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Loading and Friction of Elastoplastic Line Contact with Thermal Deformation

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1. Introduction

Line contact is a popular mechanical contact mode occurring on gears, cams and roller bearings ect [1]. It is often lubricated by oils or greases for wide operating conditions. For ideally smooth line contacts, it was shown that when the rolling speed is critically low, the Hertzian zone of a line contact will be in physically adsorbed layer boundary lubrication and carries almost all of the load [2]. Boundary lubrication has been addressed long time in the research [3, 4]. On the solid surface the lubricant molecule with a long chain should be aligned in the sliding direction of the contact and the boundary film only with mono or several lubricant molecule layers should be treated as a solid layer [3, 5]. For widely varying loads, the contact may undergo the elastic, elastoplastic or fully plastic deformations. In the boundary film lubricated line contact, the friction coefficient is usually over 0.1, and the frictional temperature rise in the contact is high for heavy loads and high sliding speeds. This unavoidably causes the significant contact thermal deformation, which is not negligible in the analysis of the contact load and friction. Although there are a lot of studies on the contact thermal deformation [6-8], no studies were seen on the effect of the contact thermal deformation in the elastoplastic line contact in boundary lubrication.

Actually, the contact surface is somewhat rough. Before the whole Hertzian zone of a line contact enters the boundary lubrication, the surface asperity contact will first be in boundary lubrication and it can be considered as a micro line contact. In this condition, the whole line contact may be in the mixed lubrication with the load carried by both the fluid film and the boundary film lubricated asperity contact or even the fresh metal to metal asperity contact. Johnson et al. [9] proposed the mixed lubrication model with the whole contact consisting of both the fluid film lubricated contact and the solid asperity contact. They showed that in a line contact the local asperity contact may undergo the elastic or elastoplastic deformations depending on both the plasticity index and the ratio of the nominal surface separation to the surface roughness, however usually only a very small portion of the asperity contacts may be in the fully plastic deformation [9]. For a local asperity contact, the subject of the influence of the contact thermal deformation also arises and it concerns the local boundary film failure, the local friction, wear and scuffing.

The present paper attempts to study the influence of the contact thermal deformation on the loading and friction of the elastoplastic line contact which is in boundary lubrication. The obtained results are new, not shown by the existing contact mechanics theories. The present study should better the modeling of a lubricated line contact particularly in the mixed or boundary lubrications.

2. Studied Line Contact

Fig. 1 shows the studied line contact which is in physically adsorbed layer boundary lubrication. The two contact surfaces are assumed as perfectly smooth; the contact can also be considered as an asperity contact. Although there are lubricating fluids in the inlet and outlet zones of the contact, the flattened Hertzian-zone-like area in the contact carries almost all of the external load. The studied contact can also be in chemical layer boundary lubrication or the fresh metal-to-metal contact. The upper contact body is the cylinder with the radius R and rotating with the circumferential speed u, and the lower contact body is the stationary plane.

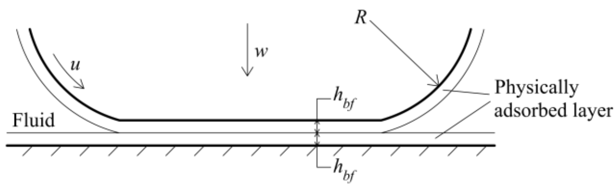


Fig. 1 The studied simple sliding line contact in boundary lubrication

3. Analysis

3.1. For the elastic line contact

With the effect of the contact thermoelastic deformation, the pressure in the Hertzian zone is [10]:

$$p(x) = p_{max,0}\left(1+K_R\right)\sqrt{1-\left(\frac{x}{b_e}\right)^2}\,, \tag{1}$$

where $p_{max,0}$ is the maximum Hertzian contact pressure without the effect of the contact thermoelastic deformation, b_e is the half contact width of elasticity, x is the coordinate the origin of which is at the Hertzian contact center, and $K_R = R / R_t$; Here, R_t is the thermal radius which is expressed as:
$R_t = R_{t,a}R_{t,b} / (R_{t,a} + R_{t,b})$, $R_{t,a} = k_a\rho_a c_a / (u\lambda_a\tau_{av}\alpha_a(1+\nu_a))$, and $R_{t,b} = k_b\rho_b c_b / (u\lambda_b\tau_{av}\alpha_b(1+\nu_b))$. Of the contact surface, k is the thermal diffusivity, c is the specific heat, ρ is the

density, ν is the Poisson's ratio, α is the linear thermal expansion coefficient, and λ is the frictional heat input rate. The subscripts "*a*" and "*b*" respectively refer to the cylinder surface and the plane surface.

