Determination of the digitally filed geodesics on a cross-section of an ellipsoidal mandrel

F.E. Betene*, P. Kah**, J.A. Atangana***, J. Martikainen****, A. Ohandja*****, B. Mvola******

*University of Douala, ENSET, B.P. 2701 Douala, Cameroon, E-mail: fabetene@yahoo.fr
**Lappeenranta University of Technology, 53850 Lappeenranta, Finland, E-mail: paul.kah@lut.fi
***University of Douala, ENSET, B.P. 2701 Douala, Cameroon, E-mail: aajean2003@yahoo.fr
****Lappeenranta University of Technology, 53850 Lappeenranta, Finland, E-mail: jukka.martikainen@lut.fi
*****University of Douala, IUT, B.P. 2701 Douala, Cameroon, E-mail: oayina@yahoo.fr
******Lappeenranta University of Technology, 53850 Lappeenranta, Finland, E-mail: Eric.Mvola.Belinga@lut.fi

1. Introduction

Filament winding is a method of forming composite structures using automated machines. A mandrel is pre-manufactured matching the geometry of the structure to be produced and mounted between the jaws of a coil comprising a control system rotation of the mandrel and a control system of the guide wire. The wire that wraps around the mandrel, generally along geodesic paths, goes across the guide wire or the eye's guide. Timing of the eye guide's movements and rotation of the mandrel permit to define the paths of the filament [1, 2].The following steps are required for the winding:

Step 1: mathematical analysis of the structure to be achieved;

Step 2: programming of the equations obtained for the simulation of the winding on the computer screen and the preparation of the control parameters of the winder;

Step 3: development of the control program of the winder by a digital control using the appropriate command parameters;

Step 4: transfer of the control program to the CNC (Computer Numerical Control);

Step 5: control of the winder by the CNC for the winding.

This work, which falls within Step 2 above, sets out to determine the control parameters of the winder and store them in a database to develop the control program of the winder for a defined CNC. The parameters to be determined numerically are rotation of the chuck and translation of the eye guide. Synchronization of these two movements allows a removal of the tape on the mandrel according to geodesic lines. To prevent slippage of the tape, which can cause wrinkling around the eye guide, a rotation of the eye guide around the axis of symmetry of its light is needed and its numerical values are to be determined. Various numerical values, block by block determined from angular increments in the rotation of the mandrel, are stored in a file for controlling the coil movement.

The winding under consideration may be wet or dry: it is called wet when the tape is impregnated with a resin solution and then wound onto the mandrel, and dry when the tape is pre-impregnated with resin (impregnated already at the time of manufacture).

Such a study has been conducted regarding the structures of revolution [3, 4]; the prototype machine was controlled by CNC CYBER 3000 works at the Institute of Automotive and Transport (ISAT) of Nevers in France.

2. Mathematical analysis

The mathematical analysis is performed in two main stages.

2.1. Equations

Setting of the problem in Fig. 1 allows us to define the equations necessary to control the machine for winding an elliptical cross-section structure [5]. The parameters to be monitored are the rotation of the mandrel on its axis, the translation of the eye guide along an axis parallel to the mandrel and rotation of the eye guide around an axis perpendicular to the chuck, to prevent wrinkling of the ribbon.

Cross-section of the structure is elliptical relative to the axes (o, x). If b denotes semi-minor axis of the ellipse and c semi-major axis of the ellipse, then the coordinates of point M will be defined in the following Eq. (1):

$$M \begin{vmatrix} x \\ U \sin \beta_0 \\ U \cos \beta_0 \end{vmatrix} ,$$

where

$$U = \frac{bc}{\sqrt{c^2 \sin^2 \beta_0 + b^2 \cos^2 \beta_0}} . \tag{1}$$

From the relation of Clairaut [6] which defines the equation of the geodesic on the surface of revolution of radius R, if ψ is the angle of removal and $R \sin \psi$ = constant, we obtain the following expression of the geodesic on the ellipsoidal surface:

$$tg\psi = \frac{b \sin \psi_{MAX}}{\sqrt{U^2 - b^2 \sin^2 \psi_{MAX}}} . \tag{2}$$

