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A probabilistic design of sacrificial cladding for a blast wall using limited statistical information on blast loading

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1. Introduction

A blast wall is a physical barrier separating a vulnerable object from a potential explosion which produces a blast loading capable to damage the object [1]. Blast walls are normally deployed to provide structural protection against military weapons or improvised explosive devices. However, blast walls are in principle suitable to mitigate the level of blast loading generated by accidental explosions occurring in industrial facilities and during a transportation of hazardous goods. Such blast loading is sometimes accompanied by impact of projectiles and spread thermal radiation (e.g., [2]).

Blast wall can be relatively lightweight and weak and still offer some degree of protection because a high level of deformation can absorb a significant amount of the blast wave energy. The cost of rigid, non-destructible walls is often prohibitive and a significant mitigation of blast can be achieved using relatively lightweight frangible or sacrificial walls [1, 3]. The energy of blast loading can be absorbed by lightweight systems used as sacrificial cladding (SC). They can be mounted on the front of a non-sacrificial structure to be protected or serve as a component of a blast wall [4-7].

Studies concerned with the performance of blast walls in providing protection against the damaging effect of blast loading deal, almost exceptionally, with two problems: (i) developing deterministic models of blast-wall interaction and wall behaviour under the blast loading; and (ii) verification of blast wall designs in highly specific experimental set-ups. In either case, characteristics of blast loading and structure subjected to it are (assumed to be) known in advance. It is a paradox that in fact no attention was paid to uncertainties related to this type of loading and structures exposed to it. In other words, the field remains almost fully deterministic.

It is obvious that blast loading generated by attack weapons, terrorist devices and industrial accidents is uncertain to a large measure. Uncertainties of certain degree will be always inherent in mechanical models describing behaviour of blast walls. A consistent quantification of the uncertainties related to blast loading and protective structures subjected to it is possible by a combined application of structural reliability analysis (SRA) and methodological tools developed in the field of quantitative risk assessment (QRA) [8-11].

The problem of uncertainty quantification in the case of blast loading generated during industrial accidents is that such accidents are unique and unexpected events, to a large margin. Post mortem statistical data on blast loading characteristics can be either unavailable or not representative. However, a design of a blast wall can be based on an experimental simulation of an accident, in which blast loading to be mitigated by the wall will be imitated either physically or numerically. A series of such experiments may yield a statistical sample of blast loading characteristics. This sample will contain information on the variability of these characteristics and, indirectly, variability of potential damage to the wall and effects on the object to be protected. With such a sample, a design of blast wall will be possible even in the case when the size of the sample will be small from the standpoint of the classical statistics [12-14]. In addition, elements of this sample can be the so-called uncertain data, that is, data represented by probability distributions and not fixed, crisp values.

The present study describes how to design in a probabilistic way an SC of a blast wall deployed to protect vulnerable object against an accidental explosion. The basic idea is that a cladding failure probability may serve as a measure of explosive damage to the SC. It is shown how to estimate this probability by an approach which combines methods of SRA and QRA. The estimation is based on a separate treatment of stochastic (aleatory) and epistemic (state-of-knowledge) uncertainties related to a mechanical model of SC. The proposed estimation procedure allows also the data on blast loading to be uncertain in the epistemic sense. The study is aimed at increasing safety of industrial facilities and parts transportation infrastructure where accidental explosions can cause major accidents.

2. Amenability of sacrificial cladding to mathematical modelling

SC is generally designed as a multi-layered structure attached to a non-sacrificial frame [4-7, 15]. A building wall to be protected by SC serves as a typical support. A certain degree of energy absorption and dissipation can be achieved also by cladding built as a part of blast walls and supported along some contours, where cladding is attached to the frame of a blast wall [16, 17]. The frame can provide support over most of the cladding area or, alternatively, the support can be reduced to the minimum and be provided by vertical non-sacrificial or less frangible posts (Fig. 1). The configuration of a non-continuous support may influence a production projectiles which after an SC failure may damage the object protected by the wall.

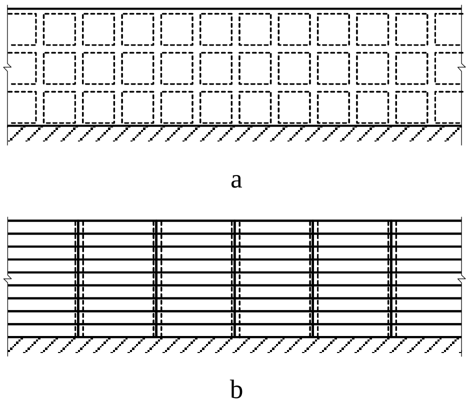

Fig. 1 Non-sacrificial frames supporting sacrificial cladding: (a) densely distributed support; (b) cladding supported by cantilever posts

The possibility to predict the behaviour of SC subjected to blast loading will depend on the presence of mathematical models which allow to obtain a deformation-time relationship and formally express a criterion for SC failure (cessation to accomplish the protective function in the course of blast loading). Studies of SCs and their components published in the recent two decades provide different possibilities of a mathematical modelling of SC behaviour. These studies can be subdivided into four groups:

1. Experimental measurements of SC deformations which do not contain any attempts to carry out a parallel analytical or numerical modelling [18, 19].

2. Experimental studies with a parallel numerical, finite element (FE) modelling [20, 21].

3. Studies on an FE analysis only [17, 22, 23].

4. Studies which develop analytical models only or in addition to experimental measurements and/or FE computations [4-6, 15, 16, 24-26].

