

# The Research on Horizontal Motion Axis Control System Error Prediction of the Gantry-Type Vertical Machining Center

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## 1. Introduction

The Gantry-type Vertical Machining (GVM) Center, as a core equipment in high-end equipment manufacturing, directly determines the machining quality of aerospace thin-walled parts, precision molds, and other complex curved surface components through the positioning accuracy and dynamic stability of its horizontal motion axes [1]. With the manufacturing industry advancing towards high precision and high efficiency, the issue of error accumulation under dynamic operating conditions using traditional control methods is becoming increasingly prominent. Specifically, during the motion of the horizontal axes, multi-source coupling effects including geometric errors [2], thermal deformation [3], load disturbances [4], and servo system nonlinearities [5] can lead to micron-level deviations, severely affecting the surface roughness and dimensional consistency of parts.

Current error compensation methods primarily fall into two categories: model-based feedforward compensation and data-driven online correction. Traditional methods such as PID control [6], model predictive control [7], and static geometric error calibration based on rigid-body kinematics [8], while capable of improving accuracy under certain conditions, rely on offline modeling and fixed parameters. This makes it difficult for them to adapt in real-time to the time-varying characteristics of thermal errors, abrupt changes in cutting forces, and dynamic coupling effects during multi-axis coordinated motion [9]. Furthermore, existing data-driven methods like neural networks [10] and Gaussian process regression [11], despite having strong fitting capabilities for error prediction, often function as "black-box" models. They offer weak interpretability regarding the mechanisms of error generation and are prone to cumulative errors in multi-step ahead prediction, making it challenging to achieve stable and reliable look-ahead compensation in high-speed, high-precision motion.

To address these limitations, this study proposes an error advance prediction and compensation method based on the Hidden Markov Model (HMM). Its novelty is mainly reflected in the following three aspects:

1. Mechanism and Data Fusion Modeling: Utilizing the "double stochastic process" characteristic of HMM, the error generation process is modeled as a Markov chain of hidden states (e.g., "high error," "normal error," "low error"). State inference is performed through observation sequences (discretized error values), which preserves the temporal dependency of error dynamic evolution while providing probabilistic description capabilities for multi-source uncertainties.

2. Advance Prediction Mechanism: By training HMM parameters ( $A, B, \pi$ ) with historical and real-time error data, it is possible to predict the probability distribution of error states for upcoming machining segments. This enables error advance compensation, meaning that feed commands are corrected before cutting commences, thereby overcoming the latency inherent in traditional compensation methods.

3. Lightweight Modeling Suitable for Multi-Degree-of-Freedom Coupled Errors: Through state discretization and probability transition matrices, HMM can model coupled errors in multi-axis coordinated motion that are difficult to fully decouple, with relatively low computational load. This is particularly applicable to dynamic interference issues, such as those between the X and Y axes in high-speed contour machining on gantry machine tools.

In this study, "error advance prediction" is defined as: based on historical machining error sequences, inferring the probability distribution of error states for future machining nodes via HMM, and accordingly correcting trajectory parameters in the command generation stage in advance to achieve feedforward suppression of errors. The "double stochastic process" refers to HMM encompassing both the stochastic process of state transitions and the stochastic mapping from states to observations, corresponding respectively to the intrinsic dynamics of error evolution and the uncertainty of external observations. The "multi-degree-of-freedom coupling mechanism" specifically refers here to the superimposed error interaction phenomenon caused by dynamic interaction, structural vibration, and control lag during coordinated interpolation of the horizontal dual axes on a gantry machine tool.

This study focuses on domestically produced mid-to-high-end the GVM Center, specifically their UMAC CNC systems and PVT velocity interpolation algorithms. The proposed method can be implemented via an external industrial PC or embedded coprocessor. It involves real-time acquisition of error data from grating scales, online updating of HMM parameters, generation of compensated position commands, and synchronization with the CNC system via a communication interface. It is anticipated to achieve a reduction in tracking error of over 60%, particularly controlling error peaks from several hundred micrometers to within several tens of micrometers during high-acceleration contour machining, significantly enhancing contour accuracy and surface quality.

This research not only provides an interpretable and predictable probabilistic modeling approach for dynamic error compensation in high-precision CNC machine tools but also offers key technical support for enhancing the

competitiveness of domestic high-end equipment in fields such as aerospace and new energy vehicle key component machining. It contributes to advancing precision manufacturing towards a "predictive intelligent manufacturing" paradigm.

