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# The estimation of time-varying thermal contact conductance between two fixed contacting surfaces

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**Nomenclature**

$c_p$ – specific heat, J/kgK; $e_{RMS}$ – root mean square error, K; $h_c$ – thermal contact conductance, W/m$^2$K; $H$ – Hilbert space of square-integrable functions; $I$ – maximum number of parameters; $k$ – thermal conductivity, W/mK; $L$ – length, m; $N$ – number of sensors; $q$ – heat flux, W/m$^2$; $S$ – objective function, K$^2$; $t$ – time, s; $t_f$ – final time, s; $T$ – temperature, K; $T_0$ – constant temperatures at specimens' end, K; $T_i$ – initial temperature, K; $Y$ – measured temperatures, K; $x$ – Cartesian spatial coordinate.
*Greek symbols*
$\alpha$ – thermal diffusivity, m$^2$/s; $\delta$ – distance from the interface, m; $\Delta$ – distance between two sensors, m; $\Delta T$ – temperature drop, K; $\rho$ – density, kg/m$^3$.
*Subscripts*
$i$ – $i$-th parameter; $j$ – $j$-th sensor; $k$ – $k$-th sensor.

**1. Introduction**

When two surfaces are in contact, the actual area of contact is much smaller than the apparent area of contact. These areas of actual contact occur where the asperities of one surface are in contact with the asperities of the other surface. Typically, there is some material or fluid in the interstitial spaces between the contacting surfaces, and heat is transferred through this interstitial material. If there is no interstitial material or fluid, then most of the heat transferred across the interface formed by the two surfaces is transferred through these small contact spots. The amount of actual contact area is also dependent on the physical properties of the contacting materials. Heat is transferred across the interfaces through some combination of three paths: conduction through contacting spots, conduction across the interstitial space through interstitial material, and radiation across the interstitial spaces. Heat transfer between surfaces of imperfect thermal contact occurs in numerous practical applications such as in thermal supervision of space vehicles and estimation of thermal contact conductance between the exhaust valve and its seat in an internal combustion engine.

Some of the techniques recently developed for estimating the thermal contact conductance are based on experimental temperature data at one or several interior positions of the contacting solids. Theoretical and experimental efforts have been continued in prediction of this prior parameter. The estimation of thermal contact conductance has been carried out while heat transfer was either in a steady-state [1] or transient condition [2, 3]. Experiments were conducted using setups that contained two specimens [2, 3]. A few numbers of these researches are related to estimation of periodic thermal contact conductance [4-10]. Components in many rotary devices and in automated processes transfer heat periodically across contact surfaces. Example includes heat transfer between a soldering iron and work piece on an assembly line. The others usually include constant contacting surfaces [11], especially in heat rejection from electronic components.

The thermal contact conductance can neither be measured nor calculated directly, so an accurate and efficient method for computing temperature distribution becomes quite important. It can be estimated from surface temperature and heat flux values computed using the linear [4] or quadratic [7-8] extrapolation method. This method utilizes multiple spatial temperature readings taken very close to each contacting surfaces. The technique of inverse heat conduction problem can be applied to solve this kind of problem. Finite difference method (FDM) and finite element method (FEM) are also widely used [12].

In this work, a linear polynomial is fitted to the spatial data and the value and slope of the extrapolated polynomial at the surface is used to estimate the time-wise variation of the thermal contact conductance between two one-dimensional solids with fixed contact. Furthermore, an inverse solution using the Conjugate Gradient Method (CGM) with adjoint problem for function estimation is presented. It is a very important class of methods of estimating functions. It is powerful and straight forward. It utilizes the ideas based on perturbation principles [13-15] to transform the inverse problem to solution of three simple problems [13]. Finally, a steady-state analysis has been successfully carried out by employing the commercial software based essentially on the FEM approach.

**2. The problem definition**

The geometry and the coordinates for the one-dimensional physical problem considered here is shown in Fig. 1. Two specimens, referred to as specimens 1 and 2, are contacting with a contact conductance $h_c(t)$ at the interface. The non-contacting ends are kept at constant, but different temperatures $T_{01}$ and $T_{02}$. It is assumed that the system does not reach the steady-state condition for the temperature distribution in the specimens, that is, the temperature distribution in the specimens is time-varying.

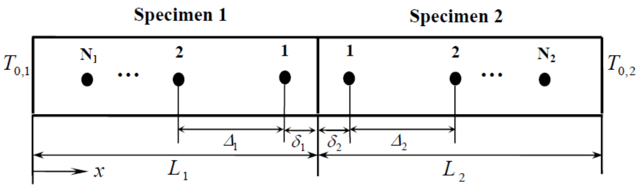


Fig. 1 Contacting specimens

The mathematical formulation of the direct heat conduction problem is given in as:

Specimen 1 ($0 \le x \le L_1$):

$$\frac{\partial^2 T_1}{\partial x^2} = \frac{1}{\alpha_1}\frac{\partial T_1}{\partial t} \quad 0 < x < L_1, \text{ for } t > 0; \tag{1, a}$$

$$T_1(0,t) = T_{0,1} \quad \text{for } t > 0; \tag{1, b}$$

$$k_1 \left.\frac{\partial T_1}{\partial x}\right|_{x=L_1} = h_c(t)\left[T_2(L_1,t) - T_1(L_1,t)\right] \text{ for } t > 0; \tag{1, c}$$

$$T_1(x,0) = T_i \quad 0 < x < 1. \tag{1, d}$$

Specimen 2 ($L_1 \le x \le L_2$):

$$\frac{\partial^2 T_2}{\partial x^2} = \frac{1}{\alpha_2}\frac{\partial T_2}{\partial t} \quad L_1 < x < L_2, \text{ for } t > 0; \tag{2, a}$$

$$k_2 \left.\frac{\partial T_2}{\partial x}\right|_{x=L_1} = h_c(t)\left[T_2(L_1,t) - T_1(L_1,t)\right] \text{ for } t > 0; \tag{2, b}$$

$$T_2(L_1,t) = T_{0,2} \quad \text{for } t > 0; \tag{2, c}$$

$$T_2(x,0) = T_i \quad L_1 < x < L_2. \tag{2, d}$$

## 2. The experiment

We consider an example using actual data taken from a thermal contact conductance experiment conducted by Moses [16].

The experimental apparatus, shown in Fig. 2, consists of two test specimens – each held at one end in a thermal reservoir – the supporting frame, and the associated equipment required to bring the test specimens uniformly into and out of contact. The specimens are in contact to reach steady-state condition.

The test specimens are caused to contact and separate by driving the lower plate with a pneumatic cylinder. The fluid reservoirs are constant-temperature baths. Each reservoir circulates fluid supplied by an external bath, which is maintained at a specified temperature. The lateral surface of the test specimens is insulated with a Teflon sleeve.

The experiment was run with mild steel test specimens having a length of 0.1524 m and a diameter of 0.0254 m. In each specimen, 5 copper-constantan thermocouples are located in centerline drillings and in corresponding surface indentations at distances of 0.0064, 0.0127, 0.0254, 0.0381 m and 0.0508 from the contact surface. Two additional thermocouples are located on the specimen centerline, at the point where the specimen enters the thermal reservoir in order to measure the temperature of circulating fluid there.

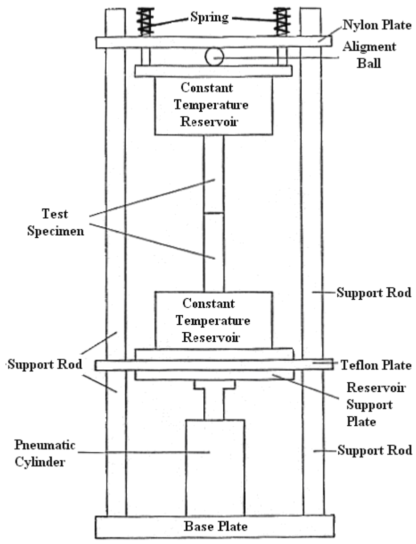


Fig. 2 Experimental setup [16]

A separate set of experiments was conducted to check for the presence of radial temperature gradients in the test specimens. These results, which are reported by Moses, indicated that, within the accuracy of the recording device, the heat flow down the rod was one-dimensional. Table 1 lists the characteristics of specimens [16].

Table 1

Characteristic of the specimens [16]

| Characteristic | Specimen 1 | Specimen 2 |
|---|---|---|
| Material | Mild steel (C = 1.5%) | Mild steel (C = 1.5%) |
| Thermal Conductivity, $k$, W/mK | 36 | 36 |
| Specific Heat, $c_p$, kJ/kgK | 0.486 | 0.486 |
| Density, $\rho$, kg/m$^3$ | 7753 | 7753 |
| Reservoir Temperature, $T_0$, °C | 52.9 | 18.7 |

## 3. The extrapolation method

The thermal contact conductance was computed by means of the following expression,

$$h_c = q/\Delta T, \tag{3}$$

where $q$ is the average of the heat fluxes of the two contacting specimens and $\Delta T$ is the temperature drop at the interface, which is computed by extrapolating the temperature profiles of each contacting specimen to the interface.

## 4. The inverse solution

For the inverse problem, the interface thermal contact conductance, $h_c(t)$ is regarded as unknown, but everything else in the system of Eqs. (1-2) is known and temperature readings taken at some appropriate locations within the medium are available, at times $t_i$, $i = 1, 2, \dots, I$. Let the temperature recordings taken with sensors to be denoted by $Y_{1j}(t) \equiv Y_{1j}$ and $Y_{2k}(t) \equiv Y_{2k}$ for specimens 1 and 2, respectively.

It is assumed that no prior information is available on the functional form of $h_c(t)$. We are after the function $h_c(t)$ over the whole time domain $(0, t_f)$, with the assumption that $h_c(t)$ belongs to the Hilbert space of square-integrable functions in the time domain [13] – denoted as $H(0,t_f)$ – in this domain, i.e., $\int_{t=0}^{t_f} \left[h_c(t)\right]^2 dt < \infty$.

The solution of the present inverse problem is to be obtained in such a way that the following functional is minimized:

$$S\left[h_c(t)\right] = \\ = \int_{t=0}^{t_f} \left[ \sum_{j=1}^{N_1} \left(T_{1j} - Y_{1j}\right)^2 \right] dt + \int_{t=0}^{t_f} \left[ \sum_{k=1}^{N_2} \left(T_{2k} - Y_{2k}\right)^2 \right] dt \,, \quad (4)$$

where $T_{1j}(t) \equiv T_{1j}$ and $T_{2k}(t) \equiv T_{2k}$ are the estimated temperatures at the measurement locations in specimens 1 and 2, respectively.

In this work, the CGM with adjoint problem for function estimation is used to solve the current inverse problem. The advantage of the present method is that no a priori information is needed on the variation of the unknown quantities, since the solution automatically determines the functional form over the domain specified. In this method, additional equations beyond the governing equation must be solved. It transforms the inverse problem to solution of three simple problems called the direct problem, the sensitivity problem and the adjoint problem together with the gradient equation. The set of three equations is iteratively solved using the method conjugate gradients for the corrections at the end of every iteration. For derivation of sensitivity and adjoint problem from the direct problem as well as the definition of iterative procedure and the computational algorithm of the method, the readers should consult references [13, 17].

## 5. The numerical analysis

The commercial program COMSOL Multiphysics 4.2.0.150 has been used to create solid and FEM of the specimens. Following steps are comprised of creating the geometry and the solid model of the specimens, defining the mesh and boundary conditions, selection of the material properties and finally, steady-state solution procedure.

The model of the specimens is one-dimensional axisymmetric. The solid model is created according to the geometry of the specimens defined before.

Fine physics controlled mesh grids are used in regions. During the creation of the solution domain, element amount are selected depending on the expectancy of the result and consists of 21 nodes. The process of mesh refinement is repeated until further mesh refinements have insignificant effects on the results.

The thermo-physical properties of the specimens shown in Table 1 are input to the analysis.

Both ends of the specimens are at constant temperature. In COMSOL Multiphysics the thermal contact resistance can be modelled by applying the "thin thermally resistive layer" boundary condition. This is a so-called slit boundary condition that allows for a discontinuity in the temperature field across the boundary. The correct thermal contact conductance is specified by applying the value obtained from the inverse solution of the problem. Slit boundary conditions are only available on assembly pair boundaries, which requires us to set up an assembly geometry to model contact resistance.

The computer program has been extensively and successfully validated by comparing the calculated temperature field with available experimental and analytical data for steady-state case.

## 6. Results and discussion

Temperature distribution at sensor locations in each specimen is illustrated in Fig. 3 in the scatter form. Note that the sensors are numbered in $x$ direction, from hottest to coldest.

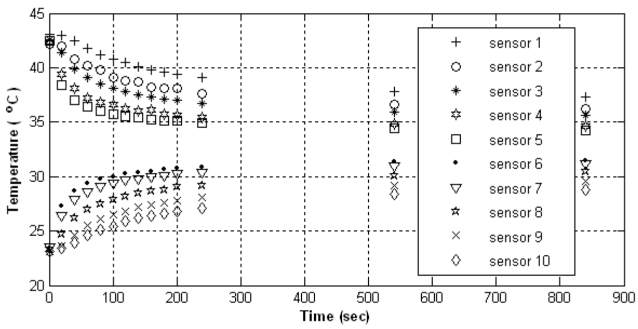


Fig. 3 Temperature distribution at sensor locations in each specimen in the scatter form

6.1. The thermal contact conductance estimation

Fig. 4 shows the estimated thermal contact conductance determined by using the present inverse method and the linear extrapolation method, both based on the experimental data supplied by Moses.

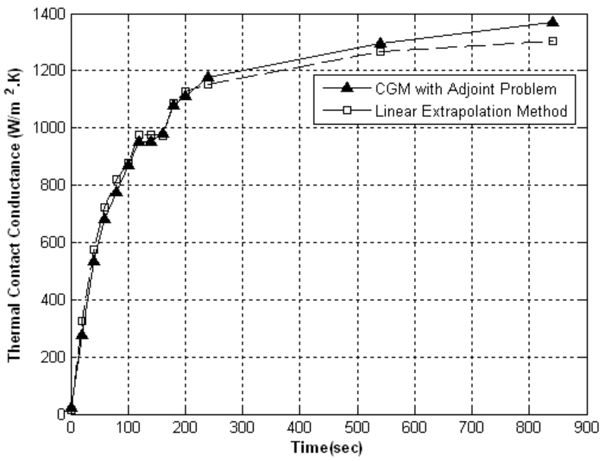


Fig. 4 Thermal contact conductance based on linear extrapolation method and CGM

Since two sensors are extremely close to the contact surface in each specimen, there is little extrapolation from the fitted curve to determine the interface temperature. So the extrapolation result appears to show damping and lagging similar to the result obtained from CGM.

The overall uncertainty analysis in the experimental results for the thermal contact conductance, based on the method of Kline and McClintock [18] indicates an uncertainty of 15.23% for the steel specimens.

As discussed before, the inverse problems are solved by minimizing an objective function with some stabilization technique used in the estimation procedure. The objective function that provides minimum variance estimates is the ordinary least squares norm i.e., the sum of the squared residuals. The objective function is a good indicator for the accuracy of the estimation. The variations of the objective function with time are illustrated in Fig. 5. Surely, the lesser the value of the objective function the more accurate estimation has been done.

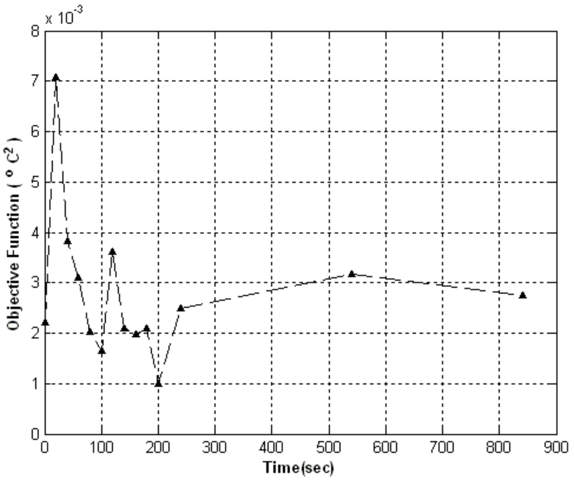


Fig. 5 The variations of the objective function with time

6.2. The steady-state analysis

The various methods used in this study can be validated by comparing the calculated temperature field with available experimental data under the s teady-state condition. The temperature distribution in $x$ direction is shown in Fig. 6. The results are obtained under steady-state condition and from using different techniques such as experimental, linear extrapolation, inverse and numerical methods. Note that the thermal contact conductance is an input to the program in the numerical analysis and must be defined by the user. The estimated thermal contact conductance obtained from the inverse solution of the problem is utilized for calculating the temperature field in the numerical analysis. The numerical results are acceptable as they cover the experimental temperatures.

We define the Root Mean Square (RMS) error as:

$$e_{RMS} = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left[T_{est}\left(t_i\right)-T_{exp}\left(t_i\right)\right]^2}\ , \tag{5}$$

where the subscripts est and exp refer to the estimated and experimental values, respectively. $N$ is the total number of the sensors which is equal to 10.

Table 2 shows the values of $e_{RMS}$. The RMS error in the estimation of the temperature at the sensor lactations is larger when the numerical method is applied. The error in the linear extrapolation results is rather negligible in comparison with those associated with the other methods as the experimental temperatures are used directly and the spatial measurements are extremely close to the contact surface. It is obviously clear in Fig. 6 as well.

Table 2

RMS error obtained by using different methods

| Methods | $e_{RMS}$, °C |
|---|---|
| Linear extrapolation method | 0.0706 |
| Inverse method (CGM) | 1.1549 |
| Numerical method | 2.9885 |

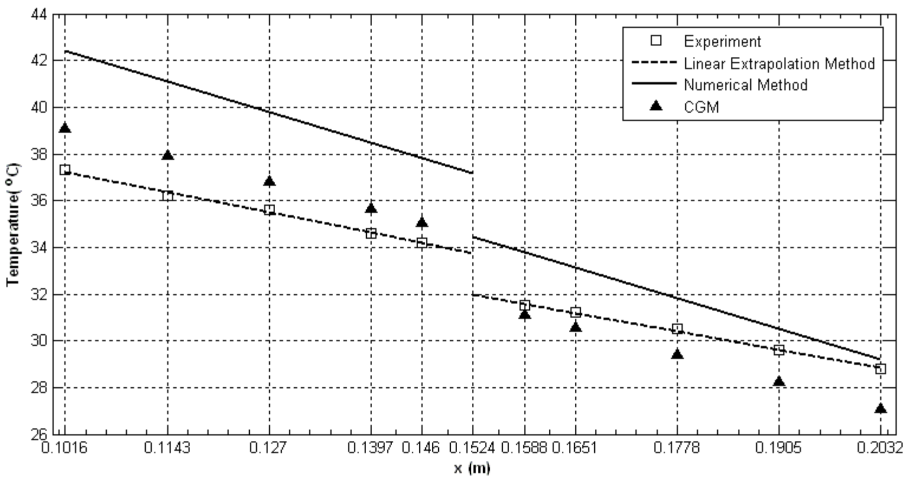


Fig. 6 The steady-state temperature distribution in axial direction

**7. Conclusions**

In this study, an inverse method in addition to the linear extrapolation method was applied for estimating the time varying thermal contact conductance. The results obtained by using data taken from an actual contact conductance experiment. The thermal contact conductance obtained from the inverse solution was validated numerically by using COMSOL Multiphysics under steady-state condition.

Generally, the results obtained with CGM and the linear extrapolation results were in good agreement. An error analysis showed that the overall uncertainty in the estimation of the thermal contact conductance by extrapolation method is considered to be 15.23%.

The steady-state analysis and its error investigation showed that the extrapolation approach is superior to the other methods used. The extrapolation method can be effective, particularly when the temperature is measured at points extremely close to the contact interface. It should be noted that the inverse solution requires experimental data;

this incorporation of the experimental temperature into the computation method prevents the inverse solution from providing a truly independent determination of the result.

**References**


1. **Jain, V.K.** 1990. Determination of heat transfer coefficient for forging applications, J Mater Shap Technol 8: 193-202. http://dx.doi.org/10.1007/BF02833814.
2. **Semiatin, S.L.; Collings, E.W.; Wood, V.E.; Altan, T.** 1987. Determination of the interface heat transfer coefficient for non-isothermal bulk forming processes, J Eng Ind 109: 49-57. http://dx.doi.org/10.1115/1.3187094.
3. **Malinowski, Z.; Lenard, J.G.; Davies, M.E.** 1994. A study of the heat-transfer coefficient as a function of temperature and pressure, J Mater Process Technol 41: 125-142. http://dx.doi.org/10.1016/0924-0136(94)90057-4.
4. **Howard, J.R.** 1976. An experimental study of heat transfer through periodically contacting surfaces, Int J Heat Mass Tranf 19: 367-372. http://dx.doi.org/10.1016/0017-9310(76)90092-2.
5. **Flach, G.P.; Ozisik, M.N.** 1988. Inverse heat conduction problem of periodically contacting surfaces, J Heat Tranf 110: 821-829. http://dx.doi.org/10.1115/1.3250580.
6. **Vick, B.; Ozisik, M.N.** 1981. Quasi-steady-state temperature distribution in periodically contacting finite regions, J Heat Tranf 103: 739-744. http://dx.doi.org/10.1115/1.3244535.
7. **Moses, W.M.; Johnson, R.R.** 1987. Experimental results for the quasi-steady heat transfer through periodically contacting surfaces, In: AIAA, Thermophysics Conference, 22nd, Honolulu, HI, June 8-10, paper No.87, 1603-1608.
8. **Moses, W.M.; Johnson, R.R.** 1988. Experimental study of the transient heat transfer across periodically contacting surfaces, J Thermophys Heat Tranf 2: 37-42. http://dx.doi.org/ 10.2514/3.59.
9. **Moses, W.M.; Dodd, N.C.; Dzenis, A.; Joshi, S.P.** 1990. Heat transfer across aluminium/steel surfaces in periodic contact, J Thermophys Heat Tranf 4: 396-399. http://dx.doi.org/ 10.2514/3.194.
10. **Goudarzi, K.; Shojaeefard, M.H.** 2009. Experimental study of the heat transfer across periodically contacting surfaces, J Mech 25: 307-311. http://dx.doi.org/ 10.1017/S1727719100002768.
11. **Ayers, G.H.** 2003. Cylindrical thermal contact conductance, MS. Thesis, Texas A&M University, Mechanical Engineering.
12. **Tsai, Y.M.; Crane, R.A.** 1992. An analytical solution of a one-dimensional thermal contact conductance problem with one heat flux and one insulated boundary condition, J Heat Tranf 114: 503-505. http://dx.doi.org/10.1115/1.2911301.
13. **Ozisik, M.N.; Orlande, H.R.B.** 2000. Inverse Heat Transfer: Fundamentals and Applications, Taylor & Francis, New York, 330 p.
14. **Alifanov, O.M.** 1994. Inverse Heat Transfer Problems, Springer-Verlag, Berlin, 348 p.
15. **Beck, J.V.; Blackwell, B.; Clair, C.R.St.** 1985. Inverse heat conduction: ill posed problems, Wiley, New York, 308 p.
16. **Moses, W.M.** 1985. An Experimental investigation of the heat transfer across periodically contacting surfaces, Ph. D. Dissertation, North Carolina State University, Raleigh, NC.
17. **Shojaeefard, M.H.; Goudarzi, K.; Mazidi, M.Sh.** 2009. Inverse heat transfer problem of thermal contact conductance estimation in periodically contacting surfaces, J Therm Sci 18: 150-159. http://dx.doi.org/10.1007/s11630-009-0150-1.
18. **Holman, J.P.** 2000. Experimental Methods for Engineers, Mc-Graw-Hill, London, 720 p.


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LAIKUI BĖGANT KINTANČIO DVIEJŲ KONTAKTUOJANČIŲ PAVIRŠIŲ ŠILUMINIO LAIDUMO NUSTATYMAS

R e z i u m ė

Siekiant nustatyti laikui bėgant kintančio terminio kontakto laidumą, tiriamas šilumos perdavimas tarp dviejų kontaktuojančių bandinių. Pasiūlytas atvirkščias šilumos perdavimo metodas. Rezultatams gauti remiantis eksperimento duomenimis taip pat taikomas gerai žinomas ekstrapoliacijos metodas. Atvirkštinio ir tiesioginio ekstrapoliavimo metodais gauti rezultatai gerai sutampa. Naudojant komercinę programinę baigtinių elementų metodo įrangą ištirta nusistovėjusi būsena, o paklaidų analizė parodė, kad ekstrapoliavimo metodas šiuo atveju yra geresnis.

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THE ESTIMATION OF TIME-VARYING THERMAL CONTACT CONDUCTANCE BETWEEN TWO FIXED CONTACTING SURFACES

S u m m a r y

This work examines the heat transfer between two fixed contacting specimens in order to estimate the time-varying thermal contact conductance. An inverse heat conduction method is presented. A well-known technique called extrapolation method is also utilized to obtain the results by using data taken from an experiment. The results achieved by the inverse and linear extrapolation methods are in good agreement. In addition, a steady-state analysis has been performed by using the commercial software based on the finite element method and an error analysis shows that the extrapolation method is superior in this analysis.

**Keywords:** thermal contact conductance, extrapolation method, inverse problem, COMSOL Multiphysics.

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