The load per unit contact length carried by the contact is:

$$w = \int_{-b_e}^{b_e} p(x)dx = \left(1 + K_R\right) w_0 \ , \tag{2}$$

where w_0 is the load per unit contact length carried by the contact without the effect of the contact thermoelastic deformation.

According to the elastic contact mechanics [11], it is obtained that:

$$\frac{b_e}{R} = \sqrt{\frac{8W}{\pi\left(1 + K_R\right)}} \text{, for } W \le W_{ec} \ , \tag{3}$$

where W is the dimensionless load, $W = w / (E_v R)$, W_{ec} is the dimensionless critical load for the contact elastic deformation, $W_{ec} = \pi K_{ec}^2 \left(1 + K_R\right) / 8$ and $K_{ec} = 2.387 \Omega_s / \left(E_v \sqrt{1 + K_R^2}\right)$; Here, Ω_s is the hardness of the softer surface, E_v is the equivalent Young's modulus of elasticity of two contact surfaces, $2 / E_v = \left(1 - \nu_a^2\right) / E_a + \left(1 - \nu_b^2\right) / E_b$, and E is the Young's modulus of elasticity of the contact surface.

The friction force per unit contact length in the contact is:

$$\begin{aligned} F_f &= \int_{-b_e}^{b_e} \tau_s dx = \int_{-b_e}^{b_e} \left[\tau_{s0} + \alpha_s p\left(x\right)\right] dx = \\ &= 2R\tau_{s0}\sqrt{\frac{8W_0}{\pi}} + \alpha_s w_0 \left(1 + K_R\right), \end{aligned} \tag{4}$$

where τ_s is the shear strength of the sliding interface, τ_{s0} is that at atmospheric pressure extrapolated, α_s is the interfacial shear strength-pressure proportionality, and $WE_vR_0 = w_0 / (E_vR)$.

The friction coefficient of the contact is:

$$f = \frac{F_f}{w} = 2\lambda_{\tau 1}\sqrt{\frac{8}{\pi W\left(1 + K_R\right)}} + \alpha_s \ , \tag{5}$$

where $\lambda\tau_1 = \tau_{s0} / E_v$.

3.2. For the elastic line contact

When $W_{ec} < W < W_{pc}$, the contact is in the elastoplastic deformation. Here W_{pc} is the dimensionless critical load for the contact fully plastic deformation, $W_{pc} = \lambda_w W_{ec}$, and $\lambda_w = 17\left(1 + K_R^2\right) / \left(1 + K_R\right)$ [10]. In this case, it was interpolated that [10]:

$$\frac{b_{ep}}{R} = K_{ep} = \frac{K_{ec}\left(\lambda_b - 1\right)}{\lambda_w - 1}\left(\frac{W}{W_{ec}} - \frac{\lambda_b - \lambda_w}{\lambda_b - 1}\right),$$

$$\text{for} \quad W_{ec} < W < W_{pc} \ , \tag{6}$$

where b_{ep} is the half contact width of elastoplasticity, and $\lambda_b = 8.5\sqrt{1 + K_R^2}$.

The friction force per unit contact length in the contact is:

$$\begin{aligned} F_f &= \int_{-b_{ep}}^{b_{ep}} \tau_s dx = \int_{-b_{ep}}^{b_{ep}} \left[\tau_{s0} + \alpha_s p\left(x\right)\right] dx = \\ &= 2b_{ep}\tau_{s0} + \alpha_s w. \end{aligned} \tag{7}$$

The friction coefficient of the contact is:

$$f = \frac{F_f}{w} = \frac{2\lambda_{\tau 1} K_{ep}}{W} + \alpha_s \ . \tag{8}$$

3.3. For the fully plastic line contact

When $W \ge W_{pc}$, the contact is in fully plastic deformation. In this case, it is obtained that:

$$\frac{b_p}{R} = \frac{\lambda_{\tau 2} W}{2\lambda_{\tau 1}} \quad \text{for} \quad W \ge W_{pc} \ , \tag{9}$$

where b_p is the half contact width of full plasticity, and $\lambda\tau_2 = \tau_{s0} / \Omega_s$.

The friction force per unit contact length in the contact is:

$$\begin{aligned} F_f &= \int_{-b_p}^{b_p} \tau_s dx = \int_{-b_p}^{b_p} \left[\tau_{s0} + \alpha_s p\left(x\right)\right] dx = \\ &= 2b_p\tau_{s0} + \alpha_s w. \end{aligned} \tag{10}$$

The friction coefficient of the contact is:

$$f = \frac{F_f}{w} = \lambda_{\tau 2} + \alpha_s \ . \tag{11}$$

4. Calculation Results

The calculation has been made for the simple sliding steel line contact in physically adsorbed layer boundary lubrication for different values of the surface hardness Ω_s with the input parameter values $E_v = 209$ GPa, $\tau_{s0} = 20$ MPa and $\alpha_s = 0.04$.

Fig. 2 shows the dependences of W_{ec} on K_R for different surface hardness Ω_s The value of W_{ec} is rapidly reduced with the increase of K_R for the practical K_R range ($K_R \le 300$). Note that a higher value of K_R indicates a more pronounced contact thermoelastic deformation and thus a more significant effect of the contact thermal deformation [10]. Fig. 2 shows that the contact thermal deformation strongly influences the initiation of the plastic deformation in the contact. For $K_R \ge 100$ and $\Omega_s \le 0.8$ GPa, the dimensionless load W less than 1.0E-6 can result in the contact elastoplastic deformation.

Figs. 3, a-c show the dependences of b / R on the dimensionless load W for different K_R values and also for the assumed elastic contact without the effect of the contact thermal deformation ($K_R = 0$). The curves in these figures show the loading performances of the contact under different K_R values i.e. for different contact thermal deformations. As $K_R = 0$ refers to no effect of the contact

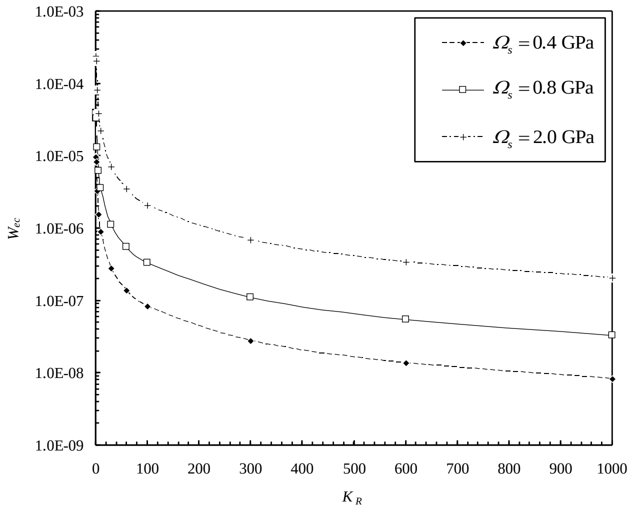
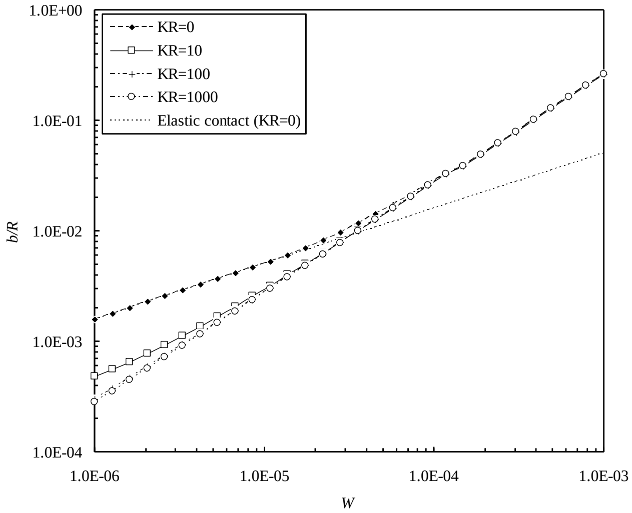


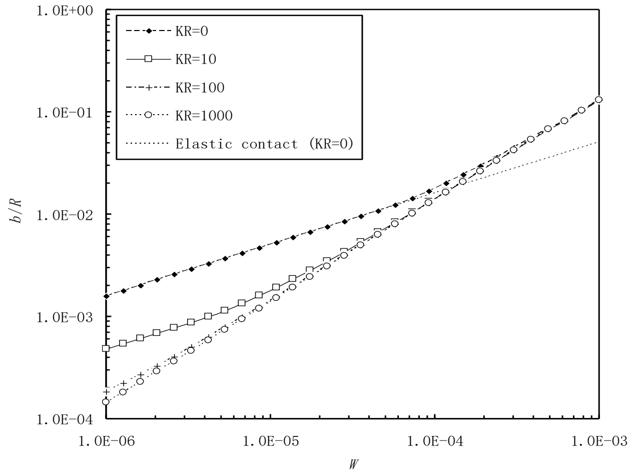
Fig. 2 The W_{ec} versus K_R curves

thermal deformation, it is found that with the increase of K_R (over zero) i.e. for a stronger effect of the contact thermal deformation, for the given load which yields the contact only or mainly in the elastic deformation, the ratio b / R i.e. the contact width is significantly reduced. However, for the load which results in the contact mainly or only in the plastic deformation, the value of K_R i.e. the contact thermal deformation has a negligible influence on the dependence of b / R on W, which follows the elastoplastic contact theory with $K_R = 0$. It also shows that if the effect of the contact thermal deformation is considered i.e. $K_R > 0$, the classical elastic contact theory (with $K_R = 0$) may often erroneously calculate the contact width (b) for a given load. The increase of the surface hardness Ω_s expands the load range in which the contact is mainly in elastic deformation and thus the influence of the contact thermal deformation on the contact width is pronounced. For $\Omega_s = 2.0$ GPa, this load range covers the practical dimensionless loads ($W \leq 1.0\text{E}–3$). In Fig. 3, c, the curve for the elastic contact assumption with $K_R = 0$ is overlaid with the curve for $K_R = 0$ based on the elastoplastic contact assumption. The overlaid curves for different K_R in Figs. 3, a-c are due to the mainly plastic or fully plastic deformations, in which the effect of the contact thermal deformation is negligible.

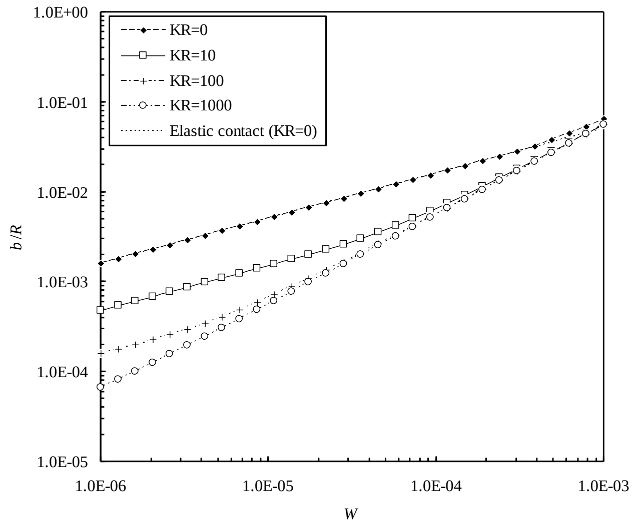
Figs. 4, a-c show the dependences of the contact friction coefficient f on the parameter K_R respectively for different surface hardness when the dimensionless load W is widely varied so that the contact is in the elastic, elastoplastic or fully plastic deformations. The overlaid curves for different K_R with nearly the constant friction coefficients in these figures are due to the mainly plastic or fully plastic deformations of the contact. It is shown that nearly or just in the fully plastic deformation regime the influence of the contact thermal deformation on the contact friction coefficient is negligible. However, when the contact is mainly or only in the elastic deformation, this influence is quite strong, and the increase of K_R significantly reduces the friction coefficient for a given load. This is due to the significant reduction of the contact width by the contact thermal deformation in the elastic deformation regime for a given load as shown in Figs. 3, a-c. In Fig. 4, c, the curve for the elastic contact assumption with $K_R = 0$ is overlaid with the curve for $K_R = 0$ based on the elastoplastic contact assumption. It is also shown in Figs. 4, a-c that the



a



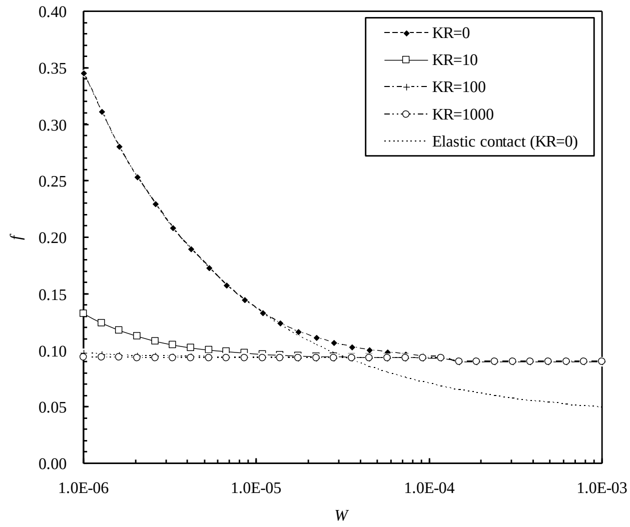
b



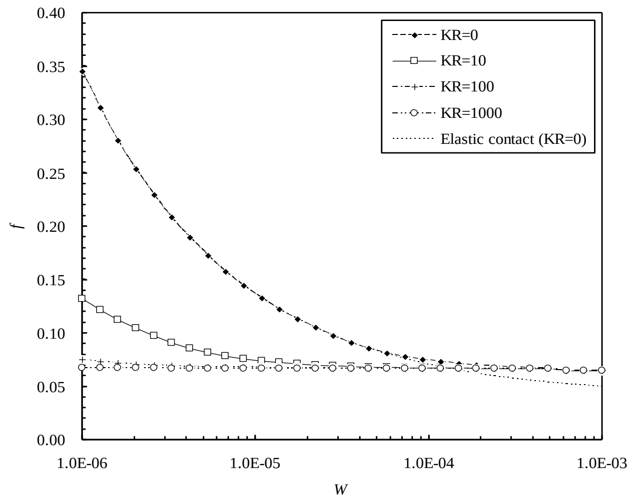
c

Fig. 3 The b/R versus W curves for different K_R values: a – Ω_s= 0.4 GPa, b – Ω_s = 0.8 GPa, c – Ω_s = 2.0 GPa

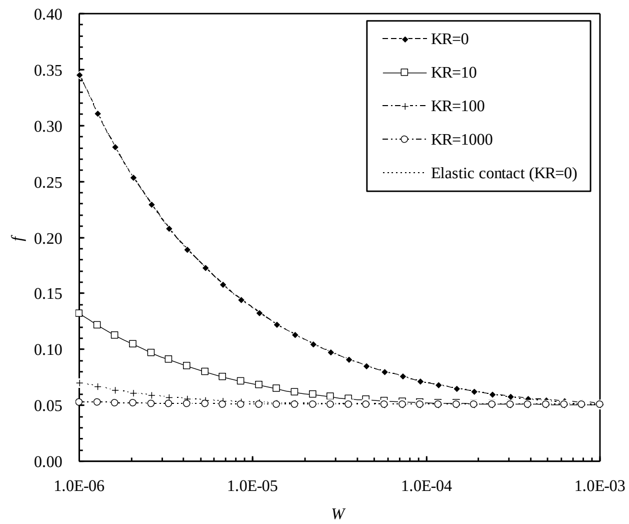
classical elastic contact theory (with $K_R = 0$ for ignoring the contact thermal deformation effect) should erroneously calculate the contact friction coefficient if the contact undergoes heavy loads or/and high sliding speeds so that the values of K_R are significantly large.



a



b



c

Fig. 4 The f versus W curves for different K_R values: a – Ω_s= 0.4 GPa, b – Ω_s = 0.8 GPa, c – Ω_s = 2.0 GPa

5. Conclusions

The influences of the contact thermal deformation on the contact width and friction coefficient are studied in the sliding elastoplastic line contact which is in physically adsorbed layer boundary lubrication, chemical layer boundary lubrication or fresh metal-to-metal contact. The contact is ideally smooth and it can also be considered as an asperity contact. The theoretical analysis is presented respectively for the contact elastic, elastoplastic or fully plastic deformations. The calculation was made for the sliding steel line contacts with different surface hardness which are in physically adsorbed layer boundary lubrication.

According to the calculation results, the conclusions are drawn as follows:

a. The contact thermal deformation significantly reduces the critical load for the contact elastic deformation; Owing to the strong effect of the contact thermal deformation, the dimensionless load W less than 1.0E-6 can practically result in the contact elastoplastic deformation.
b. When the contact is only or mainly in the elastic deformation, both the contact width and the contact friction coefficient are significantly reduced by the effect of the contact thermal deformation; However, when the contact is mainly or only in the plastic deformation, the contact thermal deformation has negligible influences on both of them.
c. The classical elastic contact theory (with K_R =0) should erroneously calculate both the contact width and the friction coefficient when the contact undergoes heavy loads or/and high sliding speeds so that the contact thermal deformation is significant.

The present obtained results and conclusions are of significant interest to the modeling and study of the gear contact in mechanical engineering which undergoes high slide-roll ratios, heavy loads and high sliding speeds.

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LOADING AND FRICTION OF ELASTOPLASTIC LINE CONTACT WITH THERMAL DEFORMATION

S u m m a r y

The loading and friction of the elastoplastic line contact in boundary lubrication is studied by accounting for the contact thermal deformation effect. When the contact is mainly in the elastic deformation, the contact thermal deformation has significant influences on both the contact width and contact friction coefficient for a given load, and it reduces their values; However when the contact is mainly in the plastic deformation, the effect of the contact thermal deformation is weak or even negligible. Owing to the contact thermal deformation, the critical load for the initiation of the contact plastic deformation is also reduced.

Keywords: boundary lubrication, elastoplastic deformation, friction, line contact, load, thermal deformation.

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