Most studies deal with the sandwich cladding which responds to blast loading by a compaction perpendicularly to the continuous base (Fig. 2, a). A closer look at the analytical models allows to conclude that the most of them are based on a single-degree of freedom (SDOF) elastic or elastic-plastic idealisation of an SC fragment which deforms axially along the blast action [15, 17, 22, 24, 25, 27, 28]. Louca et al. [16] apply an SDOF idealisation to a blast wall cladding consisting of profiled steel sections which act as one-way slabs (Fig. 2, b). Bahei-El-Din et al. [17] used an FE analysis to study blast-tolerant sandwich plates which are also idealised as beam elements (Fig. 2, c). Both claddings have some energy dissipation capability; however, their studies do not reveal how to assess the alleviation of blast action transmitted to the supporting frame.

All analytical SC models known to us attempt to predict deformations of individual cladding layers. An interaction between SC and supporting frame is not considered, and so criteria for cladding failure to accomplish the protective function are not expressed formally. However, some authors state that such a criterion should be based on a difference between the energy SC is capable to absorb, E_{absorb}, and the total energy imparted by the blast impulse, E_{blast}. Correspondingly, the failure criterion expressed through a safety margin m (a concept widely used in SRA) may have the form:

$$m = E_{absorb} - E_{blast} \leq 0. \quad (1)$$

Attempts to compute E_{absorb} and E_{blast} were made by Hansen et al. [6] and Ma and Ye [29].

A failure criterion derived from a dynamic modelling of the sandwich cladding shown in Fig. 2, a is presented by Theobald and Nurrick [26]. They relate the failure criterion to a maximum crush distance of the sandwich core, δ_{max} (Fig. 2, a). This distance is used to obtain the time at which compaction of the core occurs, t_c, and compare it to the total cladding response duration t_m. If $t_c \geq t_m$, the cladding will be able to absorb a prescribed blast load. The failure criterion can be expressed through a safety margin m as follows:

$$m = t_c - t_m \leq 0. \quad (2)$$

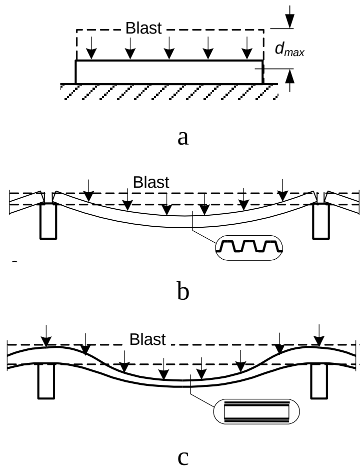

Fig. 2 Types of sacrificial cladding: (a) sandwich cladding attached to a continuous base; (b) profiled section wall supported by posts; (c) sandwich cladding

Louca et al. [16] proposed two safety margins based on a maximum dynamic response of the profiled section cladding shown in Fig. 2, b.

$$m_1 = p_R - y_1 \leq 0; \quad (3a)$$

$$m_2 = u_{pl,max} - u_{pl,dyn} \leq 0, \quad (3b)$$

where p_R is the resistance (dynamic pressure capacity) of the profiled section; y_1 is the reflected peak overpressure of uniformly distributed blast loading; and $u_{pl,max}$ and $u_{pl,dyn}$ are the maximum dynamic plastic deflection and the dynamic plastic deflection due to the blast load, respectively. The above safety margins m_i ($i = 1, 2$) were derived by considering plastic deflection limit, that is, a limit point where all the reserve strength of the profiled section have been utilised. Consequently, the negative values of m_i mean that the profiled section is "sacrificed" and this involves large plastic deformation, possible tearing of welds at supports and potential formation of a projectile.

The failure criterion expressed by Eq. (3b) can be related to the criterion based on the difference of absorbed and imparted energies expressed by Eq. (1). The maximum deflection $u_{pl,max}$ is proportional to the potential energy transferred to the cladding at the end of loading and so the absorbed energy [29]. Therefore, one can state that an occurrence of the event $E_{absorb} - E_{blast} \leq 0$ leads to an occurrence of the event $u_{pl,max} - u_{pl,dyn} \leq 0$, and vice versa.

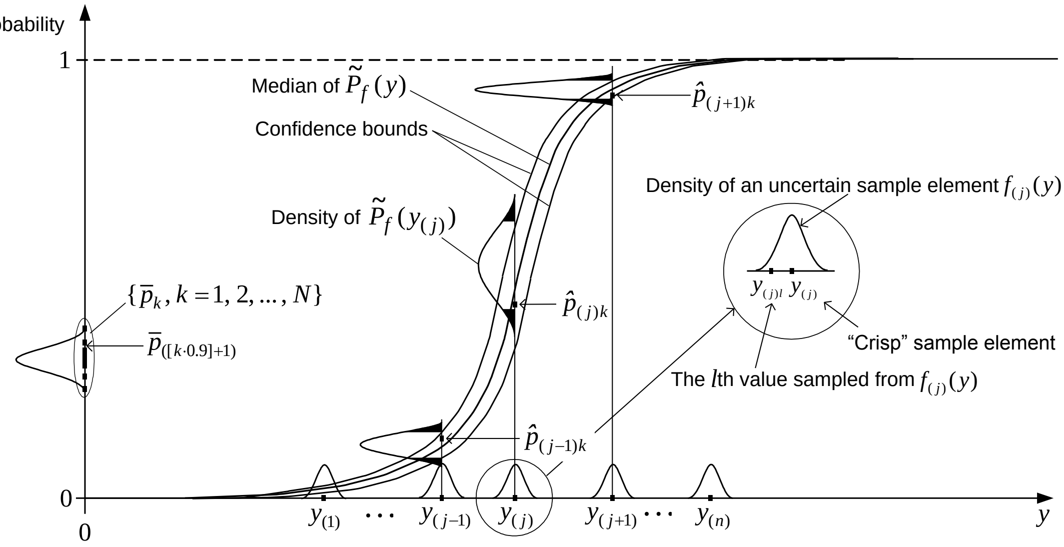

Fig. 3 A schematic illustration of the epistemic uncertainty in the value of the fragility function $\tilde{P}_f(\boldsymbol{y})$

Analytical and numerical models of SC behaviour cited above are purely deterministic on both loading and structural side. The problem of model accuracy (uncertainty) is not considered formally in the aforementioned studies. From an SRA viewpoint, the deterministic analytical and numerical models of SC behaviour, m_i, may serve as a basis for a probabilistic analysis of this protective structure. The need for such an analysis will arise in the case where blast loading can vary to a large degree and is difficult to predict it with fair degree accuracy. Uncertainties can be inherent not only in the loading but also the response of SC to it. A consistent quantification of these uncertainties will generally require to apply methods developed in the field of QRA. By the way, an assessment of potential consequences caused by an accidental explosion is in essence a problem of QRA.

