

# Material properties identification. Comparison of two techniques

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## 1. Introduction

The use of composite materials in mechanical, aerospace, automotive, ship building, and other branches of engineering, is constantly increasing [1]. Because of reductions in manufacturing costs and improvements in product quality, demand for these materials is rising at high rate.

Knowledge of the elastic properties of composite materials is essential for design and application in manufacture and the measurement of these properties during manufacturing offers the potential for improvements in material properties identification field. Moreover for the properties such as shear modulus, static tests often yield poor results. As an alternative to tensile testing, mixed numerical experimental techniques are being used. One approach to a rapid and inexpensive characterization of the elastic properties of materials involves modal vibration testing.

One version of plate vibration test for determining composite elastic constants is based on the use of impulse/frequency response method, a freely plate, and PC-based software for calculating properties from vibration data [2]. In current paper are given two approaches of PC-based software used in identification: one approach is based on the response surface approximations, which are obtained by using information on the behaviour of a structure in the reference points of experimental design [3-5]; other approach uses genetic algorithm (GA) to identify material properties. Both techniques need material mathematical model and vibration data to initiate calculations. Both techniques are not precise identification tool if taken separately, but there are some assumptions, if techniques could be combined, as result it should be fast and precise material properties identification tool. That was occasion for an idea to compare these techniques and results, obtained identifying some material properties using both techniques.

In world practice some researches are trying to combine different identification procedures or improve existing by adding up some enhancements, e.g. GA based identification technique is sensitive to initial material properties search space boundary conditions and tool itself is not very accurate, so one tries to develop a combination of GA and other stochastic search algorithm [6].

## 2. Identification procedure

Two identification techniques are basically described in this chapter: Planning of Experiments (PoE) and identification using Genetic Algorithm (GA). First one requires making an experiment plan in dependence on the number of design parameters (identifiable values) and the

number of experiments. Second methodology needs the initial optimization boundary conditions and genetic operators to be defined. In the present investigation, the finite element method is used for the modelling and dynamic analysis.

### *Finite element models*

Finite element modelling is based on the first-order shear deformation theory including rotation around the normal. FEM program ANSYS was used in calculations. Element SHELL 63 has bending and membrane capabilities, both in-plane and normal loads are permitted. The element has six degrees of freedom at each node: translations in the x, y, and z directions and rotations about the x, y, and z-axes.

### *Dynamic analysis*

The Block Lanczos eigenvalue solver is default for modal analysis [7]. This method is as accurate as the subspace method, but faster. The Block Lanczos method is especially powerful when searching for eigenfrequencies in a given part of eigenvalue spectrum of the given system. The convergence rate of eigenfrequencies will be approximately the same when extracting modes in the midrange and higher end of the spectrum as well as extracting the lowest modes.

### *ANSYS Parametric Design Language (APDL)*

APDL is used to create numerical material model. It is efficient while performing huge amount of calculations with different material properties (MP). With APDL one can automate calculation process changing variables without intervention into calculation sequence. Identifying MP using GA is convenient to change input data without process interruption each iteration creating new APDL file. Identifying material properties handling PoE technique one uses APDL to obtain spectrum of eigenfrequencies according to the plan.

### 2.1. Identification technique using plan of experiment

The proposed numerical-experimental procedure consists of five stages presented in Fig. 1. At the first stage physical or numerical experiments (using Finite Element Modelling (FEM)) are carried out and dynamic parameters of the structure are determined. In the second stage the plan of experiments should be developed in dependence on the number of design parameters (identifiable values) and number of experiments (1).

$$N = \prod_{i=1}^n \frac{K+1}{i} \quad (1)$$

where  $K$  is the number of variables,  $n$  is the degree of approximation function,  $N$  is the number of points in the plan.

Then the finite element analysis is performed in the reference points of experimental design and dynamic parameters of the structure are calculated in the third stage. In the fourth stage the numerical data obtained by the finite element method is used to determine a simple approximating functions using response surface method. An identification of the material properties is performed in the final stage minimising the error functional between experimental and numerical structural responses.

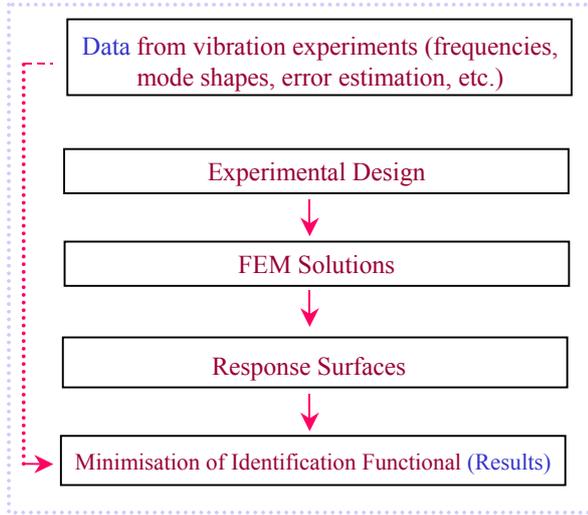


Fig. 1 Identification procedure

The initial information for elaboration of the plan is the number of factors  $n$  and the number of experiments  $k$ . The points of experiments in the domain of factors are distributed as regular as possible (Fig. 2). For this reason the following criterion is used

$$\Phi = \sum_{i=1}^{k-1} \sum_{j=i+1}^k \frac{1}{l_{ij}^2} \Rightarrow \min \quad (2)$$

where  $l_{ij}$  is the distance between the  $i$  and  $j$  ( $i \neq j$ ) points.

For each number of factors  $n$  and number of experiments  $k$  it is possible to elaborate a plan of experi-

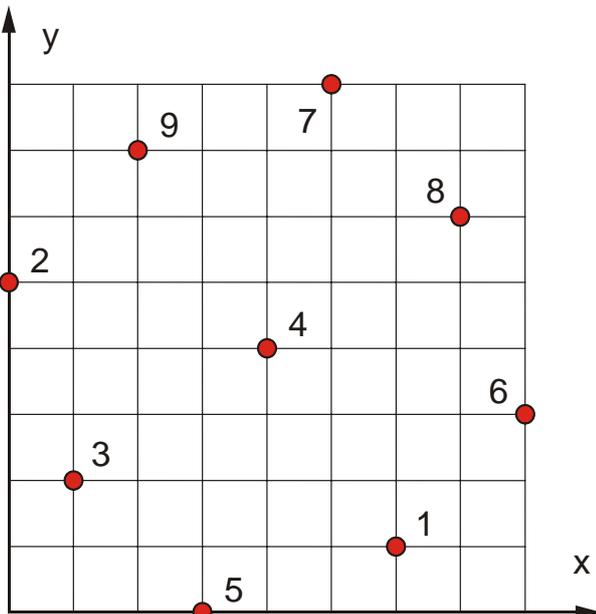


Fig. 2 2D design space

ments. But it requires huge computational resources, therefore each plan of experiment is elaborated only once and it can be used for various designing cases. The plan of experiments is characterised by the matrix of plan  $B_{ij}$ . When the domain of factors is determined as  $x_j \in [x_j^{\min}, x_j^{\max}]$ , the points of experiment are calculated by the following expression

$$x_j^{(i)} = x_j^{\min} + \frac{1}{k-1} (x_j^{\max} - x_j^{\min}) (B_{ij} - 1), \quad (3)$$

$$i = 1, 2, \dots, k, \quad j = 1, 2, \dots, n$$

Then the numerical computations are carried out in these points and the dynamic characteristics obtained by the finite element method are used to determine a simple functions using response surface method.

#### Response surfaces

Information about the behaviour of an object can be obtained by physical experiment or by computer solution in the reference points. This information can be represented as a table of data, where the response function  $y(x)$  of the object is in relationship to the variables. The goal is by using the data of experiments (in our case data are obtained by the finite element solution in the reference points) to obtain the relation  $y(x)$  in mathematical form or so called equation of regression. Details of this procedure and corresponding program RESINT were described in [8].

#### 2.2. Identification using GA

GA is an exploration algorithm based on the mechanisms of natural selection and genetics. According to these mechanisms the stronger individuals in a population survive and generate offspring, which transmit their heredity to new generations. A simple GA involves a set of individuals (population), and a set of genetic operators. Each individual (chromosome) in the population represents a solution to a problem in the code of a string. The genetic operators allow the genetic manipulation process (reproduction) to be carried out.

The GA library (obtained from Matthew Wall, Massachusetts Institute of Technology) used for elastic constants identification was developed on personal computer using the C++ language. The implementation of a service application algorithm using GA library is shown in Fig. 3.

The process of identification starts with the generation of a random initial population of sets of material properties values (Fig. 3). Each design is randomly formed by choosing the elastic constant values within particular interval of positive values. For the present investigation, the bounds on the elastic constants were set accordingly to already known material properties from static test.

Then real eigenvalue analysis is carried out. In the post-processing stage, elastic constants and the desired first nonzero fundamental frequencies extracted from the results file are saved (the first six rigid body modes for the free edge condition have zero frequency).

The objective function value is calculated in each population processing. At each iteration ANSYS is loaded to obtain natural frequencies of material specimens with certain MP. The fitness processor begins to operate at the end of the population processing, evaluating the objective

function for each design. Goal of the optimization is to find a set of elastic constants in order that the outputs from the numerical code fit the experimental results [9]. Program routine is described below in next chapter.

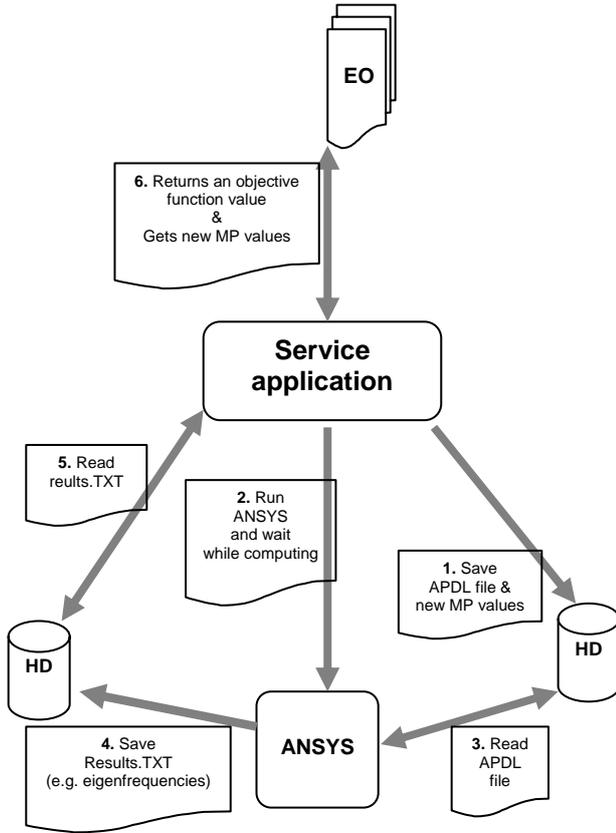


Fig. 3 Detailed optimization scheme

If the convergence criteria are not reached, the most suitable solutions are selected and then processed by means of the genetic operators, to create a new population. The process is repeated until convergence is reached.

In Fig. 4 detailed optimization scheme is presented. Main consideration should be applied to service application. This part is the main coordinating segment in identification process. It manipulates data, runs additional programs (e.g. ANSYS), and outputs information.

Using GA material properties identification method it is possible to define quite wide range of identifiable parameters, which will cause longer calculations, but it will allow revising more possible solutions. GA method might be considered to be sensitive to GA parameters (e.g. population size). Some tests were made with several generations to find out the best GA parameters ratio. From the derived solutions several better were picked, and the test was repeated with bigger generations number using obtained results. The second test was made to ensure which combination of GA parameters ratio is the best for current problem. International experience and recommendations propose 5% of mutation and 90% of crossover probabilities and as much as possible large population size. It was decided to use 40 individual population due to CPU speed and follow recommendations about GA parameters ratio. Next step in the identification is to develop mathematical model of the material and start to identify material properties.

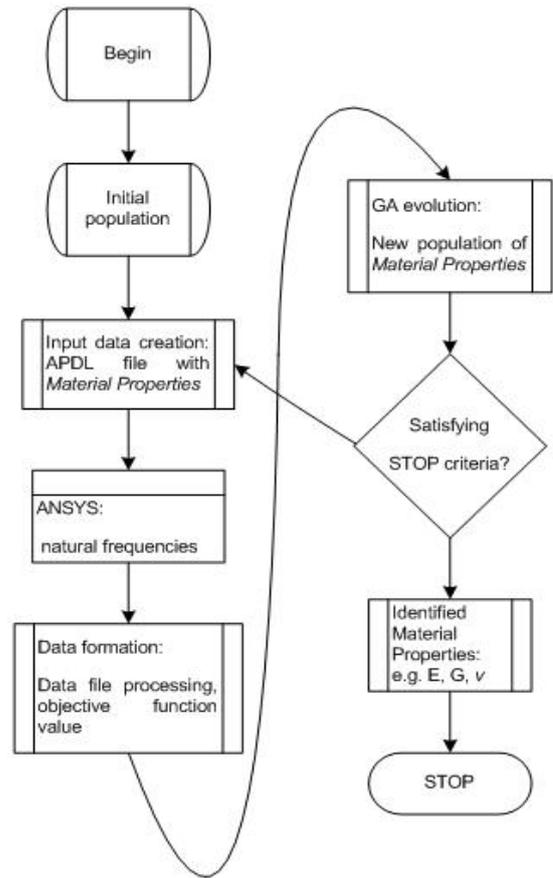


Fig. 4 Flow-chart for optimal design by GA

### 3. Identification of material properties

Testing of the developed inverse techniques based on vibration tests has been carried out identifying the material properties of homogeneous isotropic aluminium plates. Identification was performed in two different methods: planning of experiment and identification using GA.

Identification of isotropic material properties has been carried out for the aluminium plate (Fig. 5) with the density and geometrical parameters presented in Table 1. For new specimens the material with ISO certificate was used (Table 2). The eigenfrequencies have been measured using **POLYTEC** (*PSV-400-H4-S Scanning Vibrometer*) and **Pulse LabShop** (*OFV-5000 Modular Vibrometer Controller*) technique. Two different measuring techniques were used to ensure that the obtained eigenfrequencies are correct.

Table 1

Geometric dimensions

Specimen	SP1	SP2	SP3
$a$ , mm	300	300	300
$b$ , mm	300	200	200
$h$ , mm	2.3	2.0	2.0
$\rho$ , kg/m <sup>3</sup>	2700	2700	2700

To describe isotropic properties of a material only two material constants are necessary:  $E$  - modulus of elasticity and  $G$  - shear modulus. Shear modulus is taken instead of Poisson's ratio  $\nu$  to get approximately close values for the material constant limits (Table 3) used in the identification process. In this case an application of any scaling technique is not required.

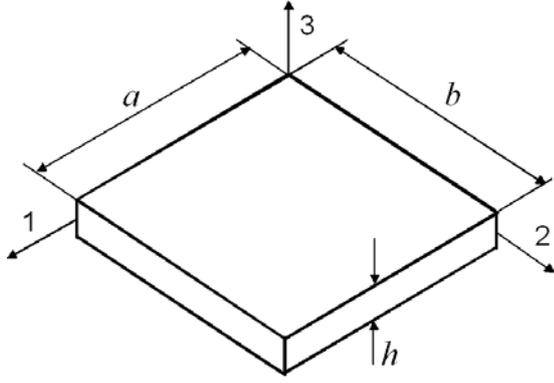


Fig. 5 Homogeneous plate model

Table 2

Properties of aluminum plates

Specimen	SP1, SP2	SP3
Manufacturers code name	SAPA	Sever Stall
Keywords	6082-T6	1060-H18
Young's modulus $E$ , GPa	68.9	68.9
Shear modulus $G$ , GPa	26	26
Poisons ratio $\nu$	0.33	0.33

Table 3

Ranges of identifiable parameters

Parameters of identification	min	max
$E$ , GPa	60	80
$G$ , GPa	22	30

The plan of experiments has been produced for 2 design parameters and 38 experiments (1). Then finite element analysis has been performed in 38 experimental points and 17 first eigenfrequencies have been determined. The number of eigenfrequencies was determined according to minimum required amount of noncoupled modes. Employing these numerical values, the approximating functions (response surfaces) for all eigenfrequencies (excluding coupled eigenfrequencies) were obtained. The objective function in this case can be written in the following form

$$\Phi(X) = \sum_{i=1}^9 \frac{(f_i^{exp} - f_i^{FEM})^2}{(f_i^{exp})^2} \Rightarrow \min \quad (4)$$

Minimising (4) function, the elastic material constants have been identified. The results have been verified comparing the experimentally measured eigenfrequencies with the numerically obtained using the identified elastic properties. The residuals are calculated by the following expression

$$\Delta_i = \frac{f_i^{exp} - f_i^{FEM}}{f_i^{exp}} \times 100 \quad (5)$$

Results of verification of the aluminium plate are given in Table 3. One can see that the eigenfrequencies calculated by the finite element method using the elastic properties obtained performing identification procedure are in good agreement with the experimental results. The difference in terms of residuals is less than 1% in most cases. It is necessary to note that the eigenfrequencies located in

the Table cells highlighted by grey only have been taken into identification process. In Table 4 only SP1 verification is shown, but the same procedure was applied to SP2 and SP3.

Table 4

Verification of eigenfrequencies (Hz) using the identified parameters, specimen SP1 ( $f^{EXP}$  – experimental,  $f^{FEM}$  – finite element solution data)

Mode No.	POLYTEC			PULSE LabShop		
	$f^{EXP}$	$f^{FEM}$	$\Delta$ , %	$f^{EXP}$	$f^{FEM}$	$\Delta$ , %
1	83	82	1.20	83	83	0
2	118	120	1.69	120	120	0
3	154	153	0.65	154	153	0.65
4	212	214	0.94	214	215	0.47
5	215	214	0.47	216	215	0.46
6	379	382	0.79	381	382	0.26
7	384	382	0.52	383	382	0.26
8	390	392	0.51	394	394	0
9	428	426	0.47	428	428	0
10	486	482	0.82	485	482	0.62
11	636	652	2.52	649	654	0.77
12	655	652	0.46	657	654	0.46
13	734	729	0.68	721	730	1.25
14	761	768	0.92	764	767	0.39
15	818	819	0.12	808	820	1.49
16	825	819	0.73	821	820	0.12
17	948	946	0.21	953	949	0.42

While working out a plan of the experiment for  $E$  and  $G$  parameters using isotropic material constant dependency some nonphysical solutions were found (e.g. if  $E = 78.5$  GPa,  $G = 22.2$  GPa, then  $\nu = 0.76$ ), because Poisson's ratio exceeds real bounds. Due to this new plan of experiment with different variables was created:  $E$  and  $\nu$  (Table 5), respectively logically arranged between two parallels (Fig 6, b).

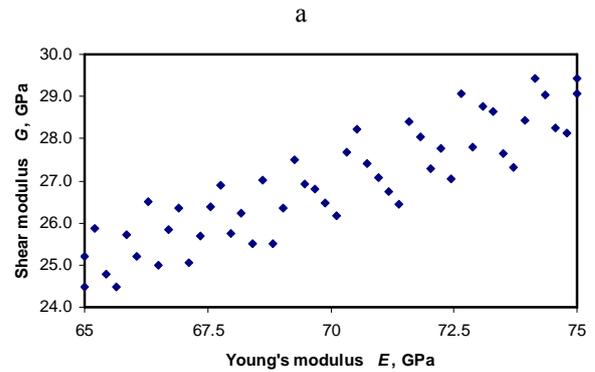
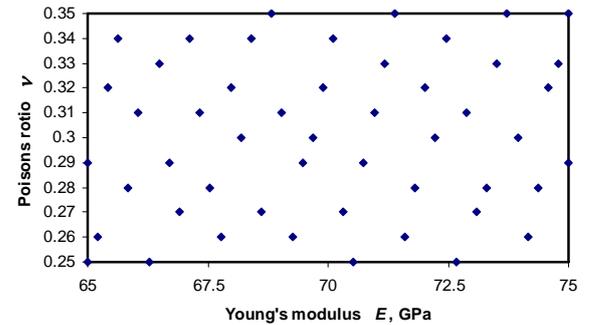
Fig. 6 Plan of experiments: a – plan explication using  $E$  and  $\nu$ ; b – plan explication using  $E$  and  $G$

Table 5  
Ranges of identifiable parameters

Parameters of identification	min	max
$E$ , GPa	65	75
$\nu$	0.25	0.35

While performing experiments a phenomenon was noticed, if a specimen is square, it has duplex eigenfrequencies. That means one eigenfrequency of the specimen has two same shapes, rotate at  $90^\circ$  respectively. Trying to avoid this phenomenon [10, 11] shape of the speci-

mens was selected with different side dimensions: 300x200 mm.

Initially there were more specimens, but only those with the best objective function value were chosen to identify their material properties using GA. Specimens with worse objective function values were rejected due to unsuccessful natural experiment or not correct specimen geometrical form (e.g. form of parallelepiped or shell). Identification data is given in Table 6. There can be seen a good conformity of those identification approaches.

Table 6

Identification data

Mode No.	Specimen SP1					Specimen SP2					Specimen SP3				
	$f^{EXP}$	PoE		GA		$f^{EXP}$	PoE		GA		$f^{EXP}$	PoE		GA	
		$f^{FEM}$	$\Delta$ , %	$f^{FEM}$	$\Delta$ , %		$f^{FEM}$	$\Delta$ , %	$f^{FEM}$	$\Delta$ , %		$f^{FEM}$	$\Delta$ , %	$f^{FEM}$	$\Delta$ , %
1	106.9	107.5	0.58	107.7	0.75	105.6	106.9	<b>1.21</b>	105.7	0.07	110	108.8	<b>1.09</b>	108.6	<b>1.27</b>
2	117.5	116.5	0.85	116.9	0.51	116.9	116	0.75	115.9	0.83	116.3	115	<b>1.08</b>	115.2	0.9
3	247.5	249.8	0.93	250.4	<b>1.17</b>	246.9	248.5	0.66	246.6	0.11	248.1	250.3	0.88	250.2	0.84
4	276.3	274.9	0.49	276.3	0	276.3	274.2	0.74	276.4	0.05	268.1	266.6	0.57	267.7	0.16
5	315	312.3	0.86	313.1	0.6	313	310.8	0.7	309	<b>1.28</b>	312.5	311.3	0.38	311.5	0.32
6	367.5	370.1	0.71	372.1	<b>1.25</b>	368.1	369.2	0.29	372.8	<b>1.27</b>	356.9	358.1	0.34	359.6	0.76
7	461.3	465.9	<b>1.01</b>	467.3	<b>1.3</b>	461.3	463.8	0.55	462.2	0.21	458.8	462.6	0.84	463.1	0.95
8	535.6	535.7	0.01	537.3	0.32	535	533.4	0.3	531.4	0.67	528.1	532.8	0.89	533.2	0.96
9	660.6	663	0.36	666.5	0.89	663.8	661.3	0.37	666.8	0.46	645	642.8	0.34	645.4	0.06
10	754	738.6	<b>2.04</b>	741.5	<b>1.66</b>	744.4	735.9	<b>1.14</b>	736.9	<b>1</b>	728.1	724.5	0.5	726.5	0.22
11	798.1	803.5	0.67	806.6	<b>1.07</b>	789.4	800.6	<b>1.42</b>	799.2	<b>1.24</b>	776.9	782.6	0.74	785.2	<b>1.07</b>
12	806.3	803.9	0.29	807.5	0.15	806.3	801.3	0.61	807	0.09	793.1	796.4	0.41	797.4	0.54
	$E$ , GPa	69.56		70.04			69		68.71			68		68.24	
	$G$ , GPa	25.83		25.89			25.52		24.85			26.58		26.49	
	$\nu$	0.3465		0.3526			0.3519		0.3825			0.2792		0.288	
	$\phi$	9.1e4		10.5e4			8.8e4		8.6e4			6.4e4		7.1e4	

The comparison of graphical results between approaches is presented in Fig. 7. Specimens with high PoE objective function values were rejected and retained specimens were renamed SP1-SP3. In this graph can be seen good coincidence of identification techniques.

