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Analysis of equivalent substitution of rectangular stress block for nonlinear stress diagram

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1. Introduction

In addition to parabola-rectangular, bilinear and rectangular concrete stress-strain diagrams for the analysis of reinforced concrete flexural members according to STR 2.05.05:2005 [1] and EC2 [2] parabola stress-strain diagram with descending branch can be used as well. It is well-known that parabola stress-strain diagram with descending branch provides the most precise description of stress-strain behaviour of the concrete in comparison with the others. Direct application of such nonlinear diagrams for engineering analysis of cross-sections of flexural and eccentrically compressed reinforced concrete members is inconvenient. Therefore in analysis of rectangular cross-section the rectangular stress block (RSB) is substituted for nonlinear stress diagram (NSD). The said substitution should be equivalent which means that the carrying capacity of reinforced concrete member determined using NSD should be equal to that determined using RSB. This equivalence is provided by coefficients which depend not only on deformative properties of the concrete, i.e. stress-strain character of the diagram, but on the method of diagrams replacement also. The coefficients, obtained according to various methods are different for identical replaced diagrams [3-6]. However the said substitution should be equivalent. In literature we found only few methods dealing with substitution of RSB for NSD [7-9] in analysis of cross-sections of flexural members. Therefore in present publication the principles of substitution of rectangular stress block for nonlinear stress diagram has been analysed.

On the other hand, methods of analysis according to EC2 [2] that we came across in publications, for example [3, 4, 6], are based only on substitution of RSB for parabola-rectangular and bilinear diagrams or on direct application of RSB without any explanation [1, 2]. In publications we were not able to find any simple and convenient engineering method of analysis according to EC2 [2] based on substitution of equivalent RSB for parabola diagram with descending branch. Therefore in this article a method for equivalent substitution of rectangular stress block for nonlinear stress diagram with descending branch is developed when the stress-strain relationship for the concrete in compression is described according to EC2 [2]. Analytical relationships, in explicit form, for area, the first moment of area and coordinate of centroid of the nonlinear stress diagram with descending branch was obtained. An explicit analytical relationship for the ratio between the depth of the rectangular stress block and that of the equivalent nonlinear stress diagram with descending branch in respect to the concrete strength was obtained. A linear approximation of the ratio between the depths of these diagrams in relation to the concrete strength was proposed as well. Coefficients suitable for substitution of equivalent rectangular stress block for parabola stress diagram with descending branch given in EC2 and STR 2.05.05:2005 are presented. Qualitative and quantitative analysis of the coefficient for substitution of diagrams given in various codes is performed as well.

2. An existing methods for replacement of diagrams

According to [7] the substitution of RSB to NSD is accomplished by multiplying the area of NSD by coefficient α

$$\alpha=\frac{\int_0^{x_w}\sigma_c(x)dx}{x_w f_c} \quad (1)$$

where x_w is the depth of a nonlinear diagram, f_c and $\sigma_c(x)$ are concrete strength and concrete stress function in relation to the depth of the cross-section respectively. This substitution is not equivalent because it does not provide equal coordinates for centroids of nonlinear diagram and RSB. This substitution gives only equal areas of mentioned diagrams. Methods allowing determination of a coefficient for substitution of RSB for parabola-rectangle diagram are presented in [3, 8]. Other method for substitution of diagrams is given in [9]

$$\alpha=\alpha_1/(2\beta_1) \quad (2)$$

$$\beta=2\beta_1 \quad (3)$$

where

$$\alpha_1=\frac{1}{f_c}\int_0^{x_w}\sigma_c(x)dx \quad (4)$$

$$\beta_1=1-\int_0^{x_w}x\sigma_c(x)dx \bigg/ \int_0^{x_w}\sigma_c(x)dx \quad (5)$$

This method provides equivalent substitution of diagrams, since both areas and coordinates of centroids for the diagrams involved are obtained equal. Partial analysis of the substitution of the diagrams is performed in [6,10,11]. In [6] analysis of the substitution of rectangular stress block for rectangular-parabola diagram is given and [10,11] deals with the analysis of substitution of RSB for nonlinear stress diagram with descending branch. It is shown in these investigations that calculation methods for cross-sections of flexural reinforced concrete members according to EC2 [2], STR 2.05.05:2005 [1] and SNiP [12] are incompatible with nonlinear stress diagram with descending branch defined in EC2 [2] and STR 2.05.05:2005 [1].

3. The main relationships

Let us assume that a nonlinear stress-strain relationship of the concrete in compression is described by a function

$$\sigma_c = f(\varepsilon_c) \tag{6}$$

where ε_c is strain of the concrete in compression. If the hypothesis of plane sections is valid then failure of a bending member occurs when the strain in concrete under the highest compression, in the compression zone, achieves its ultimate value. Then strain in the concrete under compression may be described by the following linear function (Fig. 1, a)

$$\varepsilon_c(x) = \varepsilon_{cu} - \frac{\varepsilon_{cu} x}{x_w}, \; 0 \le x \le x_w \tag{7}$$

where ε_{cu} and x_w are ultimate compressive strain in the concrete and the depth of concrete compression zone respectively. Putting Eq. (7) in to Eq. (6) it is obtained the general relationship between the stress in compression zone of a flexural member and coordinate x (see Fig. 1, b)

$$\sigma_c(x) = f\left(\varepsilon_{cu} - \frac{\varepsilon_{cu} x}{x_w}\right) \tag{8}$$

As it was mentioned above, for simplification of the calculations the equivalent RSB is substituted for NSD of the concrete in compression zone (Fig. 1, b)).

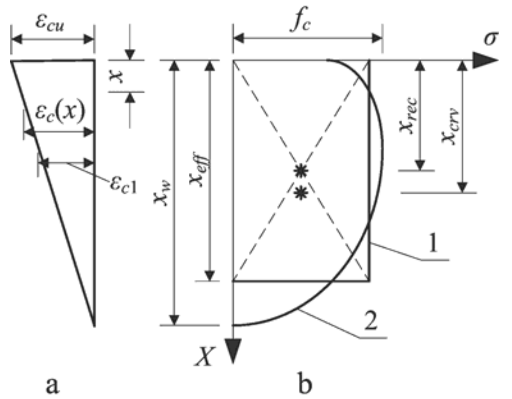

Fig. 1 Ultimate stress-strain state for a flexural member: a - distribution of strains along the compression zone depth; b - *1* and *2* are rectangular stress block and nonlinear stress diagrams respectively, x_{eff} and x_w are depths of rectangular stress block and of nonlinear diagram respectively, x_{rec} and x_{crv} are coordinates of centroids for rectangular stress block and for nonlinear diagram respectively

Diagrams are equivalent if the areas and coordinates of their centroids are equal respectively, i.e. when the following conditions are satisfied

$$A_{rec} = A_{crv} \tag{9}$$

$$x_{crv} = x_{rec} \tag{10}$$

where A_{rec} and A_{crv} are the areas of the RSB and NSD, while x_{rec} and x_{crv} are the coordinates of their centroids, respectively (Fig. 1).

According to EC2 [2] and STR [1] the parabolic relationship between stress $\sigma_c(\varepsilon_c)$ and strain ε_c for the concrete in compression are as follows

$$\sigma_c(\varepsilon_c) = \frac{\varepsilon_c (k\varepsilon_{c1} - \varepsilon_c) f_{cm}}{\varepsilon_{c1}(k\varepsilon_c - 2\varepsilon_c + \varepsilon_{c1})} \tag{11}$$

where $k = 1.1 E_{cm} |\varepsilon_{c1}| / f_{cm}$ and ε_{c1} is the concrete strain at the maximum stress, i.e. when $\sigma_c = f_{cm}$ [1,2]

$$\varepsilon_{c1}(f_{cm}) = -0.7 f_{cm}^{\;0{,}31} \cdot 10^{-3} \tag{12}$$

where f_{cm}=f_{ck}+8 and E_{cm} (in MPa) are mean value of concrete cylinder strength in compression and secant elasticity modulus of the concrete, respectively, while ε_c is the concrete strain in compression which varies within the limits of $0 \le \varepsilon_c \le \varepsilon_{cu1}$. Ultimate strain defined by the descending branch of the stress-strain diagram for the concrete in compression in EC2 [2] and STR 2.05.05:2005 [1] is denoted by ε_{cu1}

$$\left.\begin{array}{l} \varepsilon_{cu1} = -3.5 \cdot 10^{-3}, \text{when } 8\,\text{MPa} \le f_{ck} \le 50\,\text{MPa} \\ (8) \\ \varepsilon_{cu1}(f_{cm}) = -2.8 - 27\left(\dfrac{98 - f_{cm}}{100}\right)^4 10^{-3} \\ \qquad \text{when } f_{ck} \ge 50\,\text{MPa} \end{array}\right\} \tag{13}$$

where f_{ck} is concrete characteristic cylinder strength in compression.

Putting of (7) in too (11), taking $\varepsilon_{cu} = \varepsilon_{cu1}$ according to notations used in EC2[2] and STR 2.05.05:2005 [1] and collecting of terms the following relationship between the stress and the coordinate x of the cross-section depth is obtained [10]

$$\sigma_c(x) = \frac{\varepsilon_{cu1}(x_w - x)(k\varepsilon_{c1} x_w - \varepsilon_{cu1}(x_w - x)) f_{cm}}{x_w \varepsilon_{c1}(\varepsilon_{cu1}(x_w - x)(k-2) + \varepsilon_{c1} x_w)} \tag{14}$$

or

$$\sigma_c(x) = f_{cm} k(x) \tag{15}$$

where

$$k(x) = \frac{\varepsilon_{cu1}(x_w - x)(-\varepsilon_{cu1}(x_w - x) + k\varepsilon_{c1} x_w)}{x_w \varepsilon_{c1}(\varepsilon_{cu1}(x_w - x)(k-2) + \varepsilon_{c1} x_w)} \tag{16}$$

For the modification of relationship (14) relative parameters, namely, the maximum relative strain

$$\omega_{cu1} = \varepsilon_{c1} / \varepsilon_{cu1} \tag{17}$$

and the relative coordinate for the layer of concrete compression zone

$$\omega_x = x/x_w \tag{18}$$

are introduced.

Then the relationship (14) can be modified in the following form

$$\sigma_c(\omega_x) = f_{cm}\frac{(1-\omega_x)(k\omega_{cu1}-(1-\omega_x))}{\omega_{cu1}((1-\omega_x)(k-2)+\omega_{cu1})} \tag{19}$$

Using notation

$$k(\omega_x) = \frac{(1-\omega_x)(k\omega_{cu1}-(1-\omega_x))}{\omega_{cu1}((1-\omega_x)(k-2)+\omega_{cu1})} \tag{20}$$

the function of stress for the concrete compression zone is finally expressed in the following way

$$\sigma_c(\omega_x) = f_{cm}k(\omega_x) \tag{21}$$

The area and the first moment of area of the nonlinear diagram can be calculated using the well-known relationships

$$A_{crv} = \int_0^{x_w}\sigma_c(x)dx \tag{22}$$

$$S_{crv} = \int_0^{x_w}x\sigma_c(x)dx \tag{23}$$

Putting $x = \omega_x x_w$ in the above integrals and taking into account condition (21) the following forms for relationships (22) and (23) are obtained

$$A_{crv} = \int_0^1\sigma_c(\omega_x)d(\omega_x x_w) = f_{cm}x_w\int_0^1 k(\omega_x)d\omega_x = = f_{cm}x_w A_{k,crv} \tag{24}$$

$$S_{crv} = \int_0^1 x_w\omega_x\sigma_c(\omega_x)d(x_w\omega_x) = = f_{cm}x_w^2\int_0^1\omega_x k(\omega_x)d\omega_x = f_{cm}x_w^2 S_{k,crv} \tag{25}$$

where

$$S_{k,crv} = \int_0^1\omega_x k(\omega_x)d\omega_x \tag{26}$$

$$A_{k,crc} = \int_0^1 k(\omega_x)d\omega_x \tag{27}$$

In the case when the stress diagram is described by the relationship (14) then integration of integrals (26) and (27) gives

$$A_{k,crv} = \frac{(k-1)^2}{(k-2)^2} - \frac{1}{2(k-2)\omega_{cu1}} + \frac{\omega_{cu1}(k-1)^2(ln(\omega_{cu1})-ln(Y))}{(k-2)^4} \tag{28}$$

$$S_{k,crv} = \frac{(k-1)^2(k+2\omega_{cu1}-2)}{(k-2)^3} - \frac{1}{6(k-2)\omega_{cu1}} + \frac{\omega_{cu1}(k-1)^2(ln(\omega_{cu1})-ln(Y))}{(k-2)^4} \tag{29}$$

where $Y = k + \omega_{cu1} - 2$.

Coordinate x_{crv} of the centroid for NSD area A_{crv} is determined from relationships (24) and (25)

$$x_{crv} = \frac{S_{crv}}{A_{crv}} = x_w\frac{\int_0^1\omega_x k(\omega_x)d\omega_x}{\int_0^1 k(\omega_x)d\omega_x} = x_w\frac{S_{k,crv}}{A_{k,crv}} \tag{30}$$

and then the relative coordinate ω_{crv} of the centroid for NSD area A_{crv} can be expressed by equation

$$\omega_{crv} = \frac{x_{crv}}{x_w} = \frac{\int_0^1\omega_x k(\omega_x)d\omega_x}{\int_0^1 k(\omega_x)d\omega_x} = \frac{S_{k,crv}}{A_{k,crv}} \tag{31}$$

Values of ω_{crv} are presented in Table 1 in relation to the characteristic strength of the concrete and they are plotted in Fig. 2 as well.

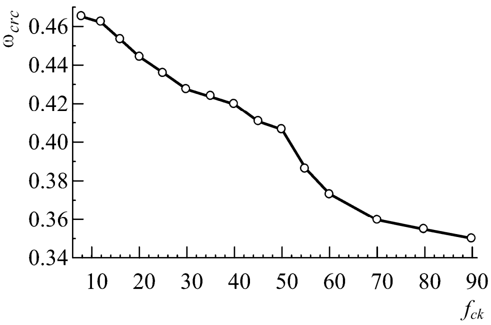

Fig. 2 Relation between relative coordinate of the centroid ω_{crv} and characteristic strength of the concrete

It can be seen see that ω_{crv} decreases and consequently the centroid of NSD moves upwards with the increase in concrete strength. Since condition (10) is valid and the depth of equivalent RSB is equal to $2\omega_{crv}$ then the depth of RSB decreases with the increase in concrete strength.

4. Analysis of equality of areas of nonlinear stress diagram and rectangular stress block

Replacement of NSD by the equivalent RSB should meet condition (10). In such case the depth x_{eff} and the area A_{rec} of the RSB should be determined as follows

$$x_{eff} = 2x_{crv} = 2\omega_{crv}x_w \tag{32}$$

$$A_{rec} = x_{eff}f_{cm} = 2f_{cm}\omega_{crv}x_w \tag{33}$$

Then taking into consideration Eq. (31) the ratio between the areas of RSB and NSD can be expressed by

$$\frac{A_{rec}}{A_{crv}} = \frac{2 f_{cm} x_w \omega_{crv}}{f_{cm} x_w \int_0^1 k(\omega_x) d\omega_x} = \frac{2\omega_{crv}}{A_{k,crv}} = \frac{2S_{k,crv}}{A_{k,crv}^2} \quad (34)$$

In STR 2.05.05:2005 [1] replacement of NSD with RSB is performed by multiplication of the concrete strength by coefficient $\alpha<1$. From Eqs. (21) and (22) it is obvious that in fact the area A_{rec} of RSB is decreased by the coefficient α. In STR 2.05.05:2005 [1] coefficient α is determined by

$$\alpha = 0.9, \text{when } f_{ck} \leq 50\text{MPa} \quad (35)$$

$$\alpha = 0.9 - \frac{f_{cm}-58}{200}, \text{when } f_{ck} > 50\,\text{MPa} \quad (36)$$

According to EC2 [2] for replacement of NSD by RSB two coefficients η and λ are used. Physical meaning of the coefficient η is the same as that of α in STR [1]. It is expressed by the following formulae

$$\eta = 1, \text{when } f_{ck} \leq 50\text{MPa} \quad (37)$$

$$\eta = 1 - \frac{f_{cm}-58}{200}, \text{when } f_{ck} > 50\,\text{MPa} \quad (38)$$

Further the influence of coefficients η and α on the ratio between the areas of diagrams expressed by Eq. (34) will take place. In addition this ratio will be investigated using coefficient 0.9. It will be shown that when coefficients η, α and 0.9 are used, the ratio expressed by Eq. (34) equals to: $\eta A_{rec}/A_{crv}$, $\alpha A_{rec}/A_{crv}$, $0.9 A_{rec}/A_{crv}$. Let us assume that $f_{cm,\theta} = f_{cm}\theta$, where $\theta \in \{\alpha, \eta, 0.9, 1\}$. Then the area of RSB in calculation of the ratio according to Eq. (34) should be taken with the coefficient θ. Using Eq. (34) it can be written

$$\frac{2 f_{cm,\theta} x_w \omega_{crv}}{f_{cm} x_w \int_0^1 k(\omega_x) d\omega_x} = \theta \frac{2 f_{cm} x_w \omega_{crv}}{f_{cm} x_w \int_0^1 k(\omega_x) d\omega_x} = \theta \frac{2\omega_{crv}}{\int_0^1 k(\omega_x) d\omega_x} = \theta \frac{A_{rec}}{A_{crv}} \quad (39)$$

Values of $\theta A_{rec}/A_{crv}$ ratio, ω_{crv} and $A_{k,crv}$ determined according to Eqs. (34), (31) and (27) respectively are presented in Table 1 and Fig. 3. It is indicated here that the variation of $\theta A_{rec}/A_{crv}$ and $0.9 A_{rec}/A_{crv}$ ratios with characteristic strength of the concrete is not uniform. For all concrete classes the ratio $\theta A_{rec}/A_{crv}>1$ with exception of C55/67 and C60/75 classes for which the ratio $0.9A_{rec}/A_{crv}<1$ shall be taken. Therefore in general it can be concluded that the substitution of RSB for NSD, when condition (10) is valid, requires that $A_{rec} > A_{crv}$. Declination from 1 of the ratio $\theta A_{rec}/A_{crv}$ grows with the increase in concrete strength starting at the class of C60/75. Thereby the quality of equivalence for substitution of the diagrams decreases.

Table 1

Values of $\theta A_{rec}/A_{crv}$, A_{crv} and ω_{crv}

Concrete class	A_{rec}/A_{crv}	$0.9 A_{rec}/A_{crv}$	$\alpha A_{rec}/A_{crv}$	$\eta A_{rec}/A_{crv}$	$A_{k,crv}$	ω_{crv}	$1/(A_{rec}/A_{crv})$
C8/10	1.173	1.056	1.056	1.173	0.796	0.467	0.852
C12/15	1.201	1.081	1.081	1.201	0.769	0.462	0.832
C16/20	1.185	1.067	1.067	1.185	0.765	0.453	0.844
C20/25	1.166	1.049	1.049	1.166	0.762	0.444	0.858
C25/30	1.158	1.042	1.042	1.158	0.753	0.436	0.863
C30/37	1.143	1.029	1.029	1.143	0.748	0.427	0.875
C35/45	1.151	1.036	1.036	1.151	0.736	0.424	0.869
C40/50	1.157	1.041	1.041	1.157	0.725	0.420	0.864
C45/55	1.136	1.022	1.022	1.136	0.723	0.411	0.880
C50/60	1.138	1.024	1.024	1.138	0.714	0.406	0.878
C55/67	1.103	0.993	0.875	0.975	0.700	0.386	0.907
C60/75	1.105	0.995	0.850	0.950	0.675	0.373	0.905
C70/85	1.149	1.034	0.800	0.900	0.626	0.360	0.870
C80/95	1.176	1.059	0.750	0.850	0.603	0.355	0.850
C90/105	1.193	1.073	0.700	0.800	0.587	0.350	0.839

As it was mentioned above the substitution of the diagrams is of higher equivalency when $\theta A_{rec}/A_{crv}$ ratio is closer to 1. If the ratio $\theta A_{rec}/A_{crv}$ is an approximation of 1 then error for the said ratio may be:
the maximum error

$$d_1(1;\theta A_{rec}/A_{crv}) = \max|1-\theta A_{rec}/A_{crv}| \quad (40)$$

and the mean square error

$$d_2(1;\theta A_{rec}/A_{crv}) = \int_a^b (1-\theta A_{rec}/A_{crv})^2 df_{cm}, a<b \quad (41)$$

where $\theta \in \{\alpha, \eta, 0.9, 1\}$. Values of $d_1(1;\theta A_{rec}/A_{crv})$ determined for various values of θ and intervals of [a,b] are given in Table 2. It should be noted that the integral in Eq. (41) is solved taking values of ε_{c1} and ε_{cu1} according to Eqs. (12) and (13) but not these from Tables given in [1,2].

It can be seen from Table 2 that value RSB area is the closest to that of NSD when coefficients 0.9 and α are taken according to STR [1] in the case of $(8 \leq f_{ck} \leq 50)$ MPa. In this case the minimum values of errors $d_1(1;\theta A_{rec}/A_{crv})$ and $d_2(1;\theta A_{rec}/A_{crv})$, i.e. $\min(d_1(1;\theta A_{rec}/A_{crv})) =$ 0.081 and $\min(d_2(1;\theta A_{rec}/A_{crv})) =$ =0.07735 when $\theta = 0.9$ are obtained. The worst quality

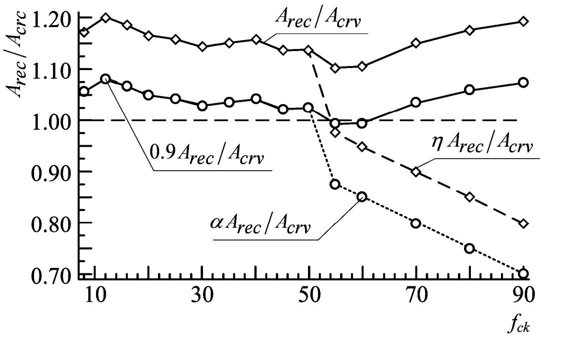

Fig. 3 Variation of ratios between the areas of rectangular stress block and nonlinear stress diagram with concrete characteristic strength

Table 2

Values of errors $d_1(1;\beta A_{rec}/A_{crv})$ and $d_2(1;\beta A_{rec}/A_{crv})$

θ	$d_1(1;\theta A_{rec}/A_{crv})$	f_{ck}^*	$d_2(1;\theta A_{rec}/A_{crv})$
8 MPa $\le f_{ck} \le$ 90 MPa			
1	0.201	12	1.8213
0.9	0.073	90	0.13856
α	0.300	90	0.47155
η	0.201	12	1.08118
8 MPa $\le f_{ck} \le$ 50 MPa			
1	0.201	12	1.00254
0.9	0.081	12	0.07735
α	0.081	12	0.07735
η	0.201	12	1.00254
50 MPa $\le f_{ck} \le$ 90 MPa			
1	0.193	90	0.81874
0.9	0.073	90	0.06121
α	0.300	90	0.3942
η	0.200	90	0.07864

*Note: in this table f_{ck} is concrete characteristic strength at which the maximum value of error d_1 is obtained

substitution of diagrams is obtained by direct replacement of nonlinear diagram with RSB without application of any coefficient i.e. $\theta=\eta=1$ according to EC2 [2]. For this case the values of the said errors are equal to 0.201 and 1.00254, and when f_{ck}=12 MPa the ratio of $\theta A_{rec}/A_{crv}$=1.201. If $(50 \le f_{ck} \le 90)$ MPa then the area of RSB is closest to that of NSD when coefficient 0.9 is considered as well. For this case $\min(d_1(1;\theta A_{rec}/A_{crv}))$ = 0.073 and $\min(d_2(1;\theta A_{rec}/A_{crv}))$ = 0.06121 when θ=0.9. The suitability of other coefficients for the substitution of diagrams cannot be unambiguously defined. In assessment according to $d_1(1;\theta A_{rec}/A_{crv})$ the coefficient 1 is in the second place, i.e. θ=1, η is in the third, and α in the fourth places respectively. In assessment according to $d_2(1;\theta A_{rec}/A_{crv})$ the coefficient η is in the second, α in the third and 1 in the fourth places respectively.

Investigations in the ratio $\theta A_{rec}/A_{crv}$ give an opportunity to state for the interval of $(8 \le f_{ck} \le 90)$ MPa that the most equivalent substitution of the diagrams is obtained when coefficient 0.9 is taken. It is clearly shown in Table 2 and Fig. 4. Coefficient 0.9 is the closest to the ratio $1/(A_{rec}/A_{crv})$. The maximum value of $0.9A_{rec}/A_{crv}$ does not reach 1.08 (see Table 1). Therefore it can be stated that for practical use the RSB can be equivalently substituted for the NSD taking coefficient 0.9. One can see in Fig. 4 that the ratio $1/(A_{rec}/A_{crv})$ decreases with on increase in concrete characteristic strength starting from 50 MPa. Tendency of changes in the values of coefficients α and η remains the same but the gradient of decrease is much greater. Values of $1/(A_{rec}/A_{crv})$ given in Table 1 can be used for equivalent substitution of RSB for NSD. Then conditions (9) and (10) will be satisfied. When intermediate concrete strength values between concrete strength classes given in codes are considered then the ratio $1/(A_{rec}/A_{crv})$ can be obtained by interpolation between the nearest values of the said ratio.

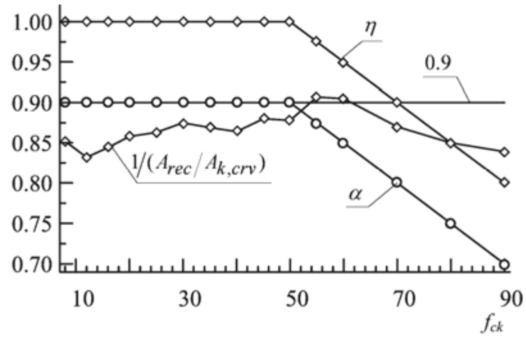

Fig. 4 Variation of $1/(A_{rec}/A_{crv})$ ratio and coefficients α and η with characteristic concrete strength

It was mentioned above that in the case of validity of conditions (9) and (10) the ratio $\beta A_{rec}/A_{crv}$ shall be equal to 1, i.e. $\beta A_{rec}/A_{crv}$=1. It means that $\beta A_{rec}/A_{crv}$ is an approximation of 1. This coefficient can be calculated by means of minimizing the mean square error Eq. (41). Let us assume that β is a constant and $\beta A_{rec}/A_{crv}$ approaches 1, the measure of error for which is Eq. (41). Then coefficient β can be calculated by means of minimization of the mean square error of the ratio $\beta A_{rec}/A_{crv}$

$$d_2(1;\beta A_{rec}/A_{crv}) = \int_a^b (1-\beta A_{rec}/A_{crv})^2 df_{cm} \tag{42}$$

Minimum value of the coefficient β will be obtained by differentiation of Eq. (42) in respect to β and by equating the obtained relationship to 0

$$2\int_a^b \frac{A_{rec}}{A_{crv}} df_{cm} - 2\beta \int_a^b \frac{A_{rec}^2}{A_{crv}^2} df_{cm} = 0 \tag{43}$$

Taking into account Eq. (34)

$$\beta = \frac{\int_a^b S_{k,crv}/A_{k,crv}^2 \, df_{cm}}{2\int_a^b \left(S_{k,crv}/A_{k,crv}^2\right)^2 df_{cm}} \tag{44}$$

Values of coefficient β calculated numerically for various intervals [a,b] are given in Table 3. Values of the maximum error $d_1(1;\beta A_{rec}/A_{crv})$ and of the mean-square error $d_2(1;\beta A_{rec}/A_{crv})$ are given in this Table as well.

Table 3 shows that the difference between the values of β calculated for various intervals is not great. Comparison of $d_1(1;\theta A_{rec}/A_{crv})$ with $d_1(1;\beta A_{rec}/A_{crv})$ and $d_2(1;\beta A_{rec}/A_{crv})$ with $d_2(1;\theta A_{rec}/A_{crv})$ points out that

$$d_1(1;\beta A_{rec}/A_{crv}) < d_1(1;\theta A_{rec}/A_{crv}) \tag{45}$$

$$d_2(1;\beta A_{rec}/A_{crv})<d_2(1;\theta A_{rec}/A_{crv}) \quad (46)$$

where $\theta\in\{\alpha,\eta,0.9,1\}$.

Table 3

Values of β, $d_1(1;\beta A_{rec}/A_{crv})$ and $d_2(1;\beta A_{rec}/A_{crv})$

Interval MPa	β	$d_1(1;\beta A_{rec}/A_{crv})$	$d_2(1;\beta A_{rec}/A_{crv})$
$8\le f_{ck}\le 50$	0.867	0.041	0.05633
$50\le f_{ck}\le 90$	0.877	0.046	0.03395
$8\le f_{ck}\le 90$	0.872	0.051	0.09028

It means that the new values of β allow performing more equivalent substitution of RSB for NSD. When the coefficient β is used, the maximum values of the ratio $\beta(A_{rec}/A_{crv})$ = 1.041 and $\beta(A_{rec}/A_{crv})$ = 1.046 are obtained when f_{ck} = 12 MPa and f_{ck} = 90 MPa respectively. The calculated values of coefficient β are close to 0.85 the value of the coefficient being used for substitution of the diagrams in ACI [13] and SNB [14,15]. According to DIN [16] coefficients used for substitution of RSB for parabola-rectangle σ_c-ε_c diagram depend on concrete characteristic strength. When $f_{ck}\le 50$ MPa then the value of coefficient for diagram substitution is equal to 0.95, when f_{ck}>50 then the value of this coefficient equals to 1.05-f_{ck}/500 [7].

More accurate description of the coefficient β is possible taking linear variation of this coefficient with the concrete strength. However, obtained accuracy for practical application is quite sufficient. Obviously the most accurate value of coefficient β is

$$\beta=\frac{1}{A_{rec}/A_{crv}}=\frac{A_{crv}}{A_{rec}} \quad (47)$$

These values are given in Table 1.

In [10] the correctness of mathematical problem formulation for the substitution of RSB for nonlinear stress diagram with descending branch and the possibility of equivalent replacement of these diagrams in cross-section carrying capacity calculations was investigated. It was determined that in case of satisfied Eq. (9) the condition (10) is not satisfied. The relative difference (ratio (x_{crv}-x_{rec})/x_{rec})) in coordinates of centroids for the said diagrams varies from 12% to 21%. If the area of RSB is determined using coefficient 0.9 then the relative difference does not exceed 9%.

5. Analysis of ratio between depths of nonlinear stress diagram and rectangular stress blocks

Below, the ratio between the depths of nonlinear stress diagram with descending branch according to EC2 [2] and STR [1] and of the equivalent RSB (see (48)) is investigated

$$x_{eff}/x_w \quad (48)$$

This ratio is compared with x_{eff}/x_w ratio used in various codes. In general the ratios x_{eff}/x_w used in various codes differ.

In STR [1] the ratio x_{eff}/x_w is noted by symbol ω

$$\omega_{STR}=a-0.008f_{cd} \quad (49)$$

where a is a coefficient depending on concrete type: for normal weight concrete a=0.85, for fine grain concrete of A group a=0.80 and for that of B group a=0.75 respectively; for lightweight concrete a=0.80, f_{cd} is design strength of the concrete in MPa.

In EC2 [2] the ratio x_{eff}/x_w is noted by λ. The physical meaning of this coefficient is the same as that of ω in STR [1] and in SNiP [12].

$$\left.\begin{aligned}&\lambda=0.8 \text{ when } f_{ck}\le 50\,\text{MPa}\\&\lambda=0.8-\frac{f_{ck}-50}{400}, \text{ when } f_{ck}>50\,\text{MPa}\end{aligned}\right\} \quad (50)$$

In SNiP [12] the ratio x_{eff}/x_w also is noted by ω

$$\omega_{SNiP}=a-0.008f_{cd,pris} \quad (51)$$

where $f_{cd,pris}$ is design value of concrete prism strength according to SNiP [12], coefficient a is the same as in Eq. (49). In mentioned above codes coefficient ω is the ratio between the depth of RSB and that of the real NSD [17]. Since in the code SNiP [12] NSD is not presented then it is possible to say that the ratio between the depths of RSB and NSD, which is not described by a function, is expressed by the Eq. (51).

If condition (10) is satisfied then for each concrete strength class the ratio between the depths of the RSB and NSD is

$$\omega=\frac{x_{eff}}{x_w}=\frac{2x_{crv}}{x_w}=2\omega_{crv} \quad (52)$$

Coefficient ω_{crv} values determined by Eq. (31) are given in Table 1. As it was shown above Eq. (9) is valid when the area of RSB is multiplied by coefficient Θ

$$\Theta x_{eff}f_{cm}=f_{cm}x_wA_{k,crv} \quad (53)$$

Then from Eq. (53) the ratio of ω is obtained

$$\omega=\Theta\frac{x_{eff}}{x_w}=\frac{1}{\Theta}A_{k,crv} \quad (54)$$

If it is assumed that $\Theta=1/(A_{rec}/A_{crv})$ then putting this expression into Eq. (54) and taking into account Eq. (34) the following is obtained

$$\omega=\frac{A_{crv}}{A_{rec}}A_{k,crv}=2\omega_{crv} \quad (55)$$

On the basis of the Eqs. (52) and (55) an important conclusion can be made that for determination of the ratio between depths of the equivalent RSB and NSD, i.e when conditions (9) and (10) are valid, calculation according to the relationship (55) is sufficient. For determination of ω ratio coefficient Θ is not required. On the basis of Eq. (55) it can be concluded too that ω directly does not depend on the concrete strength. Thus the value of $2\omega_{crv}$ and consequently that of ω for the same concrete strength class but of different design strength is the same. For example, according to EC2 [2] and STR [1] the design strengths for the same strength class concrete are different but the ratio $\omega=x_{eff}/x_w$ will be the same.

Eqs. (52), (54) and (55) indicate that coefficient ω can be considered not only as deformability characteristic of concrete in compression [17] or as ratio between the depths of RSB and NSD but as the depth of RSB in normalized coordinates as well. From Eq. (52) it is obtained that $\omega=2\omega_{crv}=2x_{rec}/x_w=2x_{eff}/x_w$. Coefficient ω can be considered as the area of nonlinear diagram described in normalized coordinates divided by the concrete compression strength (see Eq. (54)). This relationship also shows that deformability characteristic ω of concrete compression zone is described by location of resultant of stresses of that zone and evaluates relative position of gravity centre for the actual stress diagram in concrete compression zone in respect with the concrete layer in the greatest compression. The values of coefficient ω determined according to Eqs. (52), (54) and (55) are given in Table 4 and Fig. 5. The values of $\omega=A_{k,crv}/\beta$ in this table are determined using values of β from Table 3 when f_{ck} varies within the limits of $8 \le f_{ck} \le 50$ and of $50 \le f_{ck} \le 90$.

Table 4

Ratio ($\omega=x_{eff}/x_w$) between the depth of rectangular stress block and that of nonlinear stress diagram according to Eqs. (52), (54) and (55)

Concrete strength class	$\omega=2\omega_{crv}$	$\omega=A_{k,crv}$	$\omega=A_{k,crv}/0.9$	$\omega=A_{k,crv}/\alpha$	$\omega=A_{k,crv}/\eta$	$\omega=A_{k,crv}/\beta$
C8/10	0.934	0.796	0.884	0.884	0.796	0.918
C12/15	0.924	0.769	0.855	0.855	0.769	0.887
C16/20	0.907	0.765	0.850	0.850	0.765	0.882
C20/25	0.889	0.762	0.847	0.847	0.762	0.879
C25/30	0.872	0.753	0.837	0.837	0.753	0.868
C30/37	0.855	0.748	0.831	0.831	0.748	0.862
C35/45	0.847	0.736	0.818	0.818	0.736	0.849
C40/50	0.839	0.725	0.806	0.806	0.725	0.837
C45/55	0.821	0.723	0.803	0.803	0.723	0.834
C50/60	0.813	0.714	0.793	0.793	0.714	0.824
C55/67	0.773	0.700	0.778	0.800	0.718	0.799
C60/75	0.746	0.675	0.750	0.794	0.711	0.770
C70/85	0.719	0.626	0.696	0.782	0.696	0.714
C80/95	0.709	0.603	0.670	0.804	0.709	0.688
C90/105	0.700	0.587	0.652	0.839	0.734	0.669

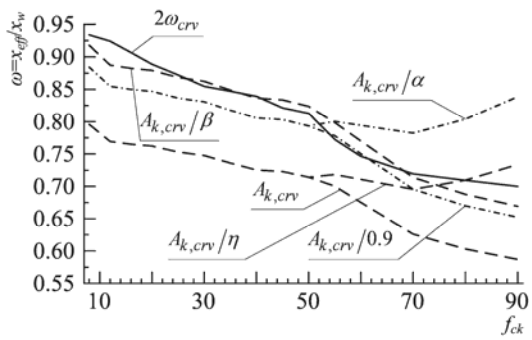

Fig. 5 Depth of rectangular stress block to that of nonlinear stress diagram ratio in relation to the concrete characteristic strength

It can be seen from the Fig. 5 and the Table 4 given above that variation of the ratio $\omega=x_{eff}/x_w$ with concrete class is almost linear. This figure also points out that agreement of the value of $\omega=A_{k,crv}/\beta$ with the exact value of the ratio of $2\omega_{crv}$ is the best of all. If the exact value of the ratio x_{eff}/x_w according to (55) is $2\omega_{crv}$ and $A_{k,crv}/\Theta$ is an approximation of $2\omega_{crv}$ then as the measure of error for $A_{k,crv}/\Theta$ can serve the following functions

the maximum absolute error

$$d_1(2\omega_{crv};A_{k,crv}/\Theta)=max|2\omega_{crv}-A_{k,crv}/\Theta| \quad (56)$$

and the mean square error

$$d_2(2\omega_{crv};\Theta 2\omega_{crv})=\int_a^b (2\omega_{crv}-A_{k,crv}/\Theta)^2\, df_{cm},\ a<b \quad (57)$$

where $\Theta\in\{\alpha,\eta,0.9,1,\beta\}$.

Table 5 shows that for the total concrete strength range the values of $\omega=A_{k,crv}/\Theta$ are the nearest to ratio $2\omega_{crv}$ or to the x_{eff}/x_w when $\Theta=\beta$ since $d_1(2\omega_{crv};A_{k,crv}/\beta)<d_1(2\omega_{crv};A_{k,crv}/\theta)$ and $d_2(2\omega_{crv};A_{k,crv}/\beta)<d_2(2\omega_{crv};A_{k,crv}/\theta)$, where $\theta\in\{1,0.9,\alpha,\beta\}$. In the interval of ($8 \le f_{ck} \le 50$) MPa according to proximity of $\omega=A_{k,crv}/\Theta$ to the ratio of x_{eff}/x_w, coefficients α and 0.9 are in the second place, 1 and η - in the third place. When ($50 \le f_{ck} \le 90$) MPa according to d_1 and d_2 the coefficients 0.9 and η are in the second and the third place respectively. In the forth place is coefficient 1, i.e. $\Theta=1$.

Table 5

Values of errors $d_1(2\omega_{crv};A_{k,crv}/\Theta)$ and $d_2(2\omega_{cvr};A_{k,crv}/\Theta)$

Θ	$d_1(2\omega_{crv};A_{k,crv}/\Theta)$	f_{ck}^*	$d_2(2\omega_{cvr};A_{k,crv}/\Theta)$
8 MPa $\le f_{ck} \le$ 50 MPa			
1	0.155	12	0.5772
0.9	0.069	12	0.0566
α	0.069	12	0.0566
η	0.155	12	0.5772
β	0.037	12	0.0131
50 MPa $\le f_{ck} \le$ 90 MPa			
1	0.113	90	0.3250
0.9	0.048	90	0.0278
α	0.139	90	0.2620
η	0.054	55	0.0410
β	0.031	90	0.0184
8 MPa $\le f_{ck} \le$ 90 MPa			
1	0.155	12	0.9020
0.9	0.069	12	0.0840
α	0.139	90	0.3190
η	0.155	12	0.6180
β	0.037	12	0.0315

*Note: f_{ck} is the value of concrete characteristic strength at which the maximum error d_1 is obtained

Now approximations of the ratio x_{eff}/x_w obtained by different methods applied in codes STR [1] (49), EC2 [2] Eq. (50) and SNiP [12] Eq. (51) will be compared between themselves and with exact values of the ratio x_{eff}/x_w according to Eq. (55). Eqs. (49), (50) and (51) point out that ω_{STR}, ω_{SNiP} and λ depend on different arguments: f_{ck}, f_{cd}, and $f_{cd,pris}$. Therefore for comparison of the said quantities their mathematical expressions are transformed in such a way that argument in all these functions is the same. In SNiP [12] the relation between design compressive strength of concrete prism and characteristic compressive cube strength $f_{ck,cube}$ is given. In STR [1] Eq. (49) and in

EC2 [2] Eq. (50) relations between design cylinder strength f_{cd}, characteristic cylinder strength f_{ck} and characteristic cube strength $f_{ck,cube}$ are given. Therefore Eqs. (49), (50) and (51) are transformed in such a way that in all of them there is only one argument $f_{ck,cube}$. Moreover, values of ω_{STR}, ω_{SNiP} and λ obtained by transformed relationships in respect of $f_{ck,cube}$ should be equal to the values obtained by not transformed relationships in respect of the original variable.

In STR [1] f_{cd} and $f_{ck,cube}$ are related by the following relationships

$$\left.\begin{aligned} f_{cd} &= \frac{\alpha}{\gamma_c} f_{ck} = 0.8\frac{\alpha}{1.5} f_{ck,cube}, \text{when } f_{ck} \le 50\,\text{MPa},\\ f_{cd} &= \frac{\alpha}{\gamma_c} f_{ck} = 0.8\frac{\dfrac{\alpha\, f_{ck,cube}}{1.5}}{1.1 - f_{ck}/500},\ f_{ck} > 50\,\text{MPa} \end{aligned}\right\} \tag{58}$$

where γ_c is safety factor for concrete strength given in [1], 0.8 is equal to the ratio of $f_{ck}/f_{ck,cube}\approx 0.8$, $f_{ck,cube}$ is characteristic cube strength used in [1], a is a coefficient defined by Eqs. (35) and (36). Putting Eq. (58) into Eq. (49) one gets

$$\omega_{STR} = a - 0.008 \cdot 0.8 \cdot \frac{\alpha}{\gamma_c} f_{ck,cube} \tag{59}$$

Putting Eqs. (35) or (36) and (58) into Eq. (59) the following relationship is obtained

$$\left.\begin{aligned} \omega_{STR} &= a - 0.00384 f_{ck,cube}, \text{when } f_{ck} \le 50\,\text{MPa}\\ \omega_{STR} &= a - 0.0054 f_{ck,cube} + 2.66\cdot 10^{-5} f_{ck,cube}^2 -\\ &\quad - 2.73\cdot 10^{-8} f_{ck,cube}^3, \text{when } f_{ck} > 50\,\text{MPa} \end{aligned}\right\} \tag{60}$$

In EC2 [2] f_{ck} and $f_{ck,cube}$ are related by

$$f_{ck} = 0.8 f_{ck,cube} \tag{61}$$

Putting Eq. (61) into Eq. (50) results

$$\left.\begin{aligned} \lambda &= 0.8,\ \text{when } f_{ck} \le 50\,\text{MPa}\\ \lambda &= 0.8 - \frac{0.8 f_{ck,cube} - 50}{400},\ \text{when } f_{ck} > 50\,\text{MPa} \end{aligned}\right\} \tag{62}$$

The relation between prism and cube characteristic concrete strengths in SNiP [12] defined by [17] is presented below

$$\left.\begin{aligned} f_{cd,priz} &= \frac{f_{ck,cube}}{\gamma_{c,SNiP}}(0.77 - 0.00125 f_{ck,cube})\\ f_{cd,priz} &\ge 0.72 f_{ck,cube}/\gamma_{c,SNiP} \end{aligned}\right\} \tag{63}$$

Expression $(0.77 - 0.00125 f_{ck,cub})$ in [17] is referred to as coefficient of the prism strength and it is indicated that the value of its coefficient of variation can reach (10-15)%. Putting Eq. (63) into Eq. (51) and collecting of terms gives

$$\left.\begin{aligned} \omega_{SNiP} &= a - 6.15\cdot 10^{-3}(0.77 - 1.25\cdot 10^{-3} f_{ck,cube}) f_{ck,cube}\\ \omega_{SNiP} &\le a - 0.00443 f_{ck,cube} \end{aligned}\right\} \tag{64}$$

The Eqs. (60), (62) and (64) show that in general the ω_{STR} and ω_{SNiP} depend on concrete characteristic cube strength in different way. These relationships, when a=0.85, are shown in Fig. 6 and Table 6. In this table values of ω_{SNiP} are given for concrete strength class up to C50/60 since in SNiP [12] characteristic cube strength for the concrete is considered just up to 60 Mpa.