## 2. Control Error Analysis

The interpolation approximation error, hardware system error and contour following error are the main reasons for the error of its control system [12].

The error term of the CNC machine tool control system is mainly reflected in the position and speed deviation. As the research object of the topic uses the velocity control system, it mainly relies on the PVT (Position, Velocity, Time) operation control algorithm of the UMAC motion controller to achieve velocity interpolation control.

The PVT trajectory algorithm has three execution parameters of  $P$ ,  $V$  and  $T$ ; the parameter  $T$  is the time base set by the electronic cam. In general,  $T$  does not appear to be in error. Parameter  $V$  has dual meanings, which are the ending speed  $V_i$  of the previous section and the initial speed  $V_o$  of the next section, respectively. To the principle of motion control, the interpolation ending speed  $V_i$  of the previous node should always be equal to the interpolation initial speed  $V_o$  of the current node ( $V_i=V_o$ ). Considering the execution performance of the controller, a speed error caused by the execution performance should exist in parameter  $V$ . The parameter  $P$  is the position variable between the interpolation nodes. Considering execution performance of the controller and the calculation formula of parameter  $P$  ( $P=2 \times (V_i^2 - V_o^2) / a$ ), then a position error caused by the speed should exist in parameter  $P$ . Therefore, through the relationship between parameters  $V$  and  $P$ , the position error can be used to achieve global compensation when the velocity interpolation control system is compensated errors, and the speed error can also be used to achieve global compensation.

During operation, the speed deviation of the node is ultimately manifested as the position being ahead or behind, so, the position compensation is adopted instead of the control compensation, that is, only the position error modification is implemented on the control system, and the look-ahead position compensation is used to make up for the deviation of the operating speed so as to achieve the improvement of the control accuracy.

Since the operational control algorithm of PVT is designed for the application of horizontal motion axis linkage, according to the above analysis, there are two kinds of control errors that need to be compensated in the GVM Center, which are  $\delta_x(p)$  and  $\delta_y(p)$ , where  $p$  is the target of the motion execution.

## 3. Prospective Prediction Theory

### 3.1. Theory basic

HMM is a probability model, represented by parameters, for describing the statistical characteristics of a stochastic process. It is a kind of Markov chain whose states cannot directly observed but can be observed through a sequence of observation vectors, each of which is generated by a state sequence with a certain probability density distribution [13, 14]. So, the hidden Markov model is a double

stochastic process, one being a hidden Markov chain with a certain number of states and the other being a set of explicitly functions.

The HMM is usually described by the following parameters:

1.  $N$ : the number of states of the Markov chain in the model, where the set of states is denoted as  $S = \{S_1, S_2, \dots, S_N\}$ , and the state at time  $t$  is denoted as  $Z_t$ , then obviously  $Z_t \in S$ .

2.  $M$ : the number of possible observation symbols corresponding to each state, and the set of observation symbols is denoted as  $O = \{O_1, O_2, \dots, O_M\}$ , and the symbol observed at time  $t$  is denoted as  $V_t$ , obviously  $V_t \in O$ .

$$3. \mathbf{A} = \begin{bmatrix} a_{S_1 S_1} & a_{S_1 S_2} & \cdots & a_{S_1 S_N} \\ a_{S_2 S_1} & a_{S_2 S_2} & \cdots & a_{S_2 S_N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{S_N S_1} & a_{S_N S_2} & \cdots & a_{S_N S_N} \end{bmatrix} : \text{state transition}$$

matrix, where  $a_{S_i S_j}$  (denoted as  $a_{ij}$ ) represents the probability of transitioning from state  $S_i$  to state  $S_j$ .

$$4. \mathbf{B} = \begin{bmatrix} b_{S_1}(O_1) & b_{S_1}(O_2) & \cdots & b_{S_1}(O_M) \\ b_{S_2}(O_1) & b_{S_2}(O_2) & \cdots & b_{S_2}(O_M) \\ \vdots & \vdots & \ddots & \vdots \\ b_{S_N}(O_1) & b_{S_N}(O_2) & \cdots & b_{S_N}(O_M) \end{bmatrix} : \text{obser-}$$

vation symbol probability matrix, where  $b_{S_i}(O_j)$  gives the probability of observing the symbol  $O_j$  when in state  $S_i$

5.  $\pi = [\pi_1, \pi_2, \dots, \pi_i, \dots, \pi_N]$ : initial state probability matrix, where  $\pi_i = P(A_1 = S_i)$ , the probability of the initial state being  $S_i$  at time  $t = 1$ .