Furthermore, the relation defining the geodesic on a surface according to the ellipsoidal rotation of the chuck elementary $d\beta_0$ around its axes (o, x) and axial translation
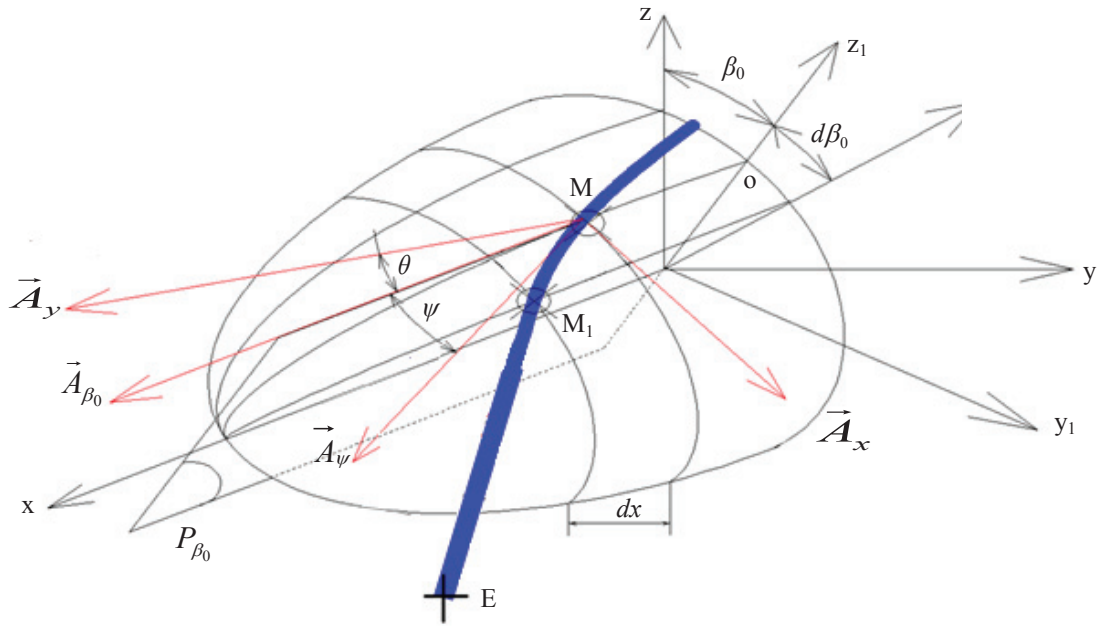
Fig. 1 Setting the structure, where Ψ is the angle of the geodesic, β_0 the angle of rotation of the mandrel (or amplitude of rotation of M around the x axis), θ the angle between $\vec{A}_{\beta_0}$ and $\vec{A}_y$, $\vec{A}_\psi$ is the unit vector tangent to the geodesic in M, $\vec{A}_{\beta_0}$ is the unit vector tangent to the meridian, $\vec{A}_x$ the unit vector tangent to the parallel M, $\vec{A}_y$ the unit vector normal $\vec{A}_x$ to M, dx the displacement along x to an angular increment of $d\beta_0$, P_{β_0} the plan files (containing the point M and the axes [o, x]), M_1 is the point of removing the tape on the mandrel, and E the position of the eye guide

dx elementary point of removal is given in Fig. 1:

$$tg\psi = \frac{1}{UH}\frac{d\beta_0}{dx} \atop H = \sqrt{\frac{cos^2\,\beta_0}{c^4}+\frac{sin^2\,\beta_0}{b^4}}\Bigg\} . \tag{3}$$

When we eliminate the current angle ψ by combining Eqs. (2) and (3), we get the final expression of the geodesic on a reference ellipsoid:

$$x_2 - x_1 = \frac{(\beta_2-\beta_1)b\,c}{2\,sin\,\psi_{Max}}\sum_{i=1}^{i=16}D_i\frac{\sqrt{c^2-sin^2\,\psi_{Max}\left(b^2\,cos^2\,\beta_{0i}+c^2\,sin^2\,\beta_{0i}\right)}}{c^4\,sin^2\,\beta_{0i}+b^4\,cos^2\,\beta_{0i}}, \tag{5}$$

where $\beta_{0i} = \frac{\beta_2+\beta_1}{2}+\frac{\beta_2-\beta_1}{2}l_i$, l_i is the roots of the Legendre polynomial of degree 16, D_i is the coefficients of the quadrature formula, β_2 is the variable and $\beta_1 = \beta_2 - d\beta_0$, $d\beta_0$ is to be chosen, values adopted π/n, $n = 12, 90$ or 180.