The standard QRA approach to uncertainty modelling is a separate treatment of stochastic and epistemic uncertainties, usually by applying a nested-loop stochastic simulation (e.g., [11, 12]). This simulation will require to evaluate the SC models m_i a large number of times and so the complexity of m_i will be an important factor influencing the computational time.

Attempts to "marry" deterministic FE analysis and uncertainty quantification are well-known in SRA (e.g., [30]). Even though blast wall cladding is a relatively simple mechanical object, FE models expressing the safety margins m_i can be too cumbersome to incorporate them into a nested loop simulation procedure used in QRA for uncertainty propagation. Therefore, the further probabilistic analysis of blast wall cladding will be based on analytical and not numerical FE models, however accurate the latter might be. The objective to be pursued by this analysis will be an estimation of an SC failure probability which can be used as a measure of explosive damage to SC.

3. Failure probability of sacrificial cladding as a measure of damage degree

In the case where all individual components of SC are nominally identical or a continuous SC can be discretised notionally into nominally identical components, a different number of them will fail (will be "sacrificed") at different intensities of reflected blast wave. Characteristics of a pressure history of this wave can be represented by a n_y-dimensional vector $\boldsymbol{y}$ with the components $y_1, y_2, y_3, \ldots, y_{n_y}$ expressing overpressure, positive duration, impulse, etc. ($n_y \geq 1$). Then the relative number of the failed components and so the degree of damage to SC can be estimated by a conditional probability of failure of an individual component:

$$P_f(y) = P(\textstyle\bigcup_i D_i \mid y), \tag{4}$$

where D_i is the random event of damage to an SC component related to the failure mode i (the ith damage event, in brief). The function $P_f(\boldsymbol{y})$ is known in SRA and QRA as a fragility function and its arguments $\boldsymbol{y}$ are called the demand variables (e.g., [31, 32]).

If the blast wave characteristics are uncertain and represented by a random vector $\boldsymbol{Y}$, the unconditional probability of SC component failure, P_f, can be expressed as a mean value of the fragility function $P_f(\cdot)$ with a random arguments $\boldsymbol{Y}$, namely:

$$P_f = \int_y P_f(\boldsymbol{y})\, f_Y(\boldsymbol{y})\, \mathrm{d}\,\boldsymbol{y} = E_Y(P_f(\boldsymbol{Y})), \tag{5}$$

where $f_{\boldsymbol{Y}}(\boldsymbol{y})$ is the joint probability density function of $\boldsymbol{Y}$. Eq. (2) is a standard definition of a failure probability widely used in SRA. The problem of estimating P_f for blast loading generated by an accidental explosion is that statistical data for fitting the model $f_{\boldsymbol{Y}}(\boldsymbol{y})$ will typically be unavailable. However, P_f can be estimated with a small-size sample consisting of observations $\boldsymbol{y}_j$ of $\boldsymbol{Y}$ obtained by experiment [10, 11]. Let this sample be:

$$\boldsymbol{y} = \{\boldsymbol{y}_1, \boldsymbol{y}_2, \ldots, \boldsymbol{y}_j, \ldots, \boldsymbol{y}_n\}. \tag{6}$$

Elements of $\boldsymbol{y}$ can be transformed into fragility function values $P_f(\boldsymbol{y}_j)$ and a new, artificial sample $\{P_f(\boldsymbol{y}_1), P_f(\boldsymbol{y}_2), \ldots, P_f(\boldsymbol{y}_j), \ldots, P_f(\boldsymbol{y}_n)\}$ formed. The latter sample can be used to compute a bootstrap confidence interval $]0, \overline{P}_f[$ for P_f. The closer is the upper limit $\overline{P}_f$ to unity, the larger number of SC components should be expected to be lost in case of an explosion. Consequently, $\overline{P}_f$ can be used as a conservative measure of the damage to a blast wall.

The interval estimate $]0, \overline{P}_f[$ comes from the classical, Fisherian statistics. If necessary, the sample $\boldsymbol{y}$ can be used to estimate P_f in a Bayesian format, namely, by a conservative percentile of a posterior distribution obtained by applying $\boldsymbol{y}$ [12-14].

The form of the sample $\mathbf{y}$ assumes that there are no uncertainties in the data $\boldsymbol{y}_j$. This assumption may not be correct in a number of cases. For example, if the blast wave characteristics are not directly recorded in experiment but are obtained by means of a mathematical modelling, the elements of $\boldsymbol{y}$ can be uncertain (fuzzy). Uncertainty in an individual element of $\boldsymbol{y}$, say, the element j can be quantified by an epistemic probability distribution with the density $f_j(\boldsymbol{y})$ [33]. A one-dimensional visualisation of a crisp and uncertain data points $\boldsymbol{y}_j$ and $f_j(\boldsymbol{y})$ is shown in Fig. 3. The interval estimation of P_f is possible also with the uncertain, as shown in the next section.