Such a hidden Markov model can be denoted as  $\lambda = (N, M, A, B, \pi)$ , and it is simply denoted as  $\lambda = (A, B, \pi)$ .

### 3.2. Mathematical modeling

The HMM error compensation is to train HMM according to the actual error data detected in the past and at present, and to clarify its five basic elements  $\lambda = (N, M, A, B, \pi)$ . The specific implementation process is as follows:

1. The error data of the moving axis of the vertical machining center is discretized.

Before error correction, the detected "node" errors are divided into three states (higher, normal, lower) with a threshold  $T_1$ ; the detected "node" are then divided into  $n$  observation values with a threshold  $T_2$ .

2. HMM parameter estimation.

When fed to the GVM center, the machining trajectory is usually divided into  $N'$  segments and then fitted in a single segment. Therefore, the trajectory to be machined is divided into  $N'$  segments, and it is necessary to establish  $N'$  different HMMs to correct their errors respectively.

Application of the HMM toolbox functions in MATLAB software to analytically determine the state transition probability matrix  $\mathbf{A}$  and the observation probability matrix  $\mathbf{B}$  to form  $N'$  HMM.

3. Error prediction.

According to the error detection data of the machined parts, the error detection data of the machined parts and the established HMM, the prediction of the max error is

made, and the error prediction value  $E'_i$  is obtained, and its prediction error calculation formula is:

$$E'_i = \zeta_i \cdot \Gamma, \quad (1)$$

$$\zeta = S \cdot P. \quad (2)$$

The meaning of the letters in Eqs. (1) and (2) is shown in Table 1.

Table 1

Application meaning of formula letters

Num	Symbol	Meaning
1	$\zeta_i$	Predict the probability distribution of the model at the time node
2	$\Gamma$	The distribution of the discretization threshold $T_2$ of the test error
3	$S^i_{d-1}$	The current state $i$ when predicting the model at the same node
4	$P_i$	Probability distribution of state $i$ transferring to other states

#### 4. Error compensation.

The actual error is compensated by using the above calculation to obtain the error prediction value, and the final feed value of the GVM Center is obtained

$$\zeta'_i = \zeta + E'_i. \quad (3)$$

#### 4. Error Compensation Implementation

The node error of the horizontal motion axis X of the GVM Center is obtained by reading the grating ruler inside the control platform UMAC ALS5000. In the HMM compensation model, the error evaluation  $e$  takes the relative error, and  $E$  takes the measured error. According to the error data, it is known that:  $-300 \mu\text{m} \leq E \leq 300 \mu\text{m}$ . The number of HMM states  $N$  is taken as 3: higher, normal, lower, and the number of possible observation values  $M$  corresponding to each state is taken as 9. The HMM initial state probability  $\pi$  is selected as  $[1/3, 1/3, 1/3]$ , the probability of occurrence of the three states is the same. According to the size of the detection error, the error state value in the discrete case is determined according to the threshold  $T_1 = 0.05$ .

$$S = \begin{cases} 1 & e < -T_1 \\ 2 & -T_1 < e < T_1 \\ 3 & T_1 < e \end{cases}. \quad (4)$$

Dividing the detection error into a discrete value according to the size of the detection error and the interval where it is located in the threshold  $T_2$ , the actual of observations can be obtained.

$$O = \begin{cases} 1 & -300 \leq E \leq -240 \\ 2 & -240 \leq E \leq -180 \\ 3 & -180 \leq E \leq -120 \\ 4 & -120 \leq E \leq -60 \\ 5 & -60 \leq E \leq 60 \\ 6 & 60 \leq E \leq 120 \\ 7 & 120 \leq E \leq 180 \\ 8 & 180 \leq E \leq 240 \\ 9 & 240 \leq E \leq 300 \end{cases}. \quad (5)$$

When the error calculation is carried out, it is necessary to restore the discretized prediction error performance value into the discretized correction error value, and the discretized distribution used for calculation by using the formula (1), (2) is:

$$\Gamma = [-270, -210, -150, -90, 0, 90, 150, 210, 270]^T.$$

The table value of the  $\Gamma$  matrix is obtained by taking the average of the left and right critical values of each section of the threshold  $T_2$ . When  $-60 \mu\text{m} \leq E \leq 60 \mu\text{m}$ , it is considered that the error value detected is already within the allowable accuracy range of the GVM Center, i.e.  $\Gamma$  takes 0 at this time.

