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Analysis the interaction of two cylindrical surfaces under shock impact load

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1. Introduction

While operating pipeline systems we do often run into a situation when a layer of transported substance (sediment) is formed on the internal surface of the pipe. Such system can be conditionally called "a pipe in a pipe". An external pipe is made of steel and the internal one – from the sediment. Thus a heterogeneous system of two cylindrical surfaces with different physical and mechanical properties is formed. The sediment hinders technological processes of transportation thus various ways of its elimination are applied [1]. Unfortunately, very often traditional ways are ineffective, especially when the sediment is elastically plastic body. Shock loadings were suggested for the renovation of the pipeline system [2]. However, the mathematical model allows approximate calculation of the elastic displacements of the sediment appeared under shock impact at a specific place of the pipe. So, we need to find the way to determine displacements of elastically plastic sediment in the pipe, impacted by an impulse shock.

It is supposed that:

- solidified sediment is a homogeneous elastically plastic body having the geometric form of the pipe;
- the pipe with the sediment is absolutely rigid and in equilibrium;
- sediment density ρ, elasticity modulus E, Poisson's ratio ν and a shear modulus G are known;
- strength characteristics of the pipe and geometric parameters are known;
- rate of external surface force, i.e. shock impact load is known;
- external volume forces (i.e. sediment mass) are ignored.

Thus we have a heterogeneous system of physical and mechanical parameters of the two cylindrical surfaces. We shall analyse the displacements of the internal layer, coming through the shock impact and dependence of displacements on the shock impact load location and parameters.

2. Mathematical model

The shock impact load and the interaction of resulting displacements have been analysed in closed space of cylindrical shape $\Omega(r, \theta, z)$ is chosen coordinate system for the mathematical model and displacements (see Fig. 1). Let's suppose that displacements in Ox direction are $u = r\cos\theta$, in Oy direction – $v = r\sin\theta$, in Oz direction – w in cylindrical system of axis.

Displacements u, v and w are to be found for the boundary conditions:

- displacements at the pipe wall are equal to zero, thus when $r = r_2$, the displacements $u = 0$, $v = 0$ and $w = 0$;
- on free surfaces of the body, radial tensions perpendicular to the free surfaces are equal to zero, i.e. $\sigma_r = 0$, when $r = r_1$;

and for the case of initial conditions:

- $u = v = w = 0$ displacements of the body of the initial instant $t = 0$ are equal to zero.

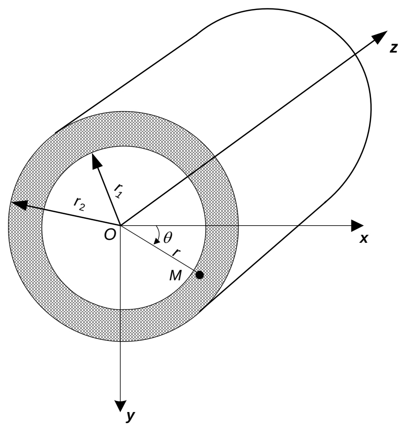

Fig. 1 Diagram for displacement analysis

Let's suppose that separation functions are

$$u=Uq;\ v=Vq\ ;\ w=Wq \tag{1}$$

where $U = U(r,\theta,z)$; $V = V(r,\theta,z)$; $W = W(r,\theta,z)$; $q = q(t)$.

Functions U, V and W are selected according to the boundary conditions, *i. e.* they should fit for the body presented in Fig. 1.

Let's suppose that stress – strain relation of elastically plastic sediment is described as

$$\sigma_i = 3G\varepsilon_i(1-\alpha) \tag{2}$$

where

$$\alpha = \frac{1-8\nu}{3(1-2\nu)} \tag{3}$$

In our case, applying Hamilton's principle, analogical description like [3] for shock impact load mathematical model is derived the following

$$m\ddot{q} + (k_T + k_P)q = P(t) \tag{4}$$

where

$$m = \rho \iiint_{(\Omega)} \left(U^2 + V^2 + W^2 \right) r dr d\theta dz \tag{5}$$

$$k_T = 2G \iiint_{(\Omega)} \left\{ \begin{array}{l} U_r^2 + \left(V_\theta + \dfrac{U}{r} \right)^2 + W_z^2 + \dfrac{\mu}{1-2\mu} \left(U_r + V_\theta + \dfrac{U}{r} \right)^2 + \\ + \dfrac{1}{2} \left[\left(U_\theta + V_r - \dfrac{V}{r} \right)^2 + \left(U_z + W_r \right)^2 + \left(W_\theta + \dfrac{W}{r} + V_z \right)^2 \right] \end{array} \right\} r dr d\theta dz \tag{6}$$

and

$$\frac{\partial U}{\partial r} = U_r, \ \frac{\partial V}{r \partial \theta} = V_\theta, \ \frac{\partial W}{\partial z} = W_z,$$

$$\frac{\partial U}{\partial z} = U_z, \ \frac{\partial V}{\partial z} = V_z, \ \frac{\partial W}{r \partial \theta} = W_\theta,$$

$$\frac{\partial U}{r \partial \theta} = U_\theta, \ \frac{\partial V}{\partial r} = V_r, \ \frac{\partial W}{\partial r} = W_r;$$

Ω is integration volume, *i. e*. the space occupied by a sediment. In Eq. (6) k_T is called a coefficient of elasticity.

$$k_P = 2 \iiint_{(\Omega)} \left\{ \begin{array}{l} \dfrac{1-2\nu}{G(1-8\nu)} \left(U_r + V_\theta + \dfrac{U}{r} + W_z \right)^2 - \dfrac{1-8\nu}{3(1-2\nu)} \left[U_r^2 + \left(V_\theta + \dfrac{U}{r} \right)^2 + W_z^2 \right] - \\ - \dfrac{G(1-8\nu)}{6(1-2\nu)} \left[\left(U_\theta + V_r - \dfrac{V}{r} \right)^2 + \left(U_z + W_r \right)^2 + \left(W_\theta - \dfrac{W}{r} + V_z \right)^2 \right] \end{array} \right\} r dr d\theta dz \tag{7}$$

In equation (7) k_P is coefficient of plasticity.

Thus, in order to find q, we have to solve the integral differential equation

$$m \frac{dq}{dt} = -\left(k_T + k_P \right) \int_0^t q dt + \int_0^t P dt \tag{8}$$

where t impulse duration.

In the case of explosion, taking into account that the pressure of explosion products (gas) is the same in all directions $(\bar{X} = \bar{Y} = \bar{Z})$, the impact effect is described by equations

$$\int_0^t P d\tau = LI \tag{9}$$

$$L = \iint_{(S)} (U + V + W) dS \tag{10}$$

where S integration area, *i. e*. the surface part subjected to external surface forces and $\bar{X} = \bar{Y} = \bar{Z}$ projections of external surface force (area unit is subjected to that force) on coordinate axes

$$I = \int_0^t \bar{X} d\tau = \int_0^t \bar{Y} d\tau = \int_0^t \bar{Z} d\tau \tag{11}$$

and according to [4]

$$I = k' \sqrt{Q_v} \tag{12}$$

where t is impact period; k' is expansion index and Q_v is explosion heat.

In this case, Eq. (8) can be rewritten

$$m \frac{dq}{dt} = -\left(k_T + k_P \right) \int_0^t q dt + LI \tag{13}$$

Approximately, by applying the iteration method the solution of Eq. (13) was derived

$$q^{(1)} = \frac{LI}{m} t - \frac{\left(k_T + k_P \right)}{6m} t^3 \tag{14}$$

$$q^{(2)} = \frac{LI}{m} t - \frac{\left(k_T + k_P \right) LI}{6m^2} t^3 + \frac{\left(k_T + k_P \right)^2}{120m^2} t^5 \tag{15}$$

$$q^{(3)} = \frac{LI}{m} t - \frac{\left(k_T + k_P \right) LI}{6m^2} t^3 + \frac{\left(k_T + k_P \right)^2 LI}{120m^3} t^5 - \frac{\left(k_T + k_P \right)^3}{5040m^3} t^7 \tag{16}$$

and displacements of the elastically plastic sediment points were calculated

$$u = Uq_i(t); \ v = Vq_i(t); \ w = Wq_i(t) \tag{17}$$

where i the number of iteration.

Hereby the theoretical study enables to solve approximately integral differential Eq. (13) and to calculate displacements of point of elastically plastic sediment at explosion. However we can not practically activate each particle of the sediment – material point, *i.e*. we can not explode charges in every point of the pipe's volume. Thus we should regard the displacements, determined in the said way, as theoretical ones and conventionally call them as test displacements. On the other hand, it is clear that it is necessary to solve the task of evaluation of the explosion action on sediment displacements within a certain distance, i.e. to find a method how to calculate real sediment displacements in the investigated volume after explosion of one charge at a specific place of the pipe. Then for the shock impact load we can write [3]

$$I_i = I_t f\left(\frac{r_u}{R_i}\right) \tag{18}$$

$$r_u \approx \sqrt[3]{G_u} \tag{19}$$

where I_t is the charge of 1 kg mass, called test shock impact load; r_u is radius of the charge; R_i is distance from the charge centre to the place of measurement; G_u is weight of the charge.

Therefore the Eq. (19) can be rewritten as follows

$$I_i = I_t\left(\frac{0.1}{0.1 + R_i}\right) \tag{20}$$

where I_i is shock impact load at the point i, the coordinates of which are r_i, θ_i, z_i; R_i is distance from point i to the real point of the charge explosion, the coordinates r_0, θ_0, z_0, *i.e.* $R_{ir} = |r_i - r_0|$, $R_{i\theta} = |\theta_i - \theta_0|$ and $R_{iz} = |z_i - z_0|$.

Having considered and evaluating the pipe's geometry, we can regard that significant difference is possible only towards z axis (see Fig. 1), i.e. alongside the pipe. Taking into account Eqs. (10) and (20), the impact load can be given as

$$\int P_{ir} dt = L U_{(i)} I_t \frac{0.1}{0.1 + R_{ir}} \tag{21}$$

$$\int P_{i\theta} dt = L V_{(i)} I_t \frac{0.1}{0.1 + R_{i\theta}} \tag{22}$$

$$\int P_{iz} dt = L W_{(i)} I_t \frac{0.1}{0.1 + R_{iz}} \tag{23}$$

For example, the earlier obtained Eq. (15), on x axis, can be rewritten as

$$q_{2r} = \frac{0.1 L U_{(i)} I_t t}{(0.1 + R_{ir}) m}\left(1 - \frac{k_T t^2}{6m}\right) + \frac{k_T^2 t^5}{120 m^2} \tag{24}$$

Then, taking into account Eq. (20) for approximate calculation along x axis of the real displacement of solidified sediment, under the action of an impact load, we get

$$u_i = q_{2r} U_i \tag{25}$$

where $U_i = U(r_i, \theta_i, z_i)$.

Thus, we have found a way to calculate the real displacements of the solidified elastically plastic sediment in the pipe after an explosion of a single charge at a specific place in the pipe.

3. Examples

Suppose that dimensions of the pipe are radii r_1 = 19.5 cm, r_2 = 22.5 cm (see Fig. 1) and length 500 cm. Physical-mechanical parameters of the material: sediment density ρ = 800 kg/m^3, shear modulus G = 20 kPa and Poisson's ratio ν = 0.4. Shock impact is obtained after the explosion of a spherical ammonite charge of 1 kg mass in the pipe at: r_0 = 20 cm, θ_0 = 60° and z_0 = 20 cm. The problem simulated numerically is sketched in Figs. 2 and 3. In this simplified model we are calculating total test and real displacement of the cross-section z_k = 10 cm, r_1 = 20 cm elastically plastic sediment.

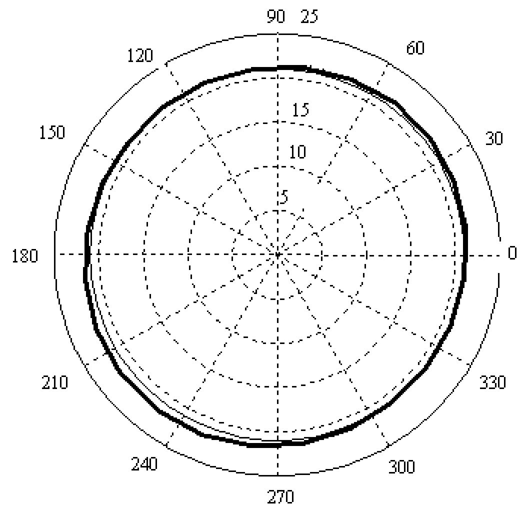

Fig. 2 Distribution of test displacement in cross-section of the pipe of elastically plastic sediment

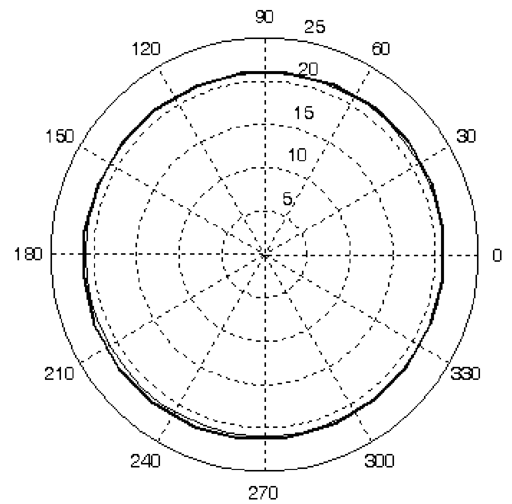

Fig. 3 Distribution of real displacement in cross-section of the pipe of elastically plastic sediment

4. Conclusions

The developed analytical method enables:

1. to calculate approximately the displacements of elastically plastic sediment appeared under shock impact at a specific place of the pipe;

2. to establish functional dependence between shock impact parameters and sediment displacements;

3. to evaluate consequences of explosion in the pipe;

4. to develop new technologies of pipeline cleaning.

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DVIEJŲ CILINDRINIŲ PAVIRŠIŲ, VEIKIAMŲ SMŪGINĖS APKROVOS, SĄVEIKOS ANALIZĖ

R e z i u m ė

Straipsnyje nagrinėjamos vamzdynų sistemų, veikiamų smūginės apkrovos, reakcija bei dinaminių procesų modeliavimo ir vertinimo galimybės. Nevienalytėms sistemoms būdingos skirtingos savybės ir ypatumai, pvz., vamzdynas su apnašomis viduje. Smūginis poveikis yra efektyvus metodas, pakeičiantis šios sistemos kokybinę ir kiekybinę būklę. Sukurtas apnašų tampriai plastiškos medžiagos poslinkių, esant smūginiam poveikiui, matematinis modelis ir atliktas modeliavimas. Tai leidžia analizuoti sistemų specifinės būklės keitimo dinamiką. Straipsnyje pateikti tampriai plastiškos medžiagos poslinkių, inicijuotų sprogimais vamzdynų sistemoje, skaičiavimų rezultatai.

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ANALYSIS THE INTERACTION OF TWO CYLINDRICAL SURFACES UNDER SHOCK IMPACT LOAD

S u m m a r y

The reactions of pipeline systems to shock impact load and the possibilities of the simulation and evaluation of dynamic processes are investigated in the paper. A heterogeneous system is composed of the elements with different properties, *e.g.* a pipeline with the sediment inside. Impact effect on it is one of the most effective methods to make substantial qualitative and quantitative changes in such system. The mathematical model for calculation of the elastically plastic displacements of the sediment was derived. It allows the analysis of dynamics of the change of the system’s specific state. The paper presents the calculation results of elastically plastic displacements of the substance caused by explosions in pipeline systems.

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АНАЛИЗ ВЗАИМОДЕЙСТВИЯ ДВУХ ЦИЛИНДРИЧЕСКИХ ПОВЕРХНОСТЕЙ ПРИ УДАРНОЙ НАГРУЗКЕ

Р е з ю м е

В статье исследуется реакция системы трубопроводов при воздействии ударной нагрузки и возможности моделирования динамических процессов. Неоднородные системы характеризуются различными свойствами и особенностями, например, трубопровод с осадками на внутренней поверхности. Ударное воздействие на систему является эффективным методом качественного и количественного изменения её состояния. Предложена математическая модель перемещений упругопластичного материала осадков при ударном воздействии и проведено математическое моделирование. Это позволяет проводить динамический анализ изменения специфического состояния системы. В статье представлены численные результаты расчета перемещений упругопластичного материала осадков, обусловленных взрывами в трубопроводной системе.

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