This equation explains axial position of point M files based on the incremental rotation of the mandrel. We deduce the axial position of the eye E guidance, given the spatial configuration of the whole (Fig. 2):

$$X_E = -\frac{1}{tan\psi}\frac{\left(U\,sin\,\theta_d - y_{E\,0}\right)H}{\left[\dfrac{cos\,\theta_d}{U^2}+U_0\,sin\,\theta_d\right]}+x, \tag{6}$$

$$\frac{dx}{d\beta_0} = \frac{\sqrt{U^2 - b^2\,sin^2\,\psi_{max}}}{UHb\,sin\,\psi_{max}}. \tag{4}$$

Integration of Eq. (4) by a numerical method with a Gauss Legendre polynomial of degree 16 [7], leads us to the following expression, which defines the equation of the geodesic on the surface of a mandrel cross-section elliptical constant:

with $U_0 = sin\,\beta_0\,cos\,\beta_0\left(\dfrac{1}{c^2}-\dfrac{1}{b^2}\right)$.

Rotation of the eye Φ_E guide E around its axis is calculated by the following expression:

$$\Phi_E = tan\,\phi = \frac{H}{tan(\psi)\left(-\dfrac{sin\,\theta_d}{U^2}+U_0\,cos\,\theta_d\right)}. \tag{7}$$

2.2. Incremental procedure

The winding operation is characterized by an incremental rotation of $d\beta_0$ the point of removal, and the resulting absolute angle is the angle of rotation of the mandrel β_0. While the fibre is wound along a sine wave around
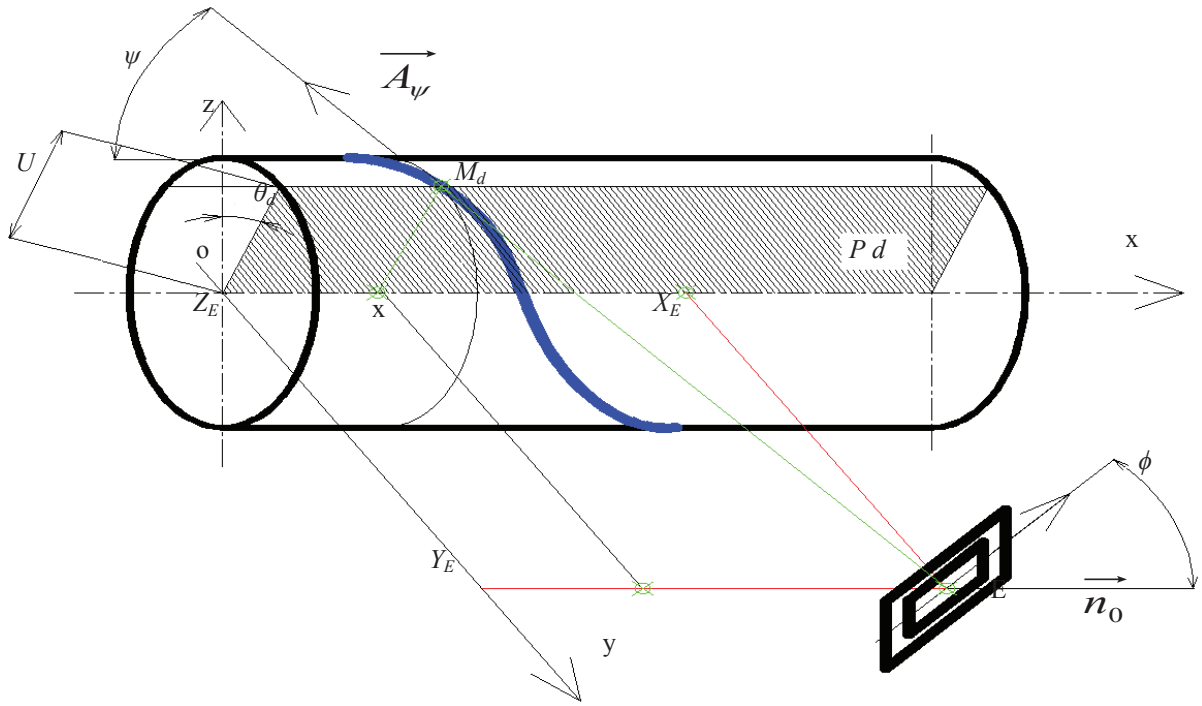
Fig. 2 Configuration of the guide of the eye, the mandrel and the geodesic in space