#### 5. Experimental Analysis

Taking the machining of a part on the GVM Center as an example, the node error with the feed 30 times ( $\varepsilon_1$ ) and 50 times ( $\varepsilon_2$ ) and 100 times ( $\varepsilon_3$ ) is used as training data to establish HMM compensation model. According to the state of the error before compensation, the performance of observation value, and the possible distribution and probability value of the possible tool feed error in machining, the expectation of prediction is calculated, and the next tool feed of the GVM Center is error compensated by using the expectation of prediction error to develop the machining and compensation program to start mach. Fig. 1 and Fig. 2 show the curve diagram before and after compensation of the control error of the horizontal moving axis X and Y of the GVM Center.

It can be analyzed and concluded from the curves before and after error compensation in Fig. 1 and Fig. 2:

1. Comparative analysis of the compensation results. After the HMM error compensation, the maximum error of node compensation value is significantly reduced. The maximum of the motion axis X before compensation is  $571 \mu\text{m}$ , and the maximum error  $\varepsilon_1$  after compensation is only  $90 \mu\text{m}$ , and the overall error is significantly improved.

2. Comparing the size of standard deviation before and after compensation, the standard deviation after HMM error compensation is smaller than that before compensation, indicating that error dispersion degree of nodes after HMM error compensation is lower. From the actual machining point of view, the lower the error dispersion degree, the smoother the trajectory to be machined.

3. With respect to the comparison of the compensation effects before and after compensation, the HMM error compensation model constructed by using the node error training data the feed 100 times ( $\varepsilon_3$ ) is more ideal compared to the node error training data of the feed 30 times ( $\varepsilon_1$ ) and 50 times ( $\varepsilon_2$ ). The variance of the error compensation value obtained by ( $\varepsilon_3$ ) is the smallest, indicating that the base

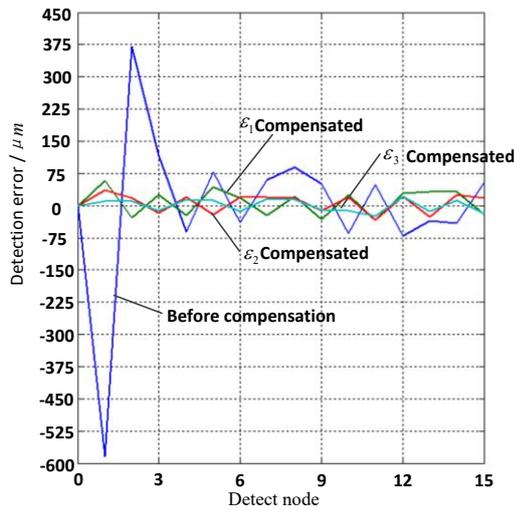


Fig. 1 Error curve before and after X-axis compensation

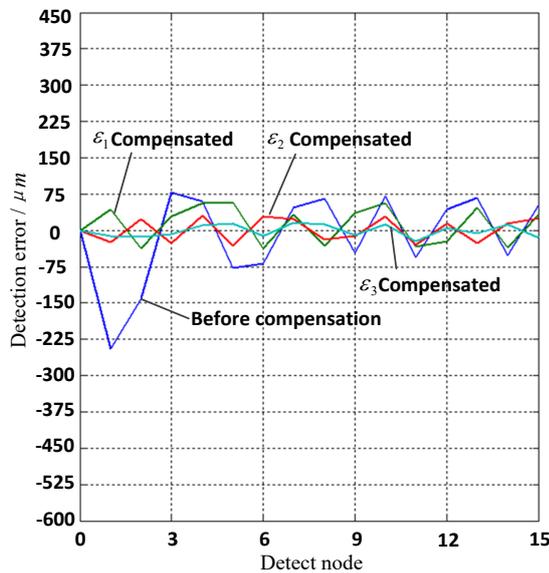


Fig. 2 Error curve before and after Y-axis compensation

number of the training model directly affects compensation effect. The larger the base number of the training model, the more precise the compensation effect of the constructed HMM error compensation model.

## 6. Conclusions

1. Aiming at the error prediction and compensation problem of the horizontal motion axis control system in the dynamic working condition the GVM Center, this study proposes an error advanced prediction method based on the hidden Markov model (HMM). By constructing an HMM dual stochastic process model integrating the error lookahead mechanism, the real-time prediction and compensation of the X, Y-axis position error are realized. The experimental results show that the proposed method can effectively the position deviation of the motion axis, where the maximum error of the X-axis is reduced from 571  $\mu\text{m}$  before compensation to 90  $\mu\text{m}$  (reduced about 84.2%), and the standard deviation of the error is significantly reduced, indicating that the trajectory tracking consistency and contour accuracy are improved.