4. Dealing with uncertainties in the mechanical model of sacrificial cladding

In the case where the damage event(s) D_i are backed by the model(s) m_i, the fragility function $P_f(\boldsymbol{y})$ can be expressed as:

$$P_f(\boldsymbol{y}) = P(\bigcup_i (m_i(\boldsymbol{Z}, \boldsymbol{y} \,|\, \boldsymbol{\theta}) \le 0)), \tag{7}$$

where $\boldsymbol{Z}$ is the vector of random input variables; $\boldsymbol{\theta}$ is the vector of parameters of the model of $m_i(\cdot)$. The random safety margin $m_i(\boldsymbol{Z}, \boldsymbol{y} \,|\, \boldsymbol{\theta})$ is a standard function of SRA, in which the vector $\boldsymbol{Z}$ and so the function m_i express the stochastic uncertainty (e.g., [34]). The uncertainty modelling prevailing in QRA requires to consider an epistemic uncertainty related to the parameter vector $\boldsymbol{\theta}$ (e.g., [35]). This uncertainty can be expressed by a random vector $\boldsymbol{\Theta}$ with a joint density $\pi(\boldsymbol{\theta})$. One or more components of $\boldsymbol{\Theta}$ can be used to express uncertainty in the accuracy of the model $m_i(\cdot)$. One can interpret the epistemic density $\pi(\boldsymbol{\theta})$ of the as a prior distribution which can be updated, at least in theory, given a new data. Then the posterior density will have the form $\pi(\boldsymbol{\theta} \,|\, \text{data})$.

With the random parameter vector $\boldsymbol{\Theta}$, the fragility function $P(D_i \,|\, \boldsymbol{y})$ becomes an epistemic random variable defined as:

$$\tilde{P}_f(\boldsymbol{y}) = P_f(\boldsymbol{y} \,|\, \boldsymbol{\Theta}) = P(\bigcup_i (m_i(\boldsymbol{Z}, \boldsymbol{y} \,|\, \boldsymbol{\Theta}) \le 0)). \tag{8}$$

An illustration of the random fragility function $\tilde{P}_f(\boldsymbol{y})$ is shown in Fig. 2. This illustration assumes that the vector $\boldsymbol{y}$ has only one component, for instance, the positive overpressure of the reflected blast wave.

The typical approach to dealing with epistemic uncertainties in fragility functions is establishing confidence bounds around the point estimates of fragility curve or median fragilities (e.g., [31]). Most authors consider the confidence bounds the final result of analysis. However, a further propagation of the epistemic uncertainty quantified by $\boldsymbol{\Theta}$ is necessary to estimate the failure probability P_f. In case where the explosion demand $\boldsymbol{y}$ is represented by the small-size sample $\mathbf{y}$, the estimation of P_f can be expressed as an estimation of a mean of fragility function values with uncertain (fuzzy) data $\tilde{P}_f(\boldsymbol{y}_j)$ (j = 1, 2, … , n). Such data can be used for updating a Bayesian prior distribution expressing epistemic uncertainty in P_f [12]. However, if a development of a prior for P_f is problematic or there is no interest in the Bayesian estimation of P_f, the failure probability can be estimated by a Fisherian confidence interval computed by means of a simulation-based procedure explained in the remainder of the present section.

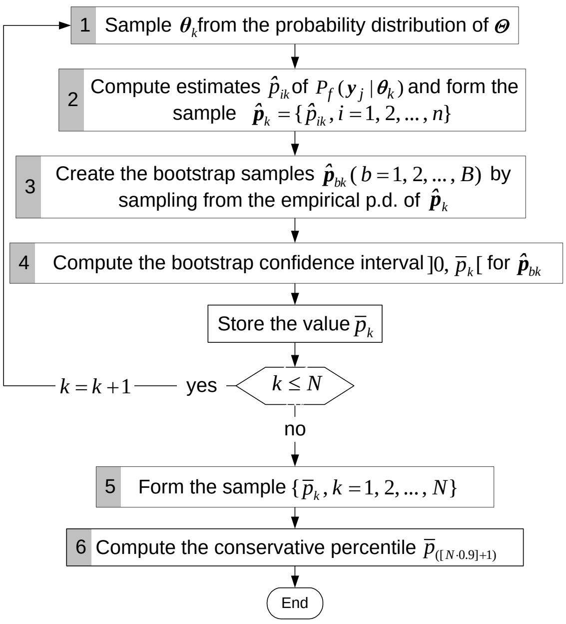

Fig. 4 The flowchart of estimating the failure probability of the sacrificial cladding, P_f

An estimate of P_f can be obtained by computing estimates of the fragility function values $P_f(\boldsymbol{y}_j \,|\, \boldsymbol{\theta}_k)$ for all n elements $\boldsymbol{y}_j$ of the sample $\boldsymbol{y}$ and the values $\boldsymbol{\theta}_k$ of the parameter vector $\boldsymbol{\Theta}$ generated from $\pi(\boldsymbol{\theta})$ or $\pi(\boldsymbol{\theta} \,|\, \text{data})$ (k = 1, 2, … , N). This will require to estimate the fragility function $n \times N$ times. The k-th loop of the estimation of P_f should start from sampling the value $\boldsymbol{\theta}_k$ (Fig. 4, block 1). For each $\boldsymbol{\theta}_k$, the estimates $\hat{p}_{jk}$ of $P_f(\boldsymbol{y}_j \,|\, \boldsymbol{\theta}_k)$ should be computed for all elements of $\boldsymbol{y}$ and grouped into the sample $\hat{\boldsymbol{p}}_k = \{\hat{p}_{jk}, j = 1, 2, ..., n\}$ (Fig. 4, block 2). An illustration of three elements of $\hat{\boldsymbol{p}}_k$ is given in Fig. 2. The sample $\hat{\boldsymbol{p}}_k$ can be used to calculate a one-sided bootstrap confidence interval $]0, \overline{p}_k[$ for P_f (Fig. 4, blocks 3 and 4, see also [11]). A repetition of this process N times will yield a sample consisting of N upper limits of the confidence interval, namely, $\{\overline{p}_k$, k = 1, 2, … , $N\}$ (Fig. 4, block 5). This sample will express the epistemic uncertainty related to the upper limit of this interval (see the abscissa axis in Fig. 2). A conservative percentile of this sample, say, $\overline{p}_{([N \cdot 0.9]+1)}$ can be used as the final result of the conservative estimation of the failure probability P_f (Fig. 4, block 6).