the mandrel, the abscise x of the filed point varies in an incremental step of Δx defined by geodesic Eq. (5). At the same time the guide of the eye performs a translational motion X_E so that the trajectory is parallel to the axes (o, x), the elementary displacement ΔX_E is proportional to the guidance of the eye that controls movement, and the coordinates Y_E and Z_E remain unchanged during the movement, including $Z_E = 0$. The eye also performs a rotational movement Φ_E around the parallel axes (o, y) to avoid fibre bandage creasing.

The control of the CNC coil is reduced to one of these elements; they therefore need to be known precisely for a better coil. Their knowledge is possible through the following expressions:

- ΔX_E, the change in the eye guide position is calculated by the program as: $\Delta X_E = X_{E\,(new)} - X_{E\,(previous)}$;
- Φ_E is calculated from Eq. (7);
- X_E is calculated by the program from Eq. (6);
- x is calculated by the system, so that $x_{(new)} = x_{(previous)} \pm \Delta x$;
- $d\beta_0$ is to be chosen: we chose the values π/n, $n = 12$, 90 or 180;
- β_0 that results, will be calculated so that $\beta_{0(new)} = \beta_{0(previous)} + d\beta_0$;
- Δx is calculated from Eq. (5) defined above.

These data will be stored in a matrix file format with n rows and seven columns as shown in the Table below.

Table

Desired presentation of the winding parameters in the result file

j	β_i	Δx	x_i	X_E	ΔX_E	Φ_E
1	$\beta_1 = 0 + \dfrac{\pi}{n}$	Δx_1	$x_1 = x_0 + \Delta x_1$	X_{E1}	ΔX_{E1}	$Tan\,\Phi$
2	$\beta_2 = \dfrac{2\pi}{n}$	Δx_2	$x_2 = x_1 + \Delta x_2$	X_{E2}	ΔX_{E2}	$Tan\,\Phi$
…	…	…	…	…	…	
i	$\beta_i = \dfrac{i\pi}{n}$	Δx_i	$x_i = x_{i-1} + \Delta x_i$	X_{Ei}	ΔX_{Ei}	$Tan\,\Phi$
n	$\beta_n = \pi$	Δx_n	$x_n = x_{n-1} + \Delta x_n = L$	X_{En}	ΔX_{En1}	$Tan\,\Phi$

3. Application example

A cross-section of the winding chuck constant in the shape of an ellipse was formed with a semi-minor axis 40 mm, semi-major axis 60 mm, length 500 mm and angular increment of rotation of the mandrel $\pi/180$. The following shows the steps of the program and the results obtained.

3.1. Data entry

Fig. 3 shows the data entry and start of the winding process which correspond to the winding simulation shown in Fig. 4.

Determination of deposit geodesic parameters
Enter data (L, b and C in milimeter)

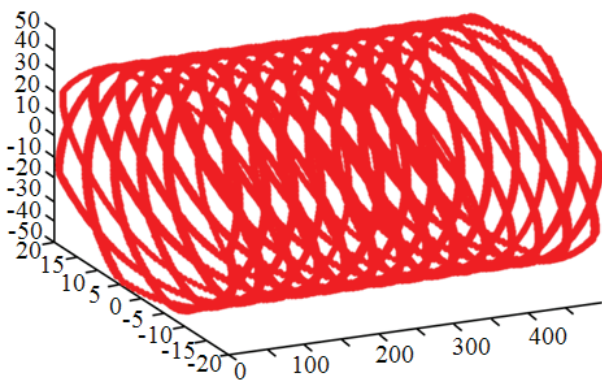|		
Long. Mandrel $L =$	500	
Nber divisions $n =$	180	
		Perform
Half small axis $b =$	40	
Half big axis $c =$	60	

Fig. 3 Data entry window and start of the execution

Fig. 4 Results of the simulation of the winding

3.2. Parameter file

The parameter file includes seven parameters in a table with seven columns where the number of rows equals the number of angular increments necessary to cycle the winding mandrel ellipsoidal:

 − J is the serial number of the angular increment;
 − $\beta_0(j)$ is the angle swept by the point of removal after d-1 angular increment;
 − $\Delta x(j)$ is the increment of the axial point of removal to move from j to j-1;
 − $x(j)$ is the point position deposited on (o, x) to the increment j;
 − $X_{E(j)}$ is the eye position of the guide (o, x) to the increment j;
 − $\Delta X_{E(j)}$ is the increment on the eye guide for the past j-1 to j, and
 − Φ_E is the rotation of the eye guide to keep the tape flat during winding.

3.3. Rotation of the chuck

Fig. 5 shows the curve values of the rotation of the mandrel. One can notice that the curve of the angle $\beta_0(i)$ is linear and increasing continually, that is to say that the core rotates in the same direction through the coil, which is what we want.
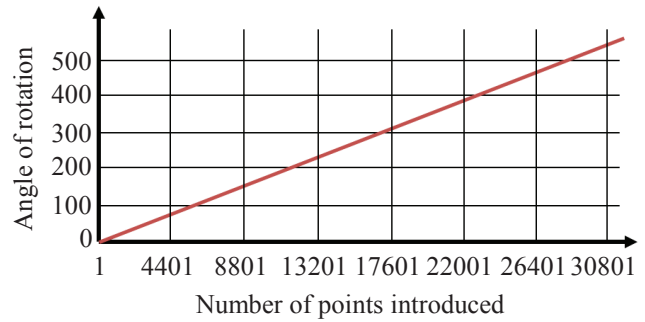

Fig. 5 Curve representing the rotation of the mandrel

3.4. Variation of the abscissa of the point of removal

The curve in Fig. 6 presents a set of the values calculated for each incremental rotation of the mandrel. For better visibility, we present the values for four turns of the mandrel.
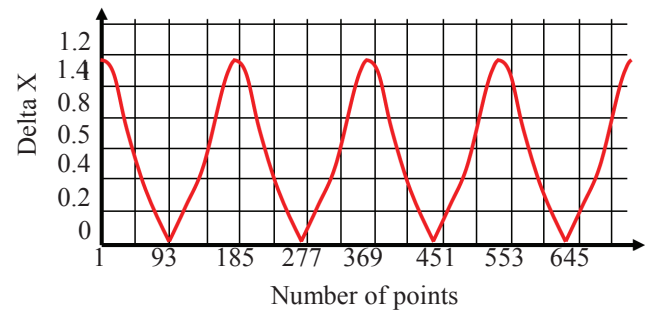

Fig. 6 Curve representing the variation of x

This parameter varies in the interval of 0 to 1.2. It decreases from 1.17060311 to 0, when the value of the angle of rotation of the mandrel is within the interval of $[0 + 2k\pi \quad (1 + 2k)\pi]$ and grows from 0 to 1.17060311 when this value is included in the interval of $[(l + 2k)\pi \quad (1 + k)2\pi]$ which shows that the cross-section of the form being wound is not circular. These values are calculated in absolute values by the system with the geodesic equation.

3.5. Abscissa of the point of removal

Fig. 7 shows the curve of the change in abscissa of the point of removal.
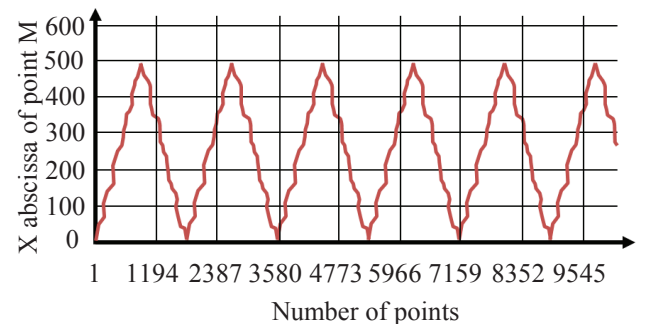

Fig. 7 Curve of the evolution of abscissa of the point of removal

The x-coordinate of the point M representing the deposit varies in the interval of $\begin{bmatrix} 0 & L \end{bmatrix}$, where L is the winding length of 500 mm in this case. This periodic variation is the movement back and forth and reflects the fact that the removal of the filament must be carried out back and forth several times until the full recovery of the surface. The undulating shape of this curve is the fact that its variation Δx is not constant.

3.6. Horizontal axis of the eye guide

The horizontal axis of the eye guide varies in the same way as that of the point of removal. Its graph has the same shape as seen in Fig. 8.
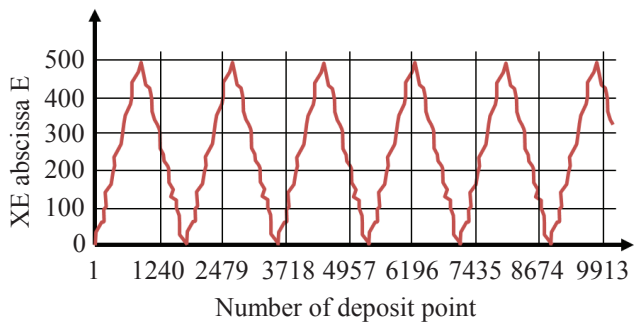

Fig. 8 Curve of the evolution of the x-axis of the eye guide

The abscissa of point X_E, where E represents the position of the eye guide along an axis parallel to the axis of rotation of the mandrel, also varies in the interval of $\begin{bmatrix} 0 & L \end{bmatrix}$. This shift similar to the M point of deposit is due to the fact that the eye must guide the filament to compel the geodesic deposit. It must also make the round trip several times until the end of winding.

3.7. Variation of the abscissa of the eye guide

The curve in Fig. 9 shows the variation of the displacement of the eye corresponding to the guide from the point of deposit.
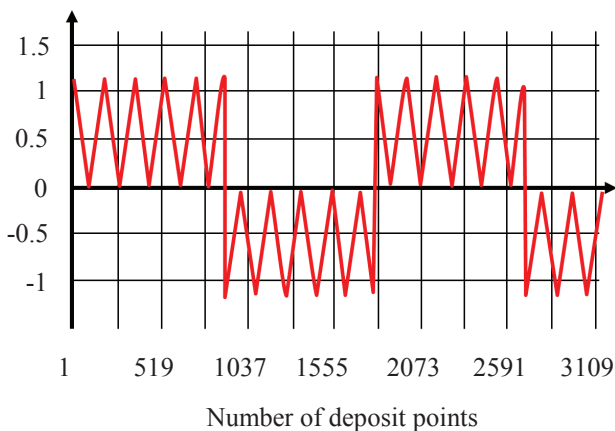

Fig. 9 Curve of the change in eye guide position

Negative values ΔX_E correspond to the values calculated during the return of the winding. This is justified by the fact that this variable is calculated by the for-

mula: $X_E(j) - X_E(j-1)$. In this phase $X_E(j)$ is always less than $X_E(j-1)$.