2. Effectiveness and data dependence of the HMM compensation model are verified by comparing experiments with different training data sizes (30, 50, 100 cutting sample). results show that as the training samples increase, the compensation accuracy is further improved: the model based on 100 cutting samples has a compensation error variance reduction of about 4% compared to 30 samples, indicating that larger training data helps to improve the model's characterization ability of the dynamic characteristics of the error, thereby improving the prediction accuracy

3. This method provides a data-driven solution for real-time error compensation of CNC machine tools, with the potential for integration into actual machining systems and to precision machining scenarios with high dimensional accuracy requirements (such as within the error range of  $\pm 60 \mu\text{m}$ ) and complex motion trajectories. However, the method still relies on a certain amount of historical error data for model training and its generalization ability under multi-axis strong coupling and extreme process parameters needs to be further verified. Future work will focus on model design, online learning mechanism, and multi-axis collaborative error prediction to enhance its applicability in the predictive control system throughout the entire life cycle

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## References

1. Meng, X. R.; Wang, Y. O.; Yin, X. L.; Fu, H. Y.; Sun, S. X.; Sun, Y. W. 2024. Investigation of Force-Controlled Polishing of Complex Curved PMMA Parts on a Machining Center, *Machines* 12(4): 259. <https://doi.org/10.3390/machines12040259>.
2. Yang, L.; Fang, J.; Zheng, M. L.; Jiang, H.; Liu, Y. F.; Shi, C. Y. 2025. Research on key geometric error identification and accuracy control of heavy-duty vertical lathes, *Sādhanā* 50(4): 324. <https://doi.org/10.1007/S12046-025-02991-9>.
3. Xu, J. T.; Tian, A. L.; Liu, H.; Liu, Y. 2023. Modeling and Prediction of Thermal Deformation Errors in Fiber Optic Gyroscopes Based on the TD-Model, *Sensors* 23(23): 9450. <https://doi.org/10.3390/s23239450>.
4. Ralhan, S.; Singh, M.; Singh, M. 2025. Design and experimental investigations of an optimal controller in PEM fuel cell for load perturbations in microgrid energy management system, *Engineering Research Express* 7(4): 045307. <https://doi.org/10.1088/2631-8695/AE09F3>.
5. Sun, Z. C.; Liu, G. P.; Cai, G. Q.; Pan, Z. M., Wang, L. B.; Fang, Q. 2025. Adaptive friction compensation for synchronous force control of dual-servo riveting systems with high-energy nonlinear disturbance, *Nonlinear Dynamics* 113(22): 31455–31477. <https://doi.org/10.1007/S11071-025-11712-0>.
6. Zhang, J.; Lv, S. T.; Pan, Y. F.; Wang, H.; Yang, R. B.; Zhao, Y.; Zhao, A.; Liu, Y. 2026. Auto-follow row control system for an autonomous maize harvester using adaptive segmented PID and row-deviation detection, *Robotics and Autonomous Systems* 196: 105252. <https://doi.org/10.1016/J.ROBOT.2025.105252>.

7. **Qian, S.; Jiang, X.; Qian, P. F.; Zi, B.; Zhu, W. D.** 2024. Calibration of Static Errors and Compensation of Dynamic Errors for Cable-driven Parallel 3D Printer, *Journal of Intelligent & Robotic Systems* 110(1): 31. <https://doi.org/10.1007/S10846-024-02062-X>.
8. **Wang, J. F.; Feng, Y. J.; Tong, X. H.; Lei, Z. K.; Chen, S. R.; Li, P. S.; Xi, M. R.** 2025. Geometric position error analysis of airborne SAR and geometric calibration with platform motion constraints, *Measurement* 241: 115650. <https://doi.org/10.1016/j.measurement.2024.115650>.
9. **Li, M. Q.; Zhang, S.; Zhao, Y. T.; Cai, Y.; Zhang, M.; Liu, Z.; Li, P. T.; Wang, B.; Xu, B. W.; Shen, J.; Feng, B.** 2025. Multifactor Coupling Effects on Permeability Evolution During Reinjection in Sandstone Geothermal Reservoirs: Insