ISSN 1392-1207. MECHANIKA. Vol. 30, No. 4, 2024: 308–317

Study on the Effect of Fiber Orientation on the Elastic Constants of Carbon Fiber Reinforced Polypropylene Composites

Shuiwen ZHU*, Shunxin WU, Zhangzheng HU***, Yu FU******
**School of Automotive Engineering, Hubei University of Automotive Technology, Shiyan 442002, China, E-mail: swzhu@huat.edu.cn*
***Hubei Key Laboratory of Automotive Power Train and Electronic Control, Shiyan 442002, China, E-mail: 202211131@huat.edu.cn*
****Hubei Key Laboratory of Automotive Power Train and Electronic Control, Shiyan 442002, China, E-mail: 20220010@huat.edu.cn*
*****College of Civil Engineering and Architecture, Southwest University of Science and Technology, Mianyang 621010, China, E-mail: fuyuyl@swust.edu.cn (Corresponding author)*

https://doi.org/10.5755/j02.mech.37069

1. Introduction

Carbon fiber reinforced polypropylene (CF/PP) composite materials, as lightweight and high-strength materials, have extensive application prospects in fields such as aerospace, automotive industry, and construction engineering. However, factors such as fiber type, content, length, distribution, orientation, and shape can all influence the mechanical properties of the material. Since fiber orientation affects the stress transfer between fibers and the matrix as well as the internal stress distribution of composite materials, selecting the appropriate fiber orientation can maximize the utilization of fiber strength and stiffness, thereby significantly improving the overall performance of composite materials. In recent years, research on fiber orientation mainly falls into two categories: one utilizes theoretical models to determine the elastic modulus of fiber composite materials, emphasizing the importance of fiber orientation averaging factors in accurately predicting material performance; the other focuses on experimental and finite element methods to explore the influence of fiber orientation distribution on the mechanical properties of composite materials.

Some researchers study the mechanical properties of composite materials through theoretical models. Shokrieh et al. [1] used a modified Halpin-Tsai method and Mori-Tanaka laminate analogy (MT-LA) analysis method to calculate the elastic modulus of nanoparticle-reinforced composite materials. They found that the new coefficient in the modified Halpin-Tsai equation is influenced by the volume fraction and stiffness ratio of the matrix and reinforcing material, further expanding the model's applicability. Montazeri et al. [2] investigated the effects of adding multi-walled carbon nanotubes (MWNT) and surface modification on the mechanical properties of composite materials. By incorporating orientation and exponent shape factors into the Halpin-Tsai equation, they calculated the Young's modulus and tensile strength of MWNT/epoxy resin composite materials, with results close to experimental values. Yeh et al. [3] proposed an improved Halpin-Tsai equation to evaluate the Young's modulus and tensile strength of MWNT/phenolic composite materials by introducing a random orientation factor and an exponent shape factor into the equation. It was found that the improved Halpin-Tsai equation agrees well with experimental values in terms of tensile strength and Young's modulus. Hassanzadeh-Aghdam et al. [4] proposed a new Halpin-Tsai micromechanical model considering the random dispersion, undulation, and agglomeration of carbon nanotubes (CNTs). They found that uniformly distributed and straight CNTs have a more significant reinforcing effect, providing new insights for the design of nanotube-reinforced composite materials, and discussed the influence of interface effects on composite material properties. Harichandra et al. [5] used the Halpin-Tsai model, fiber orientation averaged Mori-Tanaka model, as well as mixture rule model and Hashin-Shtrikman constraint to determine the elastic modulus of short fiber composite materials. The results indicate that fiber orientation averaging factors are crucial for accurately predicting the elastic properties of composite materials. Yu et al. [6] studied the influence of fiber orientation distribution on the multiscale tensile properties of ultra-high-performance concrete (UHPC). They calculated the elastic properties of UHPC considering fiber orientation using a two-step mean-field homogenization method and predicted the tensile strength of UHPC under different fiber contents and orientation tensors.

Some scholars directly investigate the influence of fiber orientation distribution on the mechanical properties of composite materials through experiments and finite element methods. Gukendran et al. [7] analyzed the effects of six different orientations including 0°, 15°, 30°, 45°, 60°, and 75° on the mechanical properties of jute fiber composite materials. By measuring the tensile strength, bending strength, and impact strength of jute fiber composite materials, it was found that the composite material with a 30° orientation of fibers exhibited better performance. Noman et al. [8] studied the mechanical properties of laminated composite materials with different carbon fiber orientations under tensile loading using finite element analysis methods. By simulating laminated structures of carbon fiber/epoxy resin composite materials with orientations of 0°, 30°, 45°, 70°, and 90°, the effects of fiber orientation on von Mises stress, total deformation, and equivalent elastic strain of composite materials were analyzed. The results showed that the composite material with a 0° orientation exhibited superior mechanical strength. H.S et al. [9] investigated the mechanical properties of glass fiber reinforced plastic (GFRP) composite materials under different fiber orientations (0°, 45°, and 90°). Composite samples with different orientations were prepared using hand lay-up technique and subjected to axial tensile testing. The results showed that the sample with a 0° orientation exhibited the best tensile performance. Aravinth et al. [10] studied the mechanical properties of sisal fiber reinforced composite materials. By changing the fiber direction, length, and volume fraction, 18 composite materials were prepared and subjected to mechanical testing. The results showed that the composite material had higher strength when using long fibers (60 cm) at a 45° orientation and 35% volume fraction. S. et al. [11] investigated the effect of fiber orientation on the tensile properties of unidirectional carbon fiber/epoxy composite materials through experimental and theoretical studies. Tensile tests were conducted on five different orientations (0°, 30°, 45°, 60°, and 90°) of carbon fiber reinforced epoxy composite materials. The results showed that the composite material exhibited the maximum tensile strength at a 0° orientation, while the minimum tensile strength was observed at a 90° orientation. Furthermore, based on four common failure theories (maximum stress, maximum strain, Tsai-Hill, and Tsai-Wu), the tensile stress of the composite material was predicted, revealing certain errors between predicted and experimental results at 30°, 45°, and 60° orientations. Shweta et al. [12] prepared mixed composite materials of carbon fiber, glass fiber, and natural jute fiber, and studied the influence of fiber orientation on various mechanical properties. The results showed that the combined use of carbon fiber and glass fiber could enhance the tensile, bending, shear, and hardness properties of composite materials, and fiber orientation also affected the performance of composite materials. Kumar et al. [13] investigated the mechanical properties of basalt fiber-reinforced epoxy composites. Composite plates with fiber orientations at 45°, 60°, and 90° were fabricated using hand lay-up technique. The results revealed that the mechanical properties of the 90° bidirectional basalt fiber-reinforced composite were superior to those of the 60° and 45° fiber orientations. Upendra et al. [14] studied the influence of fiber orientation on the mechanical properties of natural fiber-reinforced epoxy resin composites using finite element analysis. They established composite models with various orientation angles such as 0°, +22.5°/-22.5°, 45°/-45°, +67.5°/-67.5°, and 90°, and conducted tensile and bending tests. The findings demonstrated that fibers exhibited minimum stress at 0° and 90° orientations, highlighting the significant impact of fiber orientation on the mechanical properties of composites. V. et al. [15] investigated the effect of fiber orientation on the tensile strength of thin composite materials. Experimental results indicated that the 0/90° orientation exhibited the highest tensile strength, while the 30/60° and 45/-45° orientations showed lower tensile strength. Different failure modes were observed under various orientations, confirming the importance of fiber layout on composite performance. Moreover, as the fiber orientation increased from 0° to 45°, the tensile strength exhibited a decreasing trend, emphasizing the crucial role of fiber orientation in composite material design and application.

Despite extensive research on the influence of fiber orientation on composite material properties, attempts to incorporate different fiber orientation parameters into theoretical models are scarce. Existing studies mainly introduce a random orientation factor into the Halpin-Tsai model to predict the elastic properties of composites. However, this factor remains constant and fails to reflect the impact of different orientation angles on performance. In this study, based on the random orientation factor, the Mori-Tanaka method was employed to obtain transversely isotropic mechanical property data of composites instead of experimental data. This facilitated the determination of the directional factor values, and an orientation angle factor was introduced to improve the Halpin-Tsai equation. A modified exponential shape factor relationship with volume fraction and orientation angle factor was established. This approach enables the prediction of the four elastic constants (axial modulus, transverse modulus, axial shear modulus, and transverse shear modulus) of fiber-reinforced composites with different orientation angles. It not only avoids the cost and time consumption of experimental studies but also holds significant implications for understanding the relationship between the microstructure and macroscopic properties of composites.

2. Materials and methods

2.1. Materials

Carbon fiber, as a high-performance reinforcement material, possesses high linear elastic modulus and strength. On the other hand, polypropylene, as a thermoplastic polymer, has good processability, chemical stability, and low cost. By combining carbon fiber with polypropylene, the specific strength and specific modulus of the composite material can be significantly improved. In this study, carbon fiber is the reinforcing phase, while polypropylene serves as the matrix material. Furthermore, it is assumed that the matrix phase is linear elastic and isotropic, while the carbon fiber phase is linear elastic and transversely isotropic. The elastic constants of each component are shown in Table 1 [16].

Table 1

Material mechanical properties parameters

Material parameters	PP	CF
Young's modulus, GPa	1.40	-
Poisson's ratio	0.35	-
Axial Young's modulus, GPa	-	230
Transverse Young's modulus, GPa	-	15
Axial Poisson's ratio	-	0.20
Axial shear modulus, GPa	-	15
Transverse Poisson's ratio	-	0.07

2.2. Improved Halpin-Tsai model

The traditional Halpin-Tsai model [17] is a semi-empirical formula used to predict the mechanical properties of fiber-reinforced composite materials. This model takes into account the interaction between the fiber and the matrix, allowing for accurate prediction of the key mechanical properties of fiber-reinforced composite materials.

$$E = \frac{1+\xi\eta V^f}{1-\eta V^f} E^m , \tag{1}$$

$$\eta = \frac{\left(E^{\mathrm{f}} / E^{\mathrm{m}}\right) - 1}{\left(E^{\mathrm{f}} / E^{\mathrm{m}}\right) + \xi} , \tag{2}$$

where E represents the elastic modulus, V represents volume fraction, η represents the relationship parameter between the fiber, matrix, and shape factor, and ξ represents the shape factor. The superscript f denotes the fiber, while m

represents the matrix.

To investigate the influence of fiber orientation on the elastic constants of the composite material, Cox [18] introduced a parameter α to account for the randomness of discontinuous fibers. If the fiber length is greater than the specimen thickness, it is assumed that the fiber orientation is random in two dimensions, and the parameter α is set to 1/3. If the fiber length is much smaller than the specimen thickness, it is assumed that the fiber orientation is random in three dimensions, and the parameter α is set to 1/6 [3]. Cox's research demonstrated that the value of α can represent the degree of fiber orientation. However, Cox's research was limited to the case of randomly oriented fibers in two dimensions. Building upon Cox's work, this study further determines the value of α for unidirectional fibers in two dimensions (which needs to be calculated and will be provided in the next section). In order to differentiate from Cox's random orientation factor, in this study, α is referred to as the orientation factor. Thus, the traditional Halpin-Tsai equation can be modified as follows:

$$E = \frac{1+\xi\eta V^f}{1-\eta V^f} E^m, \tag{3}$$

$$\eta = \frac{\left(\alpha E^f / E^m\right)-1}{\left(\alpha E^f / E^m\right)+\xi}. \tag{4}$$

Since the carbon fiber used in this study as the reinforcement material is transversely isotropic, the carbon fiber-reinforced polypropylene composite material will have five elastic constants. By expanding Eqs. (3) and (4), the formulas for axial Young's modulus, transverse Young's modulus, axial shear modulus, and transverse shear modulus can be obtained by Eqs. from (5) to (12), while the axial Poisson's ratio can be calculated using Eq. (17), which is not discussed in this paper. Combining Eqs. (5) and (9), (6) and (10), (7) and (11), (8) and (12), Eqs. from (13) to (16) corresponding to the four shape factor equations can be derived.

$$E_{11} = \frac{1+\xi_{11}\eta_{11} V^f}{1-\eta_{11} V^f} E^m, \tag{5}$$

$$E_{22} = \frac{1+\xi_{22}\eta_{22} V^f}{1-\eta_{22} V^f} E^m, \tag{6}$$

$$G_{12} = \frac{1+\xi_{12}\eta_{12} V^f}{1-\eta_{12} V^f} G^m, \tag{7}$$

$$G_{23} = \frac{1+\xi_{23}\eta_{23} V^f}{1-\eta_{23} V^f} G^m, \tag{8}$$

$$\eta_{11} = \frac{\left(\alpha E_{11}^f / E^m\right)-1}{\left(\alpha E_{11}^f / E^m\right)+\xi_{11}}, \tag{9}$$

$$\eta_{22} = \frac{\left(\alpha E_{22}^f / E^m\right)-1}{\left(\alpha E_{22}^f / E^m\right)+\xi_{22}}, \tag{10}$$

$$\eta_{12} = \frac{\left(\alpha G_{12}^f / G^m\right)-1}{\left(\alpha G_{12}^f / G^m\right)+\xi_{12}}, \tag{11}$$

$$\eta_{23} = \frac{\left(\alpha G_{23}^f / G^m\right)-1}{\left(\alpha G_{23}^f / G^m\right)+\xi_{23}}, \tag{12}$$

$$\xi_{11} = \frac{\alpha E_{11}^f E_{11} V^m + E^m E_{11} V^f - \alpha E_{11}^f E^m}{E^m\left(\alpha E_{11}^f V^f - E^m V^f + E^m - E_{11}\right)}, \tag{13}$$

$$\xi_{22} = \frac{\alpha E_{22}^f E_{22} V^m + E^m E_{22} V^f - \alpha E_{22}^f E^m}{E^m\left(\alpha E_{22}^f V^f - E^m V^f + E^m - E_{22}\right)}, \tag{14}$$

$$\xi_{12} = \frac{\alpha G_{12}^f G_{12} V^m + G^m G_{12} V^f - \alpha G_{12}^f G^m}{G^m\left(\alpha G_{12}^f V^f - G^m V^f + G^m - G_{12}\right)}, \tag{15}$$

$$\xi_{23} = \frac{\alpha G_{23}^f G_{23} V^m + G^m G_{23} V^f - \alpha G_{23}^f G^m}{G^m\left(\alpha G_{23}^f V^f - G^m V^f + G^m - G_{23}\right)}, \tag{16}$$

$$\nu_{12} = V^f \nu_{12}^f + V^m \nu^m, \tag{17}$$

here G represents the shear modulus, ν_{12} represents the axial Poisson's ratio. The subscript 11 represents axial, 22 represents transverse, 12 represents axial shear, and 23 represents transverse shear.

In practical applications, fiber-reinforced composite materials often consist of fibers with different orientations. These different orientations can significantly affect the mechanical properties of the composite material, which the Halpin-Tsai model does not consider. In order to more accurately describe the mechanical behavior of the composite material, an orientation degree factor is introduced as part of the improved Halpin-Tsai model. This allows for a better description of the distribution of fibers in the composite material and more accurate prediction of the material's mechanical properties. Therefore, this study improves the Halpin-Tsai model to predict the elastic constants, such as axial Young's modulus, transverse Young's modulus, axial shear modulus, and transverse shear modulus, of CF/PP composite materials under different fiber orientations and volume fractions. The improved model, as shown in equation (18), modifies the shape factor to an exponential shape factor. By substituting Eq. (18) into Eqs. (3) and (4), the improved Halpin-Tsai model is obtained

$$\xi_{ij} = e^{\left(a\theta + bV^f + c\theta V^f + d\right)} \quad \left(i, j = 1, 2, 3\right), \tag{18}$$

where θ represents the orientation degree, and a, b, c, and d are constants.

2.3. Mori-Tanaka model

The Mori-Tanaka model [19] is a theoretical model used to calculate the mechanical properties of multiphase composite materials. This model takes into account the interaction between the matrix and inclusions, as well as the effects of inclusion shape, distribution, and volume fraction on the overall performance of the composite material. To accurately describe the behavior of composite materials, the model introduces the concept of a Representative Volume Element (RVE) and recognizes that the strain experienced by individual inclusions may differ from the macroscopic strain applied to the composite material. To address this issue, the method transforms the problem of multiple inclusions into a single inclusion problem, where the far-field strain acting on the inclusion is the average strain of the composite material, as shown in Fig. 1.

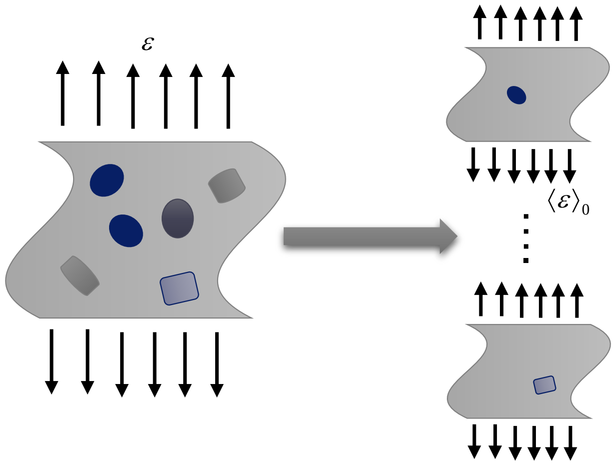

Fig. 1 Mori-Tanaka model: conversion of a multi-inclusion problem into a single-inclusion problem

In a multiphase composite material, the macroscopic stress σ and macroscopic strain ε have the following relationship:

$$\sigma = L : \varepsilon \,. \tag{19}$$

Hill and Law proposed that the overall properties of composite materials are related to the moduli and volume fractions of each phase material:

$$L = L_1 + \sum_{s=2}^{N} c_s \left(L_s - L_1\right) : A_s \,, \tag{20}$$

where A_s is the strain concentration factor, cs is the volume fraction of inclusions, and L_s is the elastic modulus of each phase material (s = 1, 2, …, N), with s = 1 representing the matrix phase.

The mechanical behavior of each phase material is determined by the following expression:

$$\sigma_s\left(x\right) = L_s : \varepsilon_s\left(x\right). \tag{21}$$

The strain concentration factor A_s is defined as:

$$\varepsilon_s\left(x\right) = A_s\left(x\right) : \varepsilon \,. \tag{22}$$

Since the Mori-Tanaka method is based on the concept of a sparse solution, which considers the multiphase material as an inhomogeneous medium with no interaction between phases, it approximates the isolated inclusions (s = 2, 3, …, N) being embedded in a matrix (s = 1) in a rotating manner. It assumes that the strain in the matrix is equal to the total strain ε of the material, while the strain in each phase is given by:

$$\varepsilon_s\left(x\right) = T_s : \varepsilon \,, \tag{23}$$

where T_s is the concentrated tensor of the sparse solution.

The sparse solution considers the interaction between two phases (s = 1 and $s \neq 1$). The overall elastic modulus can then be expressed as:

$$L = L_1 + \sum_{s=2}^{N} c_s \left(L_s - L_1\right) : T_s \,. \tag{24}$$

In the sparse solution, inclusions are considered as isolated, non-interacting phases. However, in the Mori-Tanaka method, the strain in the matrix (s = 1) is not the same as the total strain ε, but an unknown strain ε_1. Therefore, the strain in the inclusion phase is given by:

$$\varepsilon_s = T_s : \varepsilon_1 \,. \tag{25}$$

By substituting Eq. (25) into Eq. (26) and expanding, we obtain Eq. (27):

$$\varepsilon = \sum_{s=1}^{N} c_s : \varepsilon_s \,, \tag{26}$$

$$\varepsilon = \sum_{s=1}^{N} c_s T_s : \varepsilon_1 = \left[c_1 I + \sum_{s=2}^{N} c_s T_s\right] : \varepsilon_1 \,. \tag{27}$$

By transforming Eq. (27), we get:

$$\varepsilon_1 = \left[c_1 I + \sum_{s=2}^{N} c_s T_s\right]^{-1} : \varepsilon \,. \tag{28}$$

By combining Eqs. (28), (25), and (22), we obtain Eq. (29):

$$A_s = T_s : \left[c_1 I + \sum_{s=2}^{N} c_s T_s\right]^{-1} . \tag{29}$$

Finally, the effective stiffness tensor of the composite material, calculated using the Mori-Tanaka method, is given by Eq. (30):

$$L = L_1 + \left[\sum_{s=2}^{N} c_s \left(L_s - L_1\right) : T_s\right] : \left[\sum_{s=1}^{N} c_s T_s\right]^{-1} \tag{30}$$

Where $T_1 = I$, and when the inclusions have the same shape, T_s can be expressed by the following equation:

$$T_s = \left[I + P : \left(L_s - L_1\right)\right]^{-1} . \tag{31}$$

The Mori-Tanaka method provides a solution approach, considering the three-dimensional space of fiber orientation represented by ellipsoids. This study investigates the influence of different fiber orientations and volume fractions on the elastic constants of CF/PP composite materials within a two-dimensional plane. Assuming a unidirectional distribution of fibers in the matrix, with orientation degrees represented as shown in Fig. 2, and cylindrical fiber shapes, the Mori-Tanaka mechanical model is utilized to establish Representative Volume Element (RVE) models for various orientations and volume fractions. By loading in the x-axis direction and calculating the stiffness matrix of fiber-reinforced composite materials, the elastic constants of the composite materials are obtained. These values are then substituted into Eqs. (13)-(16) to compute the shape factor values of each elastic constant at different fiber orientations and volume fractions. Formula (18) is employed to fit the modified exponential shape factor using the corrected values. Subsequently, the corrected exponential shape factor is applied to Eqs. (3) and (4) to derive the improved Halpin-Tsai model.

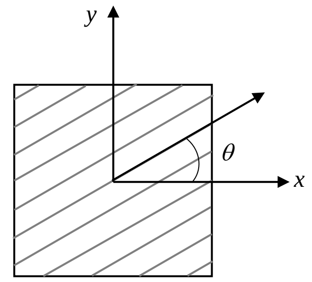

Fig. 2 Axis plot of fiber orientation degree

The value of the orientation factor α in Eqs. (13)-(16) is set to 24. This value is determined through the Mori-Tanaka method for obtaining the elastic constants of composite materials. It was found that when α is greater than or equal to 24, the shape factor values exhibit similar trends with varying orientation degrees at different volume fractions. Even when α is 1024, the change in shape factor values is minimal. When α is less than 24, the shape factor values of the transverse elastic modulus do not show a clear pattern with varying orientation degrees at different volume fractions, making it impossible to fit a surface. This may be attributed to the less pronounced effect of orientation degrees on the transverse elastic modulus when α is less than 24. Hence, α is chosen to be 24.

3. Results and discussion

3.1. Axial elastic modulus

The variation of the axial elastic modulus with different orientation angles and volume fractions is depicted in Fig. 3. Firstly, for all given orientation degrees, the axial elastic modulus increases with increasing volume fraction. This indicates an increase in material rigidity at higher fiber contents, as fibers typically possess higher elastic moduli than the matrix material. Secondly, for the same volume fraction, the dependency of the composite material's axial elastic modulus on the fiber orientation angle follows a U-shaped trend. The axial elastic modulus is highest in the 0° direction (where the fiber direction aligns with the loading direction), reflecting the high stiffness of fibers along their axial direction. As the orientation angle increases, the axial elastic modulus gradually decreases, reaching its lowest value at 60° orientation, and slightly increasing at 90° (where the fiber direction is perpendicular to the loading direction). This indicates lower stiffness of fibers in non-axial directions. These results are consistent with the trends reported in reference [11], validating the accuracy of this study.

Observing Fig. 4 reveals a certain relationship between the aspect ratio factor of the axial elastic modulus and

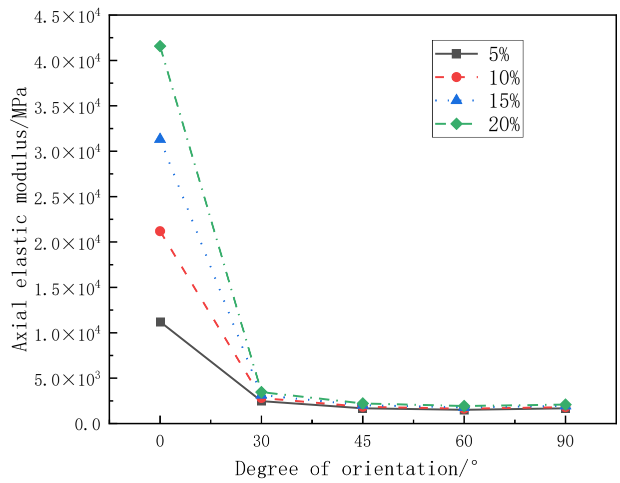

Fig. 3 Variation of axial modulus of elasticity with fiber orientation of carbon fiber reinforced polypropylene composites at different volume fractions

the fiber orientation angle and volume fraction. When the orientation angle is the same, with increasing volume fraction, the aspect ratio factor generally decreases while the axial elastic modulus increases. This suggests that for the same orientation angle, a smaller aspect ratio factor corresponds to a larger axial elastic modulus. Similarly, at the same volume fraction, the aspect ratio factor and fiber orientation angle exhibit a decreasing U-shaped relationship. Therefore, the magnitude of the aspect ratio factor can also reflect the magnitude of the axial elastic modulus. Eq. (32) represents the exponential relationship between the aspect ratio factor and fiber orientation angle, fitted from the data in Fig. 4, which can be utilized to determine the aspect ratio factor for different fiber orientations and volume fractions.

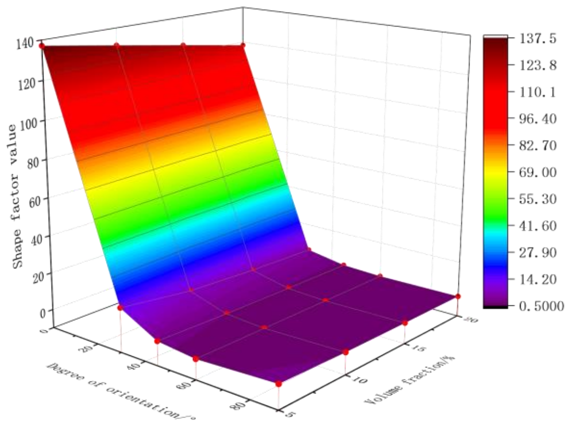

Fig. 4 Variation of axial modulus of elasticity shape factor of carbon fiber reinforced polypropylene composites with degree of fiber orientation at different volume fractions

$$\xi_{11} = e^{\left(-0.0803\theta - 0.9953V^f - 0.1277\theta V^f + 4.9727\right)}. \qquad (32)$$

Comparing the simulated values obtained from the Mori-Tanaka model with the fitted values calculated from the improved Halpin-Tsai model is depicted in Fig. 5. The error rate of the axial elastic modulus is within 10% from 0°

to 60° and does not exceed 17% at 90°. Thus, the model exhibits good fitting performance from 0° to 60°.

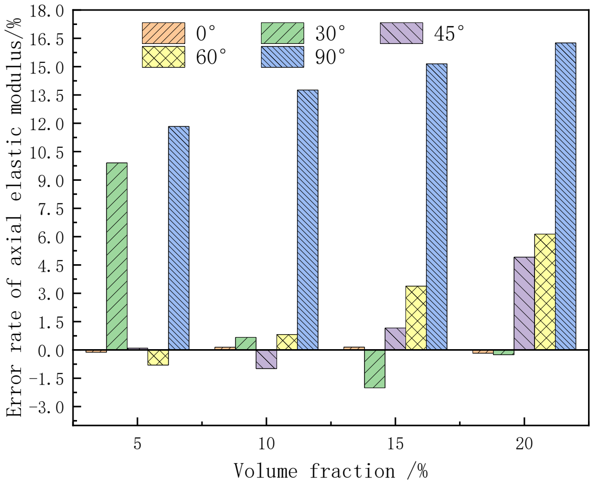

Fig. 5 Variation of the error rate of axial modulus of elasticity of carbon fiber reinforced polypropylene composites with the degree of fiber orientation at different volume fractions

3.2. Transverse elastic modulus

Fig. 6 illustrates the variation of the transverse elastic modulus with fiber orientation angle under different volume fractions. Firstly, when the volume fraction of the fiber remains constant, the transverse elastic modulus gradually increases with the fiber angle, demonstrating a nonlinear influence of fiber direction on the transverse elastic modulus. Secondly, for a fixed fiber orientation, the transverse elastic modulus increases with increasing fiber volume fraction, indicating that the addition of fibers enhances the overall rigidity of the composite material, making it more resistant to deformation in the transverse direction.

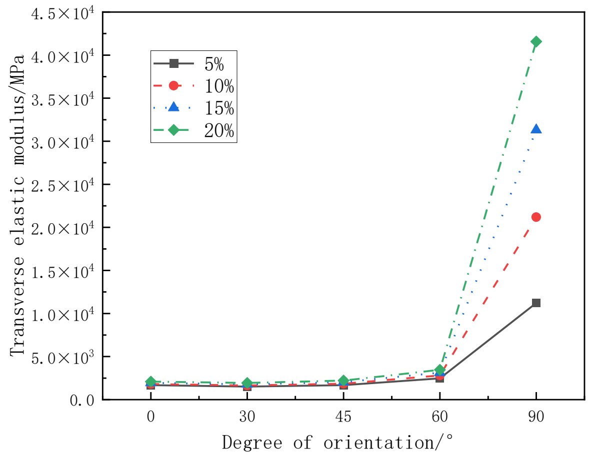

Fig. 6 Variation of transverse modulus of elasticity with fiber orientation degree in carbon fiber reinforced polypropylene composites with different volume fractions

Fig. 7 displays the relationship between the aspect ratio factor and fiber orientation angle for different fiber volume fractions, showing a U-shaped trend. The aspect ratio factor is significantly higher at 90°, reflecting the influence of fiber arrangement. Moreover, the trend in Fig. 7 is symmetric to that in Fig. 4, indicating the symmetry between the axial and transverse elastic moduli. Eq. (33) represents the relationship between the exponential aspect ratio factor and different fiber orientations and volume fractions, fitted from the data in Fig. 7

$$\xi_{22} = e^{\left(0.0905\theta - 21.0256V^{f} + 0.2246\theta V^{f} - 2.4226\right)}. \qquad (33)$$

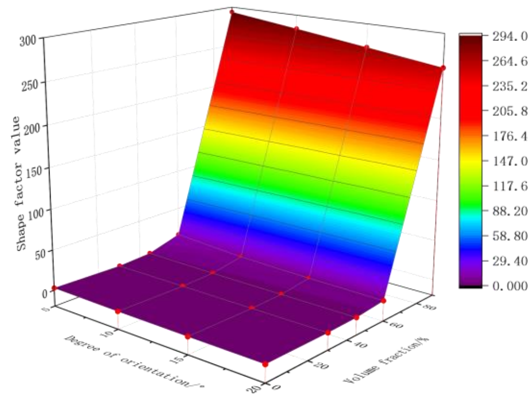

Fig. 7 Variation of transverse modulus of elasticity shape factor of carbon fiber reinforced polypropylene composites with degree of fiber orientation at different volume fractions

Fig. 8 compares the error rates of the simulated and fitted values and shows that the maximum difference in the transverse modulus of elasticity is 16.46% at 0°; the maximum error rate does not exceed 6.5% at 30° to 90° and below 15% volume fraction. Therefore, the model fits better at 30° to 90° and below 15% volume fraction.

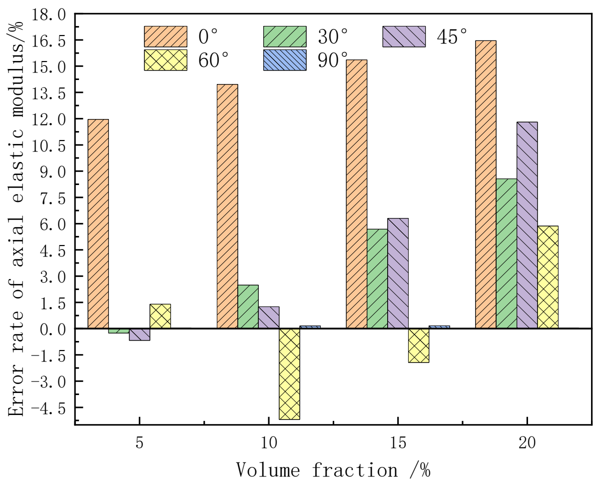

Fig. 8 Variation of transverse modulus of elasticity error rate with fiber orientation degree for carbon fiber reinforced polypropylene composites with different volume fractions

3.3. Axial shear modulus

Fig. 9 shows the relationship between the axial shear modulus of CF/PP composite materials and the orientation angle under different volume fractions. Firstly, when the fiber volume fraction remains constant, the axial shear modulus gradually decreases as the fiber orientation angle increases from 0° to 90°. This indicates that the contribution of fibers to the overall shear performance of the material is minimal when they are perpendicular to the loading direction, and maximal when parallel. Secondly, for the same fiber orientation angle, the axial shear modulus increases with increasing fiber volume fraction, suggesting that the reinforcement effect of fibers enhances with their content in the matrix.

Fig. 10 depicts the variation of the aspect ratio factor of the axial shear modulus with fiber orientation angle under different volume fractions. The aspect ratio factor decreases overall with increasing volume fraction for the same orientation angle, while the axial shear modulus increases. Additionally, the aspect ratio factor exhibits an arc-shaped relationship with fiber orientation angle at the same volume fraction. Eq. (34) represents the relationship between the aspect ratio factor of the axial shear modulus and fiber orientation angle and volume fraction, which can be utilized to determine the aspect ratio factor of the axial shear modulus

$$\xi_{12} = e^{\left(-0.0075\theta - 0.1860V^{f} - 0.0035\theta V^{f} - 0.1048\right)} \quad (34)$$

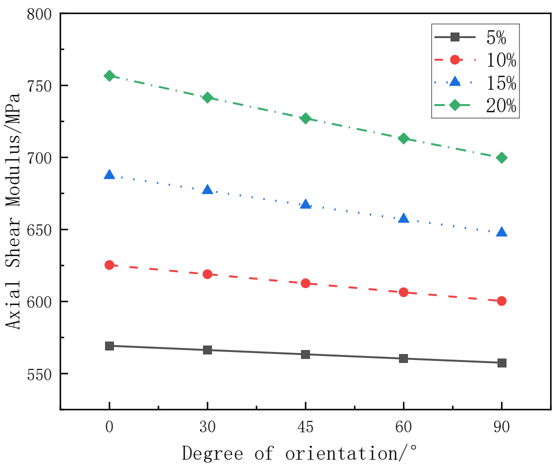

Fig. 9 Variation of axial shear modulus of carbon fiber reinforced polypropylene composites with degree of fiber orientation at different volume fractions

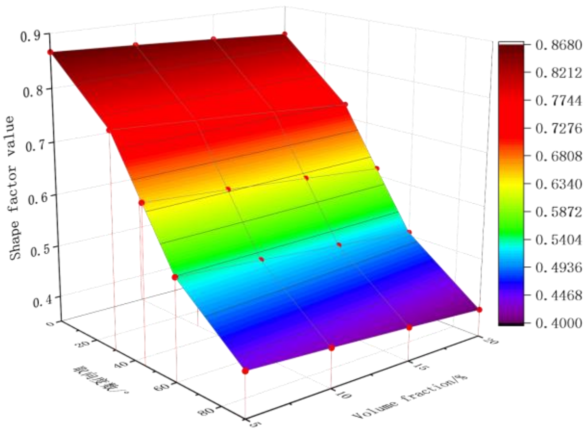

Fig. 10 Variation of axial shear modulus shape factor with fiber orientation degree in carbon fiber reinforced polypropylene composites at different volume fractions

The maximum error rate between simulated and fitted values for the axial shear modulus is only 0.83%, as shown in Fig. 11, indicating the effective prediction capability of the improved Halpin-Tsai model for CF/PP composite materials' axial shear modulus.

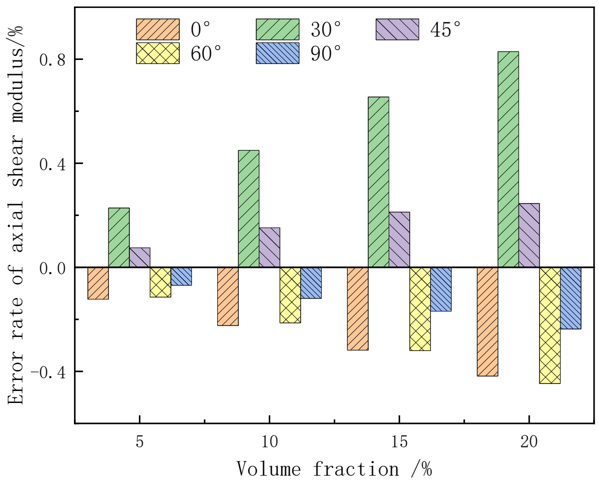

Fig. 11 Variation of axial shear modulus error rate with fiber orientation degree for carbon fiber reinforced polypropylene composites with different volume fractions

3.4. Transverse Shear Modulus

The influence of different fiber orientations and volume fractions on the transverse shear modulus is illustrated in Fig. 12. Firstly, at the same volume fraction, the transverse shear modulus initially increases and then decreases with the increase in fiber orientation angle. The transverse shear modulus reaches its maximum value when the fiber orientation angle is 45°. This is because the 45° angle optimally utilizes the anisotropy of the composite material due to the angle between the fibers and the loading direction, thus enhancing the material's shear strength. This indicates that 45° is a more ideal fiber orientation angle, maximizing the role of fibers in the composite material. Secondly, when the fiber orientation angle is fixed, the transverse shear modulus increases with an increase in fiber volume fraction. As fibers act as reinforcement, higher fiber volume fractions result in greater overall strength of the composite material, thus increasing the transverse shear modulus. When the fiber orientation angle is 0° or 90°, the transverse shear modulus remains unchanged regardless of changes in fiber volume fraction. This is because at these orientations, the fibers are parallel or perpendicular to the loading direction, contributing minimally to shear deformation.

Under different volume fractions, the shape factor of the transverse shear modulus varies with the degree of fiber orientation, as shown in Fig. 13. It can be observed that the shape factor of the transverse shear modulus is symmetric about 45° and exhibits a semi-saddle shape. This symmetry around 45° may be due to the arrangement of fibers relative to the loading direction at this angle, resulting in symmetric values of the shape factor for the transverse shear modulus. The surface in Fig. 13 is fitted separately, as shown in Eq. (35). Separate fitting is adopted because there is currently no more suitable model formula than Eq. (18), and thus, Eq. (35) can be used to determine the shape factor of the transverse shear modulus

$$\xi_{23}=\begin{cases} e^{\left(0.0507\theta-7.0238V^{f}-0.0334\theta V^{f}+1.6125\right)} & \left(0^{\circ}\le\theta\le45^{\circ}\right) \\ e^{\left(-0.0507\theta-9.9907V^{f}+0.0324\theta V^{f}+6.1756\right)} & \left(45^{\circ}\le\theta\le90^{\circ}\right) \end{cases}. \quad (35)$$

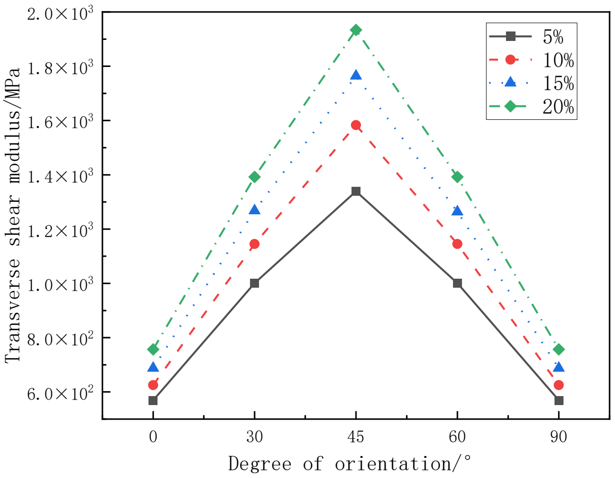

Fig. 12 Variation of transverse shear modulus of carbon fiber reinforced polypropylene composites with degree of fiber orientation at different volume fractions

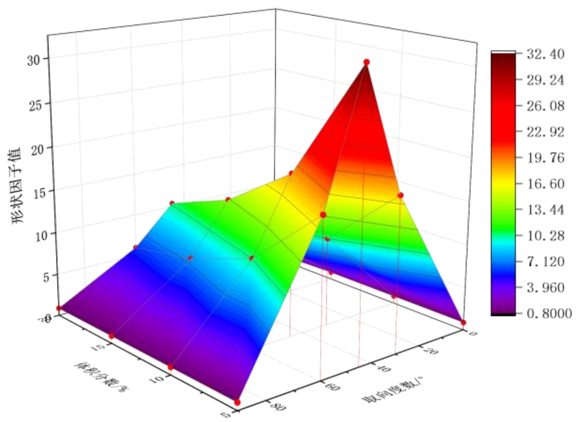

Fig. 13 Variation of transverse shear modulus shape factor with fiber orientation degree in carbon fiber reinforced polypropylene composites at different volume fractions

Fig. 14 shows the error rates of the values obtained from the Mori-Tanaka model and the modified Halpin-Tsai model, and it was found that the maximum error rate of the transverse shear modulus was more than 10% but not more than 15% at 0° and 90°, and the maximum error rate was 5.85% at 30° to 60° and below 15% volume fraction. Therefore, the model fits better at 30° to 60° and volume fraction below 15%.

3.5. Error analysis

Among the four elastic constants mentioned above, the maximum error occurs at 0° or 90°, but the maximum error does not exceed 17%. Previously, polynomial fitting was used, which required a higher order (up to 5th order)

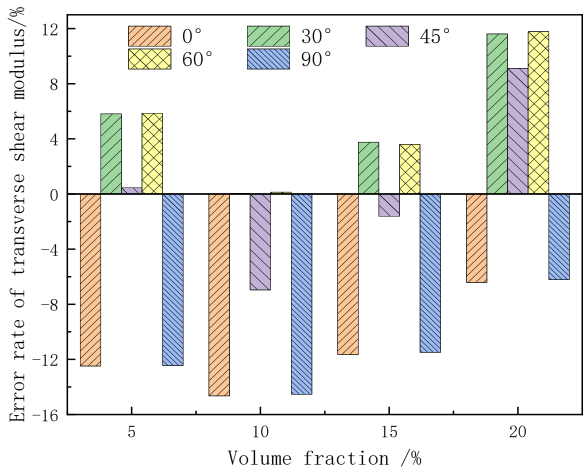

Fig. 14 Variation of transverse shear modulus error rate with fiber orientation degree for carbon fiber reinforced polypropylene composites with different volume fractions

and could potentially lead to overfitting. However, it was observed that even with polynomial fitting, the model error remained high. Therefore, exponential functions were employed for fitting, reducing the fitting coefficients and avoiding overfitting issues. Furthermore, in reference [11], the numerical values obtained using the maximum stress, maximum strain, Tsai-Hill, and Tsai-Wu theories [19] were compared with experimental results, showing a maximum error of 17%, which is similar to the error obtained from the fitted functions in this study. This to some extent also validates the applicability of the exponential model used in this study.

4 Conclusion

Based on the Mori-Tanaka method, the influence of different fiber orientation angles and volume fractions on the elastic constants of transversely isotropic fiber-reinforced composite materials was studied. Building upon the random orientation factor, the values of the directional factors were determined using an improved Halpin-Tsai model, and an analysis of the four elastic constants of transversely isotropic materials was conducted. Finally, a modified exponential shape factor relationship with volume fraction and orientation angle factor was obtained.

The study shows that, at the same orientation angle, the elastic constant values of composite materials increase with an increase in fiber volume fraction, while the shape factor values decrease with an increase in fiber volume fraction. At the same volume fraction, the shape factor values of the elastic constants exhibit U-decreasing, U-increasing, circular arc, and semi-saddle trends with changes in fiber orientation angle, with smaller shape factor values corresponding to larger elastic constant values.

By comparing the simulated values from the Mori-Tanaka model and the fitted values from the improved Halpin-Tsai model, it was found that the maximum error rate for axial elastic modulus is 9.9% from 0° to 60°; for axial shear modulus, it is 0.83%; for transverse elastic modulus and transverse shear modulus from 30° to 60° with volume fractions below 15%, the maximum error rates are 6.3% and 6.96%, respectively. Thus, the improved Halpin-Tsai model can be used within a certain range to predict the elastic constants of fiber-reinforced composite materials and provides a general method for predicting elastic constants of composite materials under different orientation angles.

Acknowledgments

This research was funded by the Doctoral Research Fund of the Hubei University of Automotive Technology, grant number BK202213, Open Fund of Hubei Key Laboratory of Automotive Power Train and Electronic Control, grant number ZDK12023B09, and Scientific Research Project Funded by Hubei Provincial Department of Education, grant number B2023076.