3.8. Rotation of the eye guide

The eye guide must occupy a position which favors the tape so that it does not wrinkle its ends. This position must be constantly adjusted to an angle depending on the position of the deposit or rotation of the mandrel. Fig. 10 shows the variation of this angle.
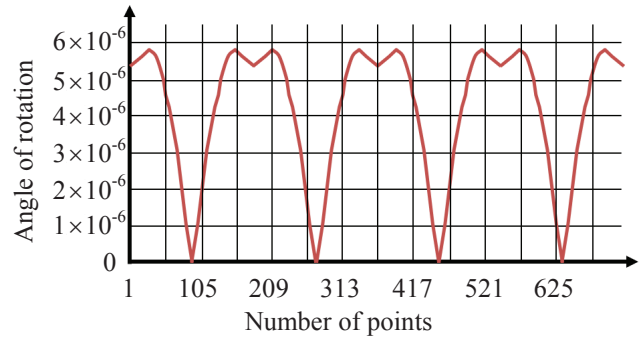

Fig. 10 Curve representing the rotation of the eye guide

The rotation of the eye guide is in the range of $\begin{bmatrix} 6.60369 \times 10^{-22} & 5.7712 \times 10^{-6} \end{bmatrix}$, and the smallest deviation occurs when the spindle is rotated $\frac{3\pi}{2} + 2k\pi$, and the largest in the proximity of $\frac{\pi}{7} + 2k\pi$. The inflection point observed on the curve corresponds to a rotation $\pi + 2k\pi$ of the chuck.

4. Conclusions

1. This work allowed us to simulate the winding structure matrix composites in thermoplastic or thermosetting resin reinforced with continuous fibers and prepare a data file to program a numerical control designed to control the winder.

2. A calculation error has to be acknowledged. For a structure of the size of the sample application above, the error recorded by a winding cycle is 0.407 mm on the movement of the eye guide. This error does not pose a problem in a cycle, given the non-zero coefficient of friction between the mandrel and the tape submitted. The programming of the CNC considers this error cancelled at the end of each winding cycle.

3. This work can be replicated for the coil structures' truncated elliptical cross-sections, and all structures of revolution, for the coil itself.

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A. Ohandja, B. Mvola

SKAITINIS GEODEZINIŲ LINIJŲ ELIPSINIO ĮTVARO SKERSINIAME PJŪVYJE NUSTATYMAS

R e z i u m ė

Straipsnyje pateikti vejamo pluošto gijų elgsenos elipsinio profilio skersiniame pjūvyje tyrimų rezultatai. Atliktas tyrimas leido palaipsniui nustatyti skaitines besisukančio įtvaro ir vediklio kreipiančiųjų poslinkio reikšmes slenkant ir sukantis, darant prielaidą, jog šie laike suderinti pokyčiai griebtuvu leidžia nuimti suvytą juostą geodezinių linijų kryptimi.

Naudojant integravimo metodą gautos įtvaro sukimąsi aprašančios skaitinės reikšmės yra kaupiamos duomenų faile. Šis programiškai sudarytas failas toliau naudojamas kaip duomenų bazė, kurios pagrindu skaitinio valdymo sistema formuoja vijimo valdymo programą.

Skaičiavimų paklaida dėl gijos susisukimo nedaro ženklesnės įtakos vediklio judesiui ir neįtakoja ciklo. Tai parodo, jog taikoma priemonė yra efektyvi ir dera su šiuo vijimo metodu.

DETERMINATION OF THE DIGITALLY FILED GEODESICS ON A CROSS-SECTION OF AN ELLIPSOIDAL MANDREL

F.E. Betene, P. Kah, J.A. Atangana, J. Martikainen,
A. Ohandja, B. Mvola

S u m m a r y

This paper presents the results of a study on the weaving beams across a straight elliptical cross-section by filament winding. The study determines, block by block, numerical values of the displacements of the rotating mandrel and guides of the eye in translation and in rotation, provided that the timing of these changes provides a removal of the tape with the chuck according to geodesic lines.

The numerical values obtained by an incremental method applied to the rotation of the mandrel are then stored in a file. This file is generated by the program as a basis for data to then develop the program executable by the digital control in order to control the weaver.

The error of the calculation recorded by filament wound is insignificant on the movement of the eye guide and does not affect the cycle. This is a demonstration of the efficiency of the application and responds to the composite winding method.