In the case of the uncertain data expressed by the densities $f_j(\boldsymbol{y})$, the procedure of the estimation of P_f can be applied in a similar way, with the difference that some number N_l of the samples $\boldsymbol{y}_l = \{\boldsymbol{y}_{1l}, \boldsymbol{y}_{2l}, \dots, \boldsymbol{y}_{jl}, \dots, \boldsymbol{y}_{nl}\}$ will have to be sampled from the distributions $f_j(\boldsymbol{y})$ ($j = 1, 2, \dots, n$). A one-dimensional illustration of the sample element $\boldsymbol{y}_{jl}$ is given in Fig. 3. The procedure shown in Fig. 4 should be applied to each $\boldsymbol{y}_l$. A repetition of this process N_l times will yield a sample of confidence interval limits, $\{\bar{p}_k,\ k = 1, 2, \dots,\ N \times N_l\}$. A percentile of this sample, say, $\bar{p}_{([N \cdot N_l \cdot 0.9]+1)}$ may serve as a conservative estimate of P_f. Clearly, the estimates $\bar{p}_{([N \cdot N_l \cdot 0.9]+1)}$ will tend to be more conservative than $\bar{p}_{([N \cdot 0.9]+1)}$, because the variability of the limits $\bar{p}_k$ will be larger in the former case than in the latter.

5. Case study

The estimation of the SC failure probability P_f will be illustrated for a blast wall intended to protect against a railway tank car explosion known as boiling-liquid expanding vapour explosion BLEVE [36, 37]. The tank car is used for a transportation of liquefied propane. Mechanical effects of BLEVE occur as blast and projectiles [2]. The present case study will consider the blast loading only whereas the protection against projectiles will be addressed in brief at the end of this section.

The object to be protected by the blast wall is a diesel fuel tank (”target”) located 63 m form external railway tracks (Figs. 5 and 6). The worst case scenario will be considered, according to which the angle of incidence of the blast wave will be equal to 90°(Fig. 7). The fuel tank is surrounded by a protective embankment used to stabilise the blast wall. The wall is to be built from non-sacrificial posts and SC consisting of profiled steel sections (Fig. 8).

Fig. 5 The areal view on the diesel fuel tanks exposed to the danger of a potential BLEVE on rail (authors' photo)

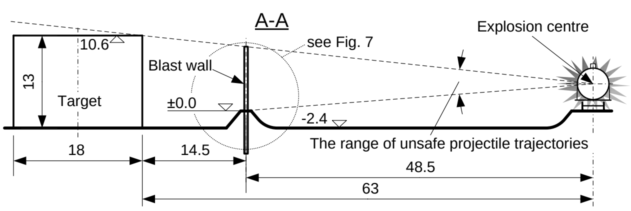

Fig. 6 The elevation of the accident situation (see Fig. 7)

The elements $\boldsymbol{y}_j = (y_{1j}, y_{2j})$ of the sample $\boldsymbol{y}$ will consist of overpressure y_{1j} and positive phase duration y_{2j} of the reflected blast wave, respectively. Experiments which could yield $\boldsymbol{y}$ are very expensive. Therefore, $\boldsymbol{y}$ was obtained by calculation and not by a direct recording $\boldsymbol{y}_j$. The real-world statistical sample used in this case study was compiled from 30 data pairs (x_{1j}, x_{2j}), where x_{1j} and x_{2j} is weight and pressure of liquefied propane in the tank car j, respectively (Table 1, Cols. 2 and 3). The pairs (x_{1j}, x_{2j}) were used to calculate the mass of trinitrotoluene (TNT) which could cause an explosion with an energy equivalent to the energy of BLEVE (Table 1, Col. 4) [2]. The TNT mass and the explosion stand-off equal to 48.5 m were used to calculate y_{1j} and y_{2j} by applying a standard empirical model developed for TNT [38] (Table 1, Cols. 5 and 6).

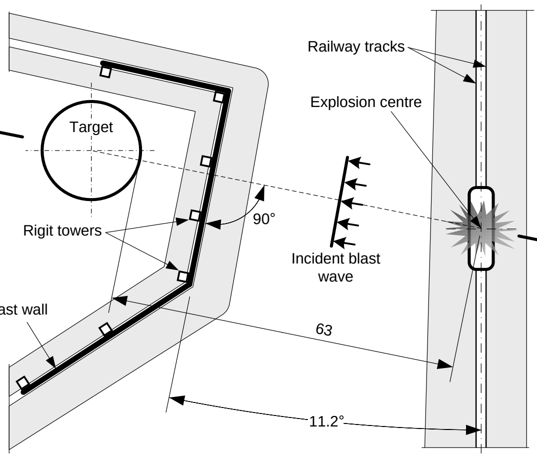

Fig. 7 The plan of the potential accident site

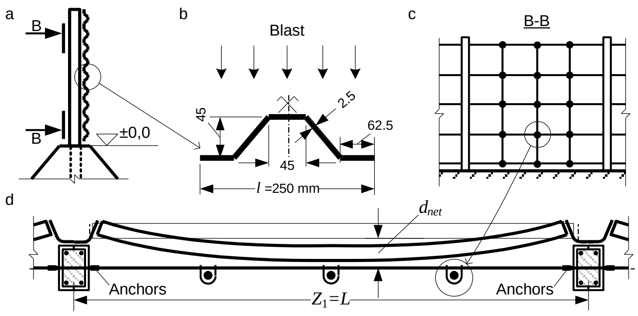

Fig. 8 Details of the blast wall: (a) vertical section; (b) profiled steel section; (c) view from the back showing a safety net; (d) plan

Two random damage events D_1 and D_2 related to the maximum dynamic response of profiled sections and backed by the respective safety margins m_1 and m_2 expressed by Eqs. (3) will be considered. The fragility function $P_f(\boldsymbol{y})$ will have the form $P(D_1 \cup D_2 \mid \boldsymbol{y})$. The safety margins expressed as functions of random variables present in the mechanical model of profiled sections have the form:

$$m_1(\boldsymbol{Z}, \boldsymbol{y} \mid \boldsymbol{\Theta}) = p_R(\boldsymbol{Z} \mid \boldsymbol{\Theta}) - y_1\,; \tag{9a}$$

$$m_2(\boldsymbol{Z}, \boldsymbol{y} \mid \boldsymbol{\Theta}) = u_{pl,\boldsymbol{max}}(\boldsymbol{Z}, \boldsymbol{y} \mid \boldsymbol{\Theta}) - u_{pl,dyn}(\boldsymbol{Z}, \boldsymbol{y} \mid \boldsymbol{\Theta}), \tag{9b}$$

where $\boldsymbol{Z} = (Z_1, Z_2, Z_3, Z_4)$ and $\boldsymbol{\Theta} = (\Theta_1, \Theta_2, \dots, \Theta_5)$ are the vectors used to model aleatory and epistemic uncertainties, respectively (Table 2); $p_R(\cdot)$, $u_{pl,max}(\cdot)$ and $u_{pl,dyn}(\cdot)$ are deterministic functions used to compute quantities given in Eqs. (3).

Probability distributions of the components of $\boldsymbol{Z}$ and $\boldsymbol{\Theta}$ were chosen partly on the basis of information on natural variability of the quantities used in the analysis and partly on the basis of subjective reasoning. Cross-sectional dimensions of profiled sections are considered to be fixed (deterministic) quantities (Fig. 8, b).

Table 1
Characteristics of the reflected blast wave, y_{1j} and y_{2j}

j	x_{1j}, kg	x_{2j},kPa	TNT, kg	y_{1j}, kPa	y_{2j}, ms
1	2	3	4	5	6
1	60939	2575	83.29	13.51	35.45
2	57566	2462	90.30	14.04	35.69
3	57419	2395	77.14	13.04	35.22
4	59472	2602	99.41	14.69	35.97
5	54108	2453	73.21	12.72	35.07
6	56751	2312	66.63	12.18	34.80
7	61307	2615	71.69	12.60	35.01
8	59950	2264	89.97	14.01	35.67
9	55176	2572	74.78	12.85	35.13
10	58094	2531	73.30	12.73	35.07
11	57839	2446	83.50	13.53	35.45
12	58116	2270	52.10	13.42	35.40
13	57777	2424	83.45	13.52	35.45
14	60724	2457	79.33	13.21	35.30
15	56333	2411	77.83	13.09	35.25
16	55878	2193	71.71	12.60	35.01
17	59339	1922	64.05	11.96	34.69
18	52549	2301	64.18	11.97	34.70
19	59697	2364	82.32	13.44	35.41
20	59215	2406	74.52	12.83	35.12
21	60088	2492	86.58	13.76	35.56
22	55379	2581	78.12	13.11	35.26
23	58567	2502	71.68	12.60	35.01
24	53204	2613	73.93	12.78	35.10
25	57594	2204	81.13	13.35	35.37
26	58586	2355	70.36	12.49	34.95
27	53499	2461	78.41	13.14	35.27
28	51802	2508	79.44	13.22	35.31
29	57106	2351	68.00	12.30	34.86
30	55286	2471	83.16	13.50	35.44

The probability distributions of the aleatory random variables Z_1 to Z_3 can be easily specified from information on random properties of steel structures (e.g., [39]). The natural period of elastic vibration, Z_4, is considered to be an aleatory quantity because it can be measured experimentally. We assumed the nominal value of this period, 3.4 ms, given by Louca et al. [16] to be a mean value of a normal distribution of Z_4. The probability distributions of the epistemic variables grouped into the vector $\boldsymbol{\Theta}$ were used to express uncertainty related to parameters of the models $p_R(\cdot)$, $u_{pl,max}(\cdot)$ and $u_{pl,dyn}(\cdot)$. These distributions quantify the doubts expressed by Louca et al. [16] and Juocevičius and Vaidogas [40] about quantities represented by $\boldsymbol{\Theta}$.

The functions on the right-hand side of Eqs. (9) are based on a mechanical model of profiled sections proposed by Louca et al. [16]. The dynamic pressure capacity is given by:

$$p_R(\boldsymbol{Z}\,|\,\boldsymbol{\Theta}) = \frac{8Z_2\Theta_1 w_{el}}{l_E^2(Z_1,\Theta_3)l}\Theta_4\Theta_5\,, \tag{10}$$

where $l_E(\cdot)$ is the effective span; w_{el} is the deterministic elastic section modulus depending on the cross-sectional dimensions; l is the cross-sectional width (Fig. 8, b).

The maximum plastic dynamic deflection capacity is given by:

$$u_{pl,max}(\boldsymbol{Z},\boldsymbol{y}|\boldsymbol{\Theta}) = \frac{5}{48}\frac{Z_2\Theta_1 l_E^2(Z_1,\Theta_3)}{0.5Z_3 h}\times$$
$$\times\frac{\Theta_4\Theta_5}{\Theta_3}\mu(Z_1,Z_2,Z_4,y_1,y_2|\Theta_1,\Theta_3,\Theta_4,\Theta_5)\,, \tag{11}$$

where $\mu(\cdot)$ is the function used to compute the ductility ratio and given by:

$$\mu(\boldsymbol{Z},\boldsymbol{Y}\,|\,\boldsymbol{\Theta}) = \Theta_2\varphi\left(\frac{y_2}{Z_4},\frac{p_R(Z_1,Z_2\,|\,\Theta_1,\Theta_3,\Theta_4,\Theta_5)}{p_r(y_1)}\right), \tag{12}$$

where $\varphi(\cdot,\cdot)$ is the function fitted to the graphs developed by in the book [27] and used for retrieving values of $\mu(\cdot)$.