Table 6

Values of coefficient ω by relationships Eqs. (60), (62) and (64) in respect to $f_{ck,cube}$ when a=0.85

Concrete class	f_{cd} by (58)	$f_{cd,priz}$ by (63)	ω_{STR} by (60)	λ by (62)	ω_{SNiP} by (64)
C8/10	4.800	5.827	0.812	0.800	0.803
C12/15	7.200	8.668	0.792	0.800	0.781
C16/20	9.600	11.462	0.773	0.800	0.758
C20/25	12.000	14.207	0.754	0.800	0.736
C25/30	15.000	16.904	0.735	0.800	0.715
C30/37	18.000	20.599	0.708	0.800	0.685
C35/45	21.000	24.923	0.677	0.800	0.651
C40/50	24.000	27.692	0.658	0.800	0.629
C45/55	27.000	30.462	0.639	0.800	0.606
C50/60	30.000	33.231	0.620	0.800	0.584
C55/67	33.000	37.108	0.600	0.788	-
C60/75	36.000	41.539	0.583	0.775	-
C70/85	42.000	47.077	0.567	0.750	-
C80/95	48.000	52.615	0.554	0.725	-
C90/105	54.000	58.154	0.545	0.700	-

Fig. 6 shows that $\lambda > \omega_{STR} > \omega_{SNiP}$ when $(15 \le f_{ck,cube} \le 105)$ MPa. In the interval of $(10 \le f_{ck,cube} \le 105)$ MPa $\omega_{STR} > \omega_{SNiP}$. It may be caused by the fact that the value of $f_{cd,}$ according to STR [1] and EC2 [2] is less than that of $f_{cd,priz}$ by SNiP [12] as it is shown in Table 6. However as it was mentioned earlier the factor in Eqs. (49) and (51) is of the same value - 0.08. Therefore actual difference between ω_{SNiP} and ω_{STR} is due to different safety factors for materials applied in SNiP [12] and in STR [1] ($\gamma_{c,SNiP}$=1.3, γ_c=1.5) and due to factor a according to Eqs. (35) and (36). When $(10 \le f_{ck,cube} \le 61)$ MPa $\omega = 2\omega_{crv} > \lambda$, and when $(62 \le f_{ck,cube} \le 105)$ MPa then $\omega = 2\omega_{crv} < \lambda$. Fig. 6 points out that in the interval of $(15 \le f_{ck,cube} \le 105)$ MPa coefficient λ in its value is the nearest to the ratio of $\omega = 2\omega_{crv} = x_{eff}/x_w$.

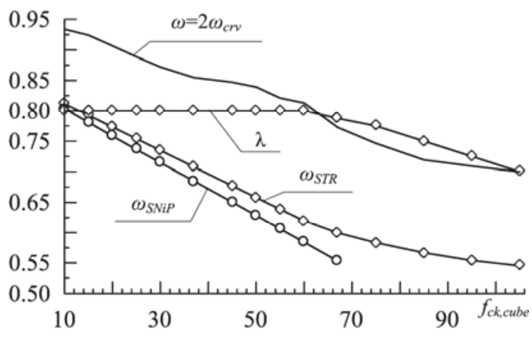

Fig. 6 Variation of coefficients λ, ω_{STR}, ω_{SNiP} and ω determined by Eqs. (62), (60), (64) and (55) in respect to charactersitic cube strength $f_{ck,cube}$

The values of ω_{STR} determined by Eq. (49) expressed via f_{cd} coincide with these values determined by Eq. (60) expressed via $f_{ck,cube}$. The values of λ calculated by Eq. (50) expressed via f_{cd} coincide with these values determined by Eq. (62) expressed via $f_{ck,cube}$ as well. Similarly the values of ω_{SNiP} calculated by Eqs. (51) and (64) expressed via $f_{cd,priz}$ and $f_{ck,cube}$ respectively coincide as well. Thus Table 6 may be used for the comparison of relationships Eqs. (60), (62) and (64) as well.

Now the values of λ, ω_{STR}, ω_{SNiP} and ω determined using relationships Eqs. (62), (60), (64) and (55) will be compared in respect to the mean concrete cube strength $f_{cm,cube}$. Therefore is necessary to change in formulae Eqs. (49), (50) and (51) the concrete design strength with the mean cube compression concrete strength $f_{cm,cube}$. As it was showed earlier in the development of Eqs. (60), (62) and (64), relationships between design strength $f_{cd,priz}$ or f_{cd} and characteristic strength $f_{ck,cube}$ of concrete according to STR [1], EC2 [2] and SNiP [12] can be found without difficulty. In publications many relationships between the cube and the prism strengths of concrete are given. In [18]

$$\left.\begin{aligned} &f_{c,pris} = f_{c,cube}\left(0.77 - 0.00125 f_{c,cube}\right) \\ &f_{c,pris} \geq 0.72 f_{c,cube} \end{aligned}\right\} \quad (65)$$

$$f_{c,pris} = f_{c,cube}\left(0.85 - 0.00585 f_{c,cube}\right) \quad (66)$$

$$f_{c,pris} = f_{c,cube}\left(0.8 - 0.0023 f_{c,cube}\right) \quad (67)$$

$$f_{c,pris} = \frac{130 + f_{c,cube}}{145 + 3 f_{c,cube}} f_{c,cube} \quad (68)$$

In [19]

$$\left.\begin{aligned} &f_{c,pris} = \left(0.93 - \frac{0.59}{100} f_{c,cube}\right) f_{c,cube} \\ &\text{when } 10\,\text{MPa} \leq f_{c,cube} \leq 60\,\text{MPa} \end{aligned}\right\} \quad (69)$$

where $f_{c,pris}$ and $f_{c,cube}$ are prism and cube compressive strengths of the concrete. On average for the low strength concrete it is possible to take that $f_{c,pris} = 0.83 f_{c,cube}$ and for the high strength concrete - $f_{c,pris} = 0.78 f_{c,cube}$ [17].

Relationships between the cylinder and the cube strength expressed by Eqs.(65) to (68) are plotted in Fig. 7. Fig. 7 shows that the difference between the cube and the prism compressive strengths increases with compressive strength of the concrete.

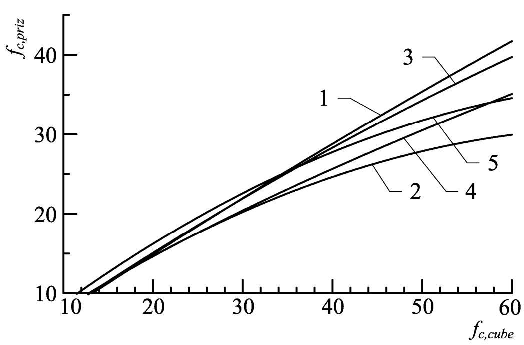

Fig. 7 Relationship between the prism and the cube compressive strengths: *1* - by Eq. (65), *2* - by Eq. (67), *3* - by Eq. (66), *4* - by Eq. (68), *5* - by Eq. (69)

In publications data about relation between the cube and the cylinder strengths of concrete are given as well.

In [19]

$$\left.\begin{aligned} &f_{c,cyl} = \left(0.94 - \frac{0.52}{100} f_{c,cube}\right) f_{c,cube} \\ &\text{when } 10\,\text{MPa} \leq f_{c,cube} \leq 60\,\text{MPa} \end{aligned}\right\} \quad (70)$$

In [20] $f_{c,cube} = 1.2 f_{c,cyl}$.

In [21]

$$\left.\begin{aligned} &f_{c,cyl} = 0.76 \text{ when } f_{c,cube} \leq 20\,\text{MPa} \\ &f_{c,cyl} = \left(0.76 - \frac{0.201}{20} f_{c,cube}\right) f_{c,cube} \\ &\text{when } f_{c,cube} > 20\,\text{MPa} \end{aligned}\right\} \quad (71)$$

However in publications it was not possible to find how in STR [1] and EC2 [2] the cylinder and the mean cube compression strengths are related.