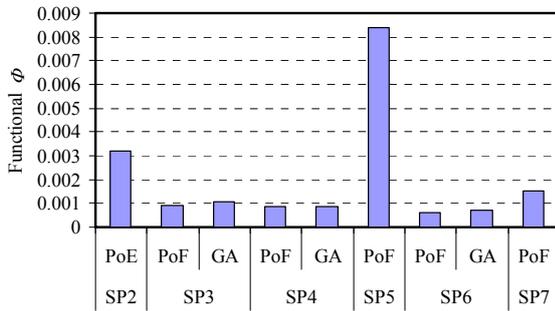


Fig. 7 Values of objective function using different identification techniques

#### 4. Conclusions

1. Preparing of specimens is significant part of the identification procedure. Some specimens appeared improper for testing, because of too high objective value (compared to other specimens). To achieve proper precision of the specimen dimensions water saw should be used to cut specimens.

2. Chosen specimens with square shape appeared to be improper for identification since they have coupled eigenfrequencies. In the next identification stage specimens with new dimensions: 200x300 mm were used. Specimens with this shape also occurred improper since there were found that this side ratio causes very low sensitivity of Poissons ratio in identification process [11, 12]. It was decided to work out an identification procedure of the specimens with proper side ratio trying to avoid this phenomenon.

3. Significant part of material properties identification procedure is detecting and excluding pairing eigenfrequencies from the identification process.

4. High sensitivity to bounds of identifiable parameters appears to be a bottle-neck of PoE method of identification. Material properties are identified at first step of approximation with wide range of search area. Second step is performed with narrower bounds of identifiable parameters taking into account results of the first step. Not less significant part of the identification is tracking identifiable parameters not to appear on the very edge of plan. Concluding, the weak part of PoE method appears to be the plan of experiment.

5. Successful matching of the plan of experiment and correlating function allows developing an engineering technology of fast material properties identification.

6. The main disadvantage of material properties

identification using GA is higher requirement of computational resources for the calculations in comparison with PoE.

7. Using combination of the both identification methods may give an advantage on material properties identification speed. First step of the identification procedure should be performed using GA to define more precise bounds of identifiable material properties. Second step (using PoE method) should realise precise approximation function dependencies for specific material. Those dependencies could be used to identify material properties quickly without additional calculations in, e.g. manufacturing process to control material quality or properties.

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## MEDŽIAGŲ TAMPRUMO CHARAKTERISTIKŲ NUSTATYMAS. DVIEJŲ METODŲ PALYGINIMAS

### R e z i u m ė

Straipsnyje lyginami du medžiagų tamprumo charakteristikų nustatymo metodai siekiant nustatyti galimybę sujungti juos į galingą ir greitaveikį medžiagų tamprumo charakteristikų nustatymo įrankį. Metodai aprašyti teoriškai, išanalizuotos pagrindinės jų galimybės. Atlikti eksperimentai, kuriais nustatytos aliuminio bandinių tamprumo charakteristikos. Abu metodai kokybiškai palyginti, pateiktos išvados ir pasiūlymai.

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## MATERIAL PROPERTIES IDENTIFICATION. COMPARISON OF TWO TECHNIQUES

### S u m m a r y

Present paper describes the comparison of two different material properties identification techniques in order to investigate a possibility of combination of two techniques into one powerful and fast tool for material properties identification. Both methods are theoretically described and main options are given in this article. Some experiments wherein aluminium material properties are identified are carried out. Methods are qualified, conclusions and proposals are given.

П. Рагаускас, Е. Скукис

## ИДЕНТИФИКАЦИЯ СВОЙСТВ МАТЕРИАЛОВ. СРАВНЕНИЕ ДВУХ МЕТОДОВ

### Р е з ю м е

В статье рассмотрено сравнение двух методов определения характеристик упругости материала. Симбиоз данных методов позволяет ускорить и упростить задачу определения свойств материала. Данные метода изложены теоретически, представлены основные характеристики. Выполнено несколько экспериментов, в которых определены механические свойства алюминия. Представлены выводы и рекомендации.

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