The dynamic plastic deflection due to the blast load is computed using the following expression:

$$u_{pl,dyn}(\boldsymbol{Z},\boldsymbol{y}\,|\,\boldsymbol{\Theta}) = \frac{p_r(y_1)l_E^4(Z_1,\Theta_1)}{384Z_3}\cdot\delta(y_2)\,, \tag{13}$$

where $\delta(y_2)$ is the dynamic loading factor computed by:

$$\delta(y_2) = \max_t\left(\begin{array}{l|l} 1-\cos(2\pi t/Z_4)+ & \\ +\dfrac{\sin(2\pi t/Z_4)}{2\pi y_2/Z_4}-\dfrac{t}{y_2} & 0\le t\le y_2 \end{array}\right). \tag{14}$$

In the present case study, the ranges of the sample components y_{1j} and y_{2j} are [11.6 kPa, 14.4 kPa] and [23.0 ms, 25.8 ms], respectively. An illustration of the fragility function $P_f(\boldsymbol{y}\,|\,\boldsymbol{\theta}_k)$ estimated for one realisation $\boldsymbol{\theta}_k$ of $\boldsymbol{\Theta}$ is shown in Fig. 9.

Fig. 10 shows a histogram of the sample $\{\bar{p}_k, k=1,2,\ldots,500\}$ obtained by generating 500 values $\boldsymbol{\theta}_k$ and applying the procedure shown in Fig. 4 (N = 500). Exceeding the maximum dynamic plastic deflection (the event D_2) was a dominating failure mode and this failure determined the confidence limits $\bar{p}_k$. The 90-th percentile of the above sample, $\bar{p}_{([N\cdot 0.9]+1)}$, is equal to 0.263. This value is a conservative estimate of the SC failure probability P_f. It means that less than 26.3% of profiled sections will be destroyed ("sacrificied") in case of an explosion. This percentage can be changed as needed by redesigning SC, say, choosing a different profiled section.

A BLEVE produces high-energy projectiles generated by a rupture of tank car vessel [2]. It is highly probable that the blast wall under study will have to sustain an impact by some of them. Therefore, the height of the wall will be governed by unsafe trajectories of potential projectiles (Fig. 6). The profiled sections will not be able to stop larger projectiles and, in our opinion, a safety net should be added behind the cladding (Fig. 8, c and d). The net can be designed to sustain not only primary projectiles from vessel rupture but also profiled sections which will fail under blast loading and/or projectile impact. The space between cladding and safety net, δ_{net}, should allow to reach the maximum dynamic plastic deflection of the profiled sections, $u_{pl,max}$ (Fig. 8, d). As this deflection is a random quantity, the value of δ_{net} can be chosen by reducing the probability $P(u_{pl,\boldsymbol{max}}(\boldsymbol{Z},\boldsymbol{y}\,|\,\boldsymbol{\Theta})\ge\delta_{max})$ to some small and tolerable value.

Table 2

Aleatory and epistemic random variables used in the analysis of the blast wall shown in Fig. 8

Description and notation (notation used this study ≡ notation from the original text by Louca *et al.* [16])	Mean/coeff. of variation	Probability distribution
Aleatory random quantities (components of $\boldsymbol{Z}$)		
Span (spacing of posts) $Z_1 \equiv L$ (m) (see Fig. 8d)	2.0/0.005*	Lognormal
Static yield strength of profiled section steel, $Z_2 \equiv p_y$ (MPa)	554/0.11*	Lognormal
Modulus of elasticity of profiled section steel, $Z_3 \equiv E$ (GPa)	200/0.06*	Normal
Natural period of elastic vibration of profiled sections, $Z_4 \equiv T$ (ms)	3.4/0.05	Normal
Epistemic random quantities (components of $\boldsymbol{\Theta}$)		
Enhancement factor for steel strength, $\Theta_1 \equiv \gamma$; the uncertainty in Θ_1 was modelled by the expression $1 + \Delta\times\xi^{**}$ ($\Delta = 0.12$)	1.012/0.011	Beta, $\xi \sim$ Be(1, 9)
The factor of uncertainty related to the model of ductility ratio μ, Θ_2	1/0.04	Normal N(1, 0.04)
Reduction factor for stiffness of profiled sheet, $\Theta_3 \equiv f_K$; the uncertainty in Θ_3 was modelled by the expression $1 - \Delta\times\xi^{***}$ ($\Delta = 0.3$); the mode of Θ_3 is equal to 0.85	0.85/0.05	Beta, $\xi \sim$ Be(6, 6)
Reduction factor for transverse stress effect, $\Theta_4 \equiv f_C$	0.99/0.085	Beta Be(70, 1)
Reduction factor for flattering of cross-section, $\Theta_5 \equiv f_F$; the uncertainty in Θ_5 was modelled by the expression $1 - \Delta\times\xi^{***}$ ($\Delta = 0.2$); the mode of Θ_5 is equal to 0.952	0.933/0.0382	Beta, $\xi \sim$ Be(2, 4)
* Spaethe [39]; ** This linear transformation is used to obtain a Beta distribution defined on the interval]1, 1.12[which covers potential values of the strength enhancement factor [40]; *** This linear transformation is used to obtain a Beta distribution defined on the interval [Δ, 1]		

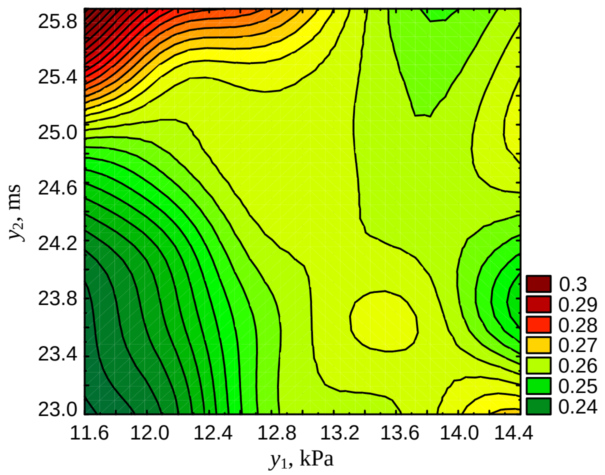

Fig. 9 The surface of the fragility function $\tilde{P}_f(\boldsymbol{y})$ with the demand variables y_1 (peak overpressure) and y_2 (positive phase duration)

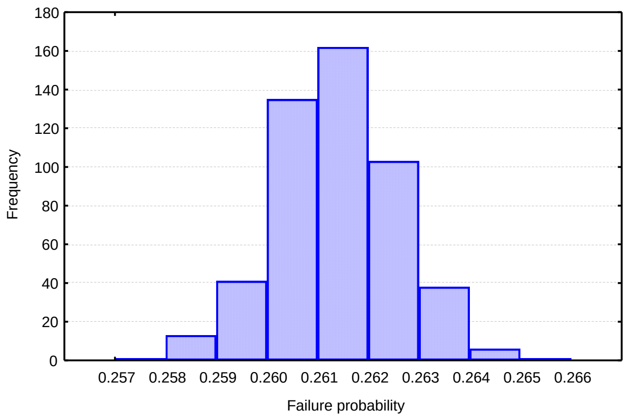

Fig. 10 Histogram of the sample $\{\bar{p}_k, k = 1, 2, \ldots, 500\}$

The horizontal cables of the net can span over several posts. Cable ends can be anchored in rigid towers distributed along the barrier (Fig. 7). Additional anchors can be added where the cables cross the posts (Fig. 8, d). This will add extra stability to the posts and so the cladding. However, a detailed design of safety net, posts, and towers was beyond the scope of this case study.

5. Conclusions

The design of SC for blast walls deployed as protection against accidental explosions has been considered. Such a design may face considerable uncertainties related to potential blast loading. The behaviour of SC components subjected to blast loading may also be uncertain to a large degree. A consistent quantification and propagation of these uncertainties is possible by combining methods of structural reliability analysis and quantitative risk assessment. An application of these methods to an analysis of SC components can yield an estimate of probability of their failure under blast loading. This probability can be used as a measure of explosive damage to SC provided that the SC consists of nominally identical components. A component failure probability will be proportional to the relative number of the components which may fail (be "sacrificed") in case of an explosion.

An estimation of the SC failure probability will require to specify a probabilistic model of blast wave characteristics. Such model can be difficult to obtain as post-mortem data on accidental explosions are rarely available in the amount allowing to compile a statistical sample for fitting the model. However, the SC failure probability can be estimated without such model. A sample of blast loading characteristics recorded in experiment or estimated by explosion simulation can be directly applied to the probability estimation. The size of this sample can be small from the standpoint of the classical statistics. Such estimation can be carried out by a simulation-based propagation of stochastic and epistemic uncertainties through a fragility function developed for an SC component. The estimate will have the form of a one-sided confidence interval of the failure probability. The upper limit of this interval can be used for making decisions concerning the degree of the damage to SC which may be caused by an explosion.

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SUNAIKINAMOJO SPROGIMO BARJERO APDARO PROJEKTAVIMAS NAUDOJANT RIBOTĄ STATISTINĘ INFORMACIJĄ APIE SPROGIMO APKROVĄ

R e z i u m ė

Nagrinėjamas sunaikinamojo apdaro, kuris yra sprogimo barjero dalis, projektavimas. Apdarui projektuoti naudojami konstrukcijų patikimumo teorijos ir kiekybinio rizikos vertinimo metodai. Pagrindinė šio projektavimo idėja – apdaro pažeidimo laipsnį išreikšti jo elementų atsako tikimybe. Ja gali būti vertinama sprogimo sunaikintų apdaro elementų dalis. Apdaro elementų atsako tikimybė yra vertinama kiekybiškai išreiškiant ir transformuojant neapibrėžtumus, susijusius su mechaniniu apdaro elementų modeliu ir statistinės sprogimo apkrovos charakteristikų imties elementais. Parodyta, kaip vertinti apdaro elementų atsako tikimybę, kai tos imties dydis klasikinės statistikos požiūriu yra mažas. Siūloma apdaro elementų atsako tikimybės vertinimo procedūra iliustruojama pavyzdžiu, kaip projektuoti sprogimo barjero, skirto kuro talpyklai apsaugoti nuo avarinio geležinkelio cisternos sprogimo, sunaikinamąjį apdarą.

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A PROBABILISTIC DESIGN OF SACRIFICIAL CLADDING FOR A BLAST WALL USING LIMITED STATISTICAL INFORMATION ON BLAST LOADING

S u m m a r y

A design of sacrificial cladding for a blast wall is considered. Methods of structural reliability analysis and quantitative risk assessment are applied to the design. The basic idea of this design is to apply a probability of failure of cladding components as a criterion of damage to the cladding. This probability is used as an estimate of the proportion of cladding components destroyed by an explosion. The cladding failure probability is estimated by quantifying and propagating uncertainties related to a mechanical model of cladding and elements of the statistical sample containing records of blast loading. It is demonstrated how to estimate the cladding failure probability when the size of this sample is small from the standpoint of classical statistics. The proposed procedure of the cladding failure probability estimation is illustrated by means of a case study. The case study considers a design of cladding for a blast wall to be deployed for a protection of a fuel tank against an explosion of a railroad tank car.

Keywords: blast wall, explosion, blast loading, sacrificial cladding, small-size sample.

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