Let us investigate the ratio between concrete characteristic strengths, f_{ck} and $f_{ck,cube}$, and between corresponding their mean strengths, f_{cm} and $f_{cm,cube}$, presented in STR [1] and EC2 [2]. The ratio between the cube $f_{ck,cube}$ and the cylinder f_{ck} characteristic strengths of concrete can be obtained from the Tables given in the codes

$$f_{ck} / f_{ck,cube} \approx 0.8 \quad (72)$$

In these codes the mean cylinder compression strength is determined by

$$f_{cm} = f_{ck} + 8\,\text{MPa} \quad (73)$$

This relationship can be obtained assuming the standard deviation of concrete compressive strength equal to 5 MPa, i.e. σ=5 MPa [22]. Then

$$f_{cm} = f_{ck} + 1.645 \cdot 5 = f_{ck} + 8.225\,\text{MPa} \approx f_{ck} + 8\,\text{MPa}$$

If the standard deviations for cylinder and cube strengths of the same value are taken, i.e. equal to 5 MPa, then

$$f_{cm,cube} = f_{ck,cube} + 1.645 \cdot 5 \approx f_{ck,cube} + 8\,\text{MPa} \quad (74)$$

From Eqs. (73) and (74) it is obtained

$$\frac{f_{cm}}{f_{cm,cube}} = \frac{f_{ck} + 8}{f_{ck,cube} + 8} \quad (75)$$

From Eq. (72) it is found that

$$f_{ck,cube} = 1/0.8 f_{ck} = 1.25 f_{ck} \quad (76)$$

Then putting Eq. (76) into Eq. (75) gives

$$\frac{f_{cm}}{f_{cm,cube}} = \frac{f_{ck} + 8}{1.25 f_{ck} + 8} = M \quad (77)$$

From here it is found that

$$f_{cm,cube} = \frac{1}{M} f_{cm} \quad (78)$$

The values of coefficient M in respect to concrete strength class are presented in Table 7. The Table shows that the coefficient M decreases from 0.89 to 0.81 with increasing in concrete strength class.

Table 7

Variation of M in respect to the concrete strength class

Concrete class														
C8/10	C12/15	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60	C55/67	C60/75	C70/85	C80/95	C90/105
M														
0.89	0.87	0.86	0.85	0.84	0.84	0.83	0.83	0.82	0.82	0.82	0.82	0.82	0.81	0.81

Table 8

Values of coefficients ω_{STR} and ω_{SNiP} determined by formulae Eq. (79) to Eq. (81) when α=0.85

Concrete class	f_{cm}	$f_{cm,cube}$ by (78)	ω_{STR} by (79)	λ by (80)	ω_{SNiP} by (81)
C8/10	16	17.978	0.799	0.800	0.785
C12/15	20	22.989	0.775	0.800	0.768
C16/20	24	27.907	0.753	0.800	0.751
C20/25	28	32.941	0.731	0.800	0.734
C25/30	33	39.286	0.707	0.800	0.712
C30/37	38	45.238	0.684	0.800	0.693
C35/45	43	51.807	0.663	0.800	0.671
C40/50	48	57.831	0.643	0.800	0.651
C45/55	53	64.634	0.626	0.800	0.627
C50/60	58	70.732	0.610	0.800	0.606
C55/67	63	76.829	0.596	0.788	0.585
C60/75	68	82.927	0.583	0.775	-
C70/85	78	95.122	0.563	0.750	-
C80/95	88	108.642	0.549	0.725	-
C90/105	98	120.988	0.541	0.700	-

Using Eqs. (60), (61) and (62) the values of coefficients ω_{STR} and λ can be determined with respect to f_{cm}. Putting $f_{ck,cube}=1/0.8 f_{ck}=1/0.8(f_{cm}-8\,\text{MPa})$ in Eqs. (60) and (62) results

$$\left.\begin{aligned} &\omega_{STR}=a-0.0048 f_{cm}+0.0384, \text{when } f_{ck}\le 50\,\text{MPa}\\ &\omega_{STR}=a+0.0567-7.423\cdot 10^{-3} f_{cm}+4.288\cdot 10^{-5} f_{cm}^2-\\ &\qquad -5.33\cdot 10^{-8} f_{cm}^3,\ \text{when } f_{ck}>50\,\text{MPa}\end{aligned}\right\} \quad (79)$$

$$\left.\begin{aligned} &\lambda=0.8,\ \text{when } f_{ck}\le 50\,\text{MPa}\\ &\lambda=0.8-\frac{f_{cm}-58}{400},\ \text{when } f_{ck}>50\,\text{MPa}\end{aligned}\right\} \quad (80)$$

Coefficient ω_{SNiP} in respect to f_{cm} can be determined by Eq. (64). Putting in this relationship $f_{ck,cube}=f_{cm,cube}(1\text{-}1.645\cdot 0.135)=0.778 f_{cm,cube}$ and Eq. (78), the following relationship between ω_{SNiP} and f_{cm} is obtained

$$\left.\begin{aligned} &\omega_{SNiP}=a-\frac{f_{cm}}{M}0.003684+0.465\cdot 10^{-5}\frac{f_{cm}^2}{M^2}\\ &\omega_{SNiP}\le a-0.00345\frac{f_{cm}}{M}\end{aligned}\right\} \quad (81)$$

The values of ω_{STR}, λ and ω_{SNiP} determined by Eqs. (79) to (81) are given in Table 8 and plotted in Fig. 7.

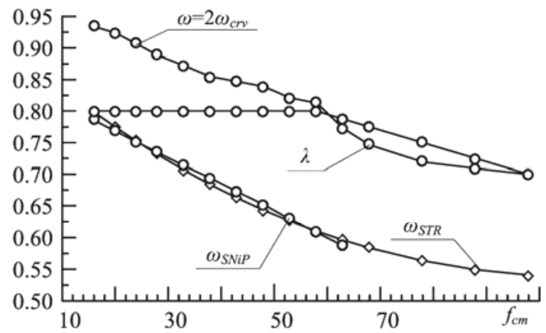

Fig. 8 Variation of coefficients ω_{STR}, ω_{SNiP}, λ and ω according to Eqs. (79), (80), (81) and (55) with the mean cylinder compressive strength f_{cm}

The data presented in Tables 6, 8 and in Figs. 6 and 8 indicate as well that the values of λ are nearer to ω than to ω_{SNiP} or ω_{SNiP}. Also $\omega_{STR}>\omega_{SNiP}$. The performed analysis shows that description of the ratio x_{eff}/x_w between the depths of RSB and NSD by coefficients ω_{SNiP} and ω_{STR} is very poor. Description of this ratio by the coefficient λ in the interval of ($8\le f_{ck}\le 50$) MPa is poor as well. Therefore in this article below a function for the ratio x_{eff}/x_w will be fitted.

6. Linear approximation of the ratio between the depths of the diagrams

The ratio x_{eff}/x_w between the depths of RSB and NSD of concrete compression zone is approximated by the linear function

$$\gamma=b_0-b_1 f_{cm} \quad (82)$$

where coefficients b_0 and b_1 are calculated in such a way, that their mean square errors would be minimum

$$d_2(\omega;\gamma)=\int_a^b(\omega-\gamma)^2 df_{cm},\ a<b \quad (83)$$

Binomial expression under the integral of Eq. (83) is expanded

$$d_2(\omega;\gamma)=-2b_0\int_a^b\omega df_{cm}+2b_1\int_a^b\omega f_{cm}df_{cm}+b_0^2\int_a^b df_{cm}-$$
$$-2b_1b_0\int_a^b f_{cm}df_{cm}+b_1^2\int_a^b f_{cm}{}^2 df_{cm}+\int_a^b\omega^2 df_{cm} \quad (84)$$

Integration by parts in respect to b_0 and b_1 of Eq. (84), collecting of terms and equating of the obtained expression to 0 results in the system of two equations

$$\begin{cases} -2\int_a^b\dfrac{2S_{k,crv}}{A_{k,crv}}df_{cm}+2b_0 82-b_1 9348=0\\ 2\int_a^b\dfrac{2S_{k,crv}}{A_{k,crv}}f_{cm}df_{cm}+b_0 9348-\dfrac{2}{3}b_1 937096=0\end{cases} \quad (85)$$

By numerical solution of the system of Eqs. (85)

the values of $b_0 = 0.98$ and $b_1 = 3{,}1\cdot10^{-3}$ were obtained for $(16 \le f_{cm} \le 98)$ MPa. Then the relationship Eq. (82) takes the form as follows

$$\gamma = 0.98 - 3.1\cdot10^{-3} f_{cm} \tag{86}$$

Relationship between the coefficient γ and f_{ck} with consideration of Eq. (73) is

$$\gamma = 0.955 - 3.1\cdot10^{-3} f_{ck} \tag{87}$$

Variation of the coefficient γ with f_{cd} is determined taking into account the relationship between characteristic and design strength of the concrete. According to STR [1] $f_{cd} = \alpha_{cc} f_{ck}/1{,}5$ when $f_{ck} \le 50$ MPa and $f_{cd} = \alpha_{cc} f_{ck}/(1{,}5/(1{,}1\text{-}f_{ck}/500)$ when $f_{ck} > 50$ MPa, where α_{cc} is the coefficient for long-term strength. Solution of these expressions in respect to f_{ck} gives

$$\left.\begin{aligned} f_{ck} &= \frac{1.5}{\alpha_{cc}} f_{cd}, \text{ when } f_{ck} \le 50\,\text{MPa} \\ f_{ck} &= \frac{1}{\alpha_{cc}}\left(275 + 5\sqrt{3025 - 30 f_{cd}}\right), \text{ when } (50 < f_{ck} \le 90)\,\text{MPa} \end{aligned}\right\} \tag{88}$$

Then coefficient γ in relation to f_{cd} can be obtained putting Eq. (88) into Eq. (87).

Since according to STR [1] coefficient α=1 in the case of NSD, and according to EC2 [2] f_{cd}=$\alpha_{cc} f_{ck}/\gamma_c$ when it is recommended to take the long term coefficient α_{cc}=1 then relationships of γ in respect to f_{cd} are as follows

$$\left.\begin{aligned} &\gamma = 0.955 - 4.65\cdot10^{-3} f_{cd}, \text{when } f_{ck} \le 50\,\text{MPa} \\ &\gamma = 0.1025 - 0.0155\sqrt{3025 - 30 f_{cd}}, \\ &\quad \text{when } (50 < f_{ck} \le 90)\,\text{MPa} \end{aligned}\right\} \tag{89}$$

In Eqs. (86) to (89) f_{cm}, f_{ck} and f_{cd} are in MPa. The ratio between the depths x_{eff}/x_w of the diagrams according to Eq. (55) and coefficient γ according Eq. (87) are plotted in Fig. 9.

The maximum error $d_1(2\omega_{crv};\gamma)=max|2\omega_{crv}\text{-}\gamma|$ and the mean square error by Eq. (83) are: in the interval of $(8 \le f_{ck} \le 50)$ MPa d_1=0.01292 and d_2=0.0012, in the interval of $(50 \le f_{ck} \le 90)$ MPa d_1=0.0241 and d_2=0.00823.

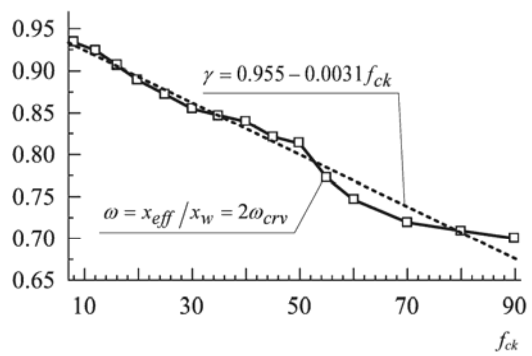

Fig. 9 Variation of coefficients ω and γ with cylinder characteristic compression strength f_{ck}

Fig. 9 shows that the relationship of γ describes the ratio between the depths of RSB and NSDs quite well. It is evident that using nonlinear relationship the ratio of these depths can be described more accurately. However, the obtained errors are sufficiently small and the proposed relationship is suitable for practical application.

In summary, the results of investigation give opportunity to state that the coefficients for substitution of the diagrams presented in the codes EC2, STR, DIN cannot provide the equivalent substitution of RSB for nonlinear stress diagram with descending branch. Therefore carrying capacity of flexural, eccentrically compressed and eccentrically tensioned members determined using RSB and these obtained using the nonlinear stress diagram with descending branch will be different.

7. Conclusions

In the following conclusions the term *nonlinear diagram* is referred to as nonlinear diagram with descending branch for concrete compression stresses according to EC2 and STR 2.05.05:2005.

1. It was determined that the substitution of rectangular stress block for nonlinear stress diagram according to EC2 and STR 2.05.05:2005 results in some inaccuracies. If the centroids of the diagrams coincide then the ratio between the areas of nonlinear stress diagram and of rectangular stress block varies in the interval of 0.103 to 1.201. If for substitution of the diagrams the coefficient of 0.9 is applied, as it is required in STR 2.05.05:2005, the interval of variation for this ratio is smaller: 0.995 to 1.081.

2. When the centroids of the diagrams coincide then coefficient ω, which is the ratio between depths of the diagrams, can be considered as the depth of rectangular stress block in normalized coordinates. When the areas of the diagrams are equal then the coefficient ω can be considered as the area of the nonlinear diagram described in normalised coordinates, divided by concrete compressive strength. Thus the said coefficient takes a new physical meaning.

3. It was determined that agreement of the ratio between the depths of the diagrams used in STR 2.05.05:2005 with the ratio between the depths of the rectangular stress block and that of the equivalent nonlinear stress diagram is very poor.

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KREIVINĖS ĮTEMPIŲ DIAGRAMOS PAKEITIMO STAČIAKAMPE ĮTEMPIŲ DIAGRAMA EKVIVALENTIŠKUMO ANALIZĖ

R e z i u m ė

Darbe analizuojama kreivinės įtempių diagramos su žemyn krintančia dalimi pakeitimo stačiakampe įtempių diagrama ekvivalentiškumas skaičiuojant stačiakampio skerspjūvio lenkiamus, ekscentriškai gniuždomus ir ekscentriškai tempiamus gelžbetoninius elementus. Pasiūlyta metodika, leidžianti ekvivalentiškai pakeisti šias diagramas. Pateiktos kreivinės diagramos su žemyn krintančia dalimi ploto ir svorio centro analizinės išraiškos. Standartinėms betono klasėms yra apskaičiuoti ir pateikti koeficientai, įgalinantys ekvivalentiškai pakeisti minėtas diagramas. Tai leidžia skaičiuoti gelžbetoninį elementą taikant stačiakampės įtempių diagramos modelį. Diagramų pakeitimas pagal EC2 ir STR 2.05.05:2005 palygintas su ekvivalentišku kreivinės diagramos su krintančia dalimi pakeitimu stačiakampe įtempių diagrama. Parodyta, kad pagal STR 2.05.05:2005 aukščių santykis labai skiriasi tikrojo nuo stačiakampės ir kreivinės įtempių diagramos aukščių santykio. Straipsnyje taip pat pateikta šio santykio tiksli analizinė išraiška bei tiesinė aproksimacija priklausomai nuo betono charakteristinio ir skaičiuojamojo stiprio.

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ANALYSIS OF EQUIVALENT SUBSTITUTION OF RECTANGULAR STRESS BLOCK FOR NONLINEAR STRESS DIAGRAM

S u m m a r y

This article deals with the analysis of equivalency of the substitution of rectangular stress block for nonlinear stress diagram with descending branch for the calculation of flexural, eccentrically compressed and eccentrically tensioned reinforced concrete members of rectangular cross-section. The method for equivalent substitution of these diagrams is proposed. Analytical relationships of area and its centre for the nonlinear diagram with descending branch are presented. Coefficients for equivalent substitution of the said diagrams for the standard concrete strength classes are determined and given. It gives the opportunity to design reinforced concrete members using rectangular stress block model. Substitution of the diagrams applied in EC2 and in STR 2.05.05:2005 is compared with the equivalent substitution of rectangular stress diagram for nonlinear stress diagram with descending branch. It is shown that in STR 2.05.05:2005 description of the ratio between the depth of the rectangular diagram and that of the equivalent nonlinear one with descending branch is very poor. An explicit analytical relationship for this ratio and its linear approximation in respect to the concrete characteristic and design strengths are presented in this article as well.

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АНАЛИЗ ЭКВИВАЛЕНТНОСТИ ЗАМЕНЫ КРИВОЛИНЕЙНОЙ ДИАГРАММЫ НАПРЯЖЕНИЙ НА ПРЯМОУГОЛЬНУЮ ДИАГРАММУ НАПРЯЖЕНИЙ

Р е з ю м е

В статье анализируется эквивалентность замены криволинейной диаграммы с ниспускающейся ветвью на прямоугольную диаграмму при расчете изгибаемых, внецентрально сжатых и внецентрально растянутых элементов прямоугольного сечения. Предложена методика, позволяющая эквивалентную замену этих диаграмм. Даны аналитические выражения площади и её центра тяжести для криволинейной диаграммы с ниспускающейся ветвью. Для стандартных прочностных классов бетона подсчитаны и даны коэффициенты, позволяющие эквивалентную замену упомянутых диаграмм. Это позволяет железобетонный элемент рассчитывать, используя модель прямоугольной диаграммы. Сравнена замена диаграмм согласно EC2 и STR 2.05.05:2005 с эквивалентной заменой криволинейной диаграммы с ниспускающейся ветвью на прямоугольную диаграмму напряжений. Показано что STR 2.05.05:2005 очень неточно описывает отношение высот прямоугольной диаграммы и эквивалентной криволинейной с ниспускающейся ветвью диаграммы. В статье также дано точное аналитическое выражение этого отношения и его линейная аппроксимация в зависимости от нормативной и расчетной прочности бетона.

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