References

1. **Shokrieh, M. M.; Moshrefzadeh-Sani, H.** 2016. On the constant parameters of Halpin-Tsai equation, Polymer 106: 14-20. http://dx.doi.org/10.1016/j.polymer.2016.10.049.
2. **Montazeri, A.; Javadpour, J.; Khavandi, A.; Tcharkhtchi, A.; Mohajeri, A.** 2010. Mechanical properties of multi-walled carbon nanotube/epoxy composites, Materials and Design 31(9): 4202-4208. https://doi.org/10.1016/j.matdes.2010.04.018.
3. **Yeh, M. K.; Tai, N. H.; Liu, J. H.** 2006. Mechanical behavior of phenolic-based composites reinforced with multi-walled carbon nanotubes, Carbon: An International Journal Sponsored by the American Carbon Society, 44(1): 1-9. https://doi.org/10.1016/j.carbon.2005.07.005.
4. **Hassanzadeh-Aghdam, M. K.; Jamali, J.** 2019.A new form of a Halpin–Tsai micromechanical model for characterizing the mechanical properties of carbon nanotube-reinforced polymer nanocomposites, Bulletin of Materials Science 42: 117. https://doi.org/10.1007/s12034-019-1784-6.
5. **Chandekar, H.; Chaudhari, V.; Waigaonkar, S.** 2022. Theoretical models for stiffness prediction of short fibre composites, Materials Today: Proceedings 57(P2): 711-714. https://doi.org/10.1016/j.matpr.2022.02.177.
6. **Yu, J.; Zhang, B.; Chen, W.; Liu, H.** 2022. Multi-scale analysis on the tensile properties of UHPC considering fiber orientation, Composite structures 280: 114835. https://doi.org/10.1016/j.compstruct.2021.114835.
7. **Gukendran, R.; Sambathkumar, M.; Kondayampalayam, S. K. S.; Alsoufi, M. S.; Ibrahim, A. M. M.; Muthusamy, S.; Panchal, H.; Sadasivuni, K. K.; Elsheikh, A. H.** 2021. An extensive analysis of mechanical, thermal and physical properties of jute fiber composites with different fiber orientations, Case Studies in Thermal Engineering 28: 101612. https://doi.org/10.1016/j.csite.2021.101612.
8. **Noman, A. A.; Shohel, S. M.; Riyad, S. H.; Gupta, S. S.** 2023. Investigate the mechanical strength of laminated composite carbon fiber with different fiber orientations by numerically using finite element analysis, Materials Today: Proceedings. https://doi.org/10.1016/j.matpr.2023.02.132.
9. **Sharath Chandra, H.S.; Lokesh, K.S.; Ravindra Babu, G.; Shrinivasa Mayya, D.; Naveen Kumar, J. R.** 2023. Impact of fibre orientation on mechanical properties of GFRP composites, Materials Today: Proceedings 92(P1): 78-83. https://doi.org/10.1016/j.matpr.2023.03.754.
10. **Aravinth, K.; Sathish, R.; Ramakrishnan, T.; Balu Mahandiran, S.; Shiyam Sundhar, S.** 2023. Mechanical investigation of agave fiber reinforced composites based on fiber orientation, fiber length, and fiber volume fraction, Materials Today: Proceedings. https://doi.org/10.1016/j.matpr.2023.05.436.
11. **Mohamed, Y.S.; Abdelbary, A.** 2023. Theoretical and experimental study on the influence of fiber orientation on the tensile properties of unidirectional carbon fiber/epoxy composite, Alexandria Engineering Journal 67: 693-705. https://doi.org/10.1016/j.aej.2022.12.058.
12. **Singh, S.; Kumar Gupta, P.** 2022. Effect of fiber orientation on mechanical properties of jute/carbon/glass hybrid composite, Materials Today: Proceedings 68(P6): 2574-2580. https://doi.org/10.1016/j.matpr.2022.09.419.
13. **Kumar, N.; Singh, A.** 2021. Study the effect of fiber orientation on mechanical properties of bidirectional basalt fiber reinforced epoxy composites, Materials Today: Proceedings, 39(4): 1581-1587. https://doi.org/10.1016/j.matpr.2020.05.707.
14. **Sharan Gupta, U.; Dharkar, A.; Dhamarikar, M.; Choudhary, A.; Wasnik, D.; Chouhan, P.; Tiwari, S.; Namdeo, R.** 2021. Study on the effects of fiber orientation on the mechanical properties of natural fiber reinforced epoxy composite by finite element method, Materials Today: Proceedings 45(P9): 7885-7893. https://doi.org/10.1016/j.matpr.2020.12.614.
15. **Patel, H. V.; Dave, H. K.** 2021. Effect of fiber orientation on tensile strength of thin composites, Materials Today: Proceedings 46(P17): 8634-8638. https://doi.org/10.1016/j.matpr.2021.03.598.
16. **Huang, Q. M.** 2018. Failure and strength of composite materials, Beijing: Science Press (in Chinese).
17. **Affdl, J. C. H.; Kardos, J. L.** 1976. The Halpin-Tsai equations: A review, Polymer Engineering and Science 16(5): 344-352. https://doi.org/10.1002/pen.760160512.
18. **Cox, H. L.** 1952. The elasticity and strength of paper and other fibrous materials, British Journal of Applied Physics 3(3): 72. https://doi.org/10.1088/0508-3443/3/3/302.
19. **Shen. G.; Hu, G.; Liu, B.** 2013. Mechanics of Composite Materials (Second Edition) [M]. Beijing: Tsinghua University Press. (in Chinese).

S. Zhu, S. Wu, Z. Hu, Y. Fu

STUDY ON THE EFFECT OF FIBER ORIENTATION ON THE ELASTIC CONSTANTS OF CARBON FIBER REINFORCED POLYPROPYLENE COMPOSITES

S u m m a r y

In order to predict the effects of different fiber orientations on the elastic constants of composites, this paper takes carbon fiber reinforced polypropylene composites as an example, and adopts the Mori-Tanaka method to establish the Random Volume Element (RVE) model of fiber composites with five different fiber orientations, namely, 0°, 30°, 45°, 60°, and 90°, and investigates the effects of the fiber The effects of fiber orientation and fiber volume fraction on their elastic constants were investigated to obtain the required data by this method instead of experiment. By improving the Halpin-Tsai model, the value of the orientation factor was determined on the basis of the random orientation factor, and the original shape factor was modified to an exponential shape factor, the orientation degree factor was introduced, and finally, the relationship equations between the modified exponential shape factor and the volume fraction and orientation degree were obtained. The results show that for the same fiber orientation and volume fraction, the smaller the value of shape factor, the higher the elastic modulus. The fitting errors of the four values of elastic constants were also analyzed, and it was found that the improved Halpin-Tsai model could be used to predict the elastic constants of carbon fiber-reinforced polypropylene composites in a certain range, thus providing a method to study the prediction of the elastic constants of composites by fiber orientation.

Keywords: Halpin-Tsai model, fiber orientation, volume fraction, Mori-Tanaka model, elastic modulus.

Received April 24, 2024
Accepted August 19, 2024