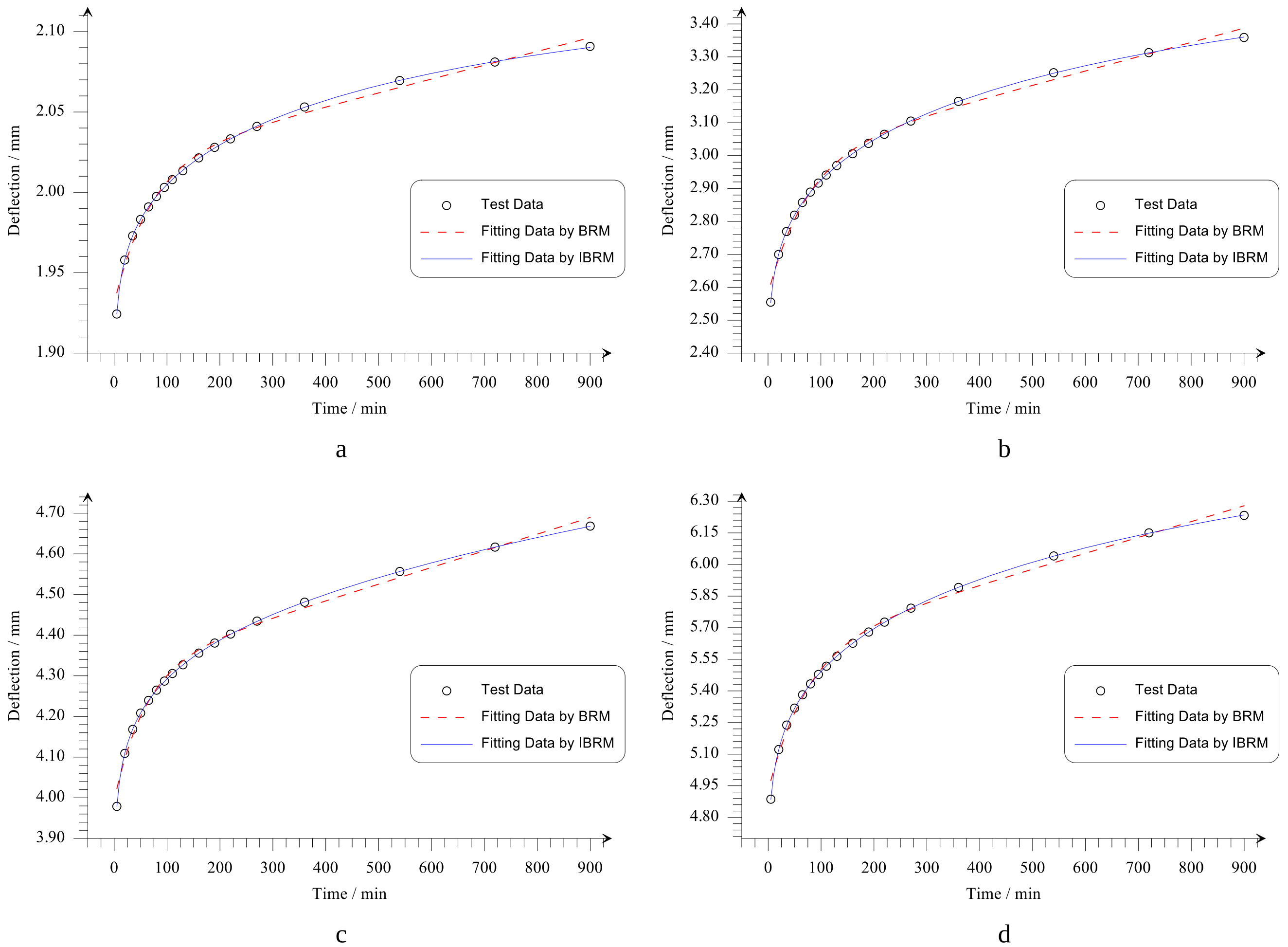


Fig. 8 The creep fitting curve of GLB under different stress levels at room temperature by IBRM. a, b, c, and d correspond to stress levels 18.06 MPa, 36.12 MPa, 54.18 MPa and 72.24 MPa respectively

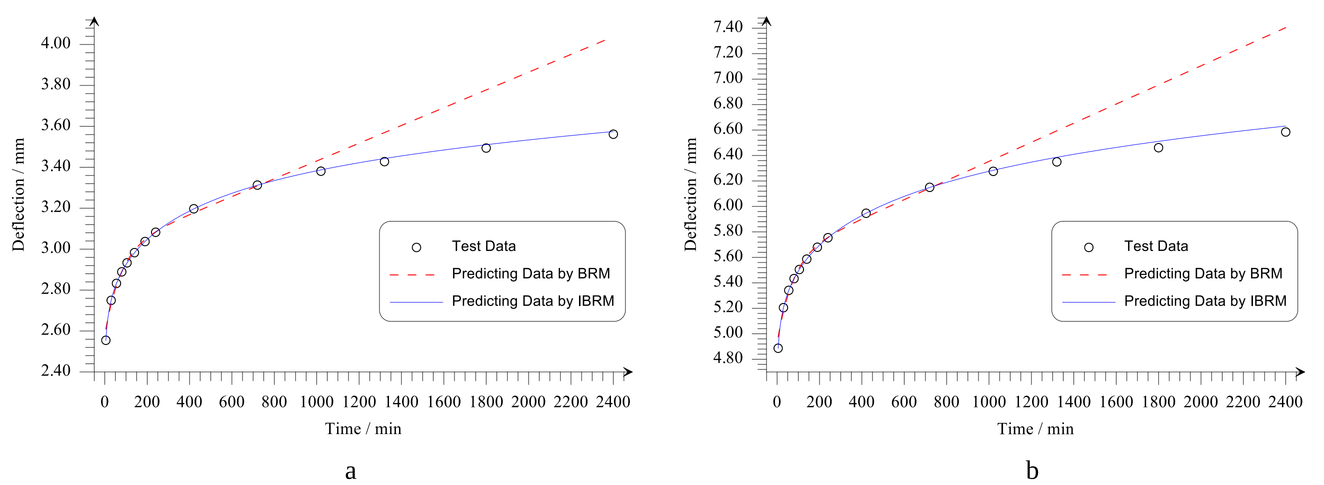


Fig. 9 The prediction results and test data of GLB: a – stress level 36.12 MPa, b –stress level 72.24 MPa

nonlinear dashpot in IBRM can better describe the viscous deformation in the second stage of creep. The IBRM can better describe the creep process of GLB materials.

4.3. Prediction for long time creep properties of GLB

It can be seen from the above analysis that IBRM can better describe the initial stage and the second stage of creep of GLB. In order to further understand the adaptability of the Burgers model, we used the parameters in table 1 and table 3 to predict the long-term creep performance of GLB under 36.12 MPa and 72.24 MPa. In order to compare with the predicted results, a longer creep experiment was carried out on GLB with the same stress level. The loading time is 40 hours. Fig. 9 shows the experimental data and the prediction results of BRM and IBRM under two stress levels. It can be seen from Fig. 9 that the prediction results by IBRM are in good agreement with the test data. It can be further explained that IBRM can describe the initial stage and the second stage of creep properties of GLB. It is proved that it is feasible to introduce nonlinear viscosity coefficient into the burgers rheological model.

5. Conclusions

In this paper, the short-term creep properties of GLB were studied experimentally under four stress levels. In order to explain the creep behavior of GLB, Burgers rheological model was introduced. At the same time, the Maxwell elements in Burgers rheological model is rewritten by introducing nonlinear viscosity coefficient. The Burgers rheological model and the improved Burgers rheological model are used to fit the creep experimental data of GLB. According to the fitting results of nonlinear least square method, both BRM and IBRM can describe the short-term creep behavior of GLB. In the initial stage of creep, the fitting results of the two models are in good agreement with the experimental data. But the fitting result of IBRM is closer to the experimental data than the fitting result of BRM in the second stage of creep. The fitting values of BRM at the end of the curve are larger than the experimental data.

The parameters of BRM and IBRM under four stress levels can be obtained by studying the short-term creep properties of GLB. According to the obtained relevant parameters, BRM and IBRM are used to predict the long-term creep performance of GLB under two stress levels of 36.12MPa and 72.24MPa. In order to compare with the predicted results, creep tests were carried out at two stress levels for 40 hours. Compared with the experimental results, the predicted value of BRM is far greater than the experimental data. And the predicted value of IBRM is more consistent with the experimental data. This shows that IBRM is more suitable than BRM to describe the creep properties of GLB. The modification to the Burgers rheological model is effective.

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ADAPTABILITY OF BURGERS RHEOLOGICAL MODEL AND ITS IMPROVED MODEL TO THE CREEP PROPERTIES OF GLUE LAMINATED BAMBOO

S u m m a r y

The proportion of bamboo material used in industrial production has increased year by year. The glue laminated bamboo (GLB) is a kind of bamboo engineering material. GLB can be used in modern buildings, bridges, furniture and other fields. It is necessary to study its mechanical properties, especially its creep properties. In the paper, the short-term creep performance of GLB is studied at four stress levels. At first, the Burgers rheological model (BRM) and the improved Burgers rheological model (IBRM) are used to study the creep properties of GLB. The parameters of the two models are obtained by fitting the experimental data. The results show that BRM is only suitable for the elastic deformation part of the GLB material. But the nonlinear viscosity coefficient is introduced into IBRM, so it can better describe the creep characteristics of GLB. On this basis, IBRM and BRM are used to predict the long-term creep performance of GLB based on the obtained creep parameters. The results show that the prediction error of BRM is large, and the prediction result of IBRM is in good agreement with the experimental data. It shows that IBRM is more suitable to describe the creep characteristics of GLB.

Keywords: glued laminated bamboo, creep, Burgers rheological model, improved Burgers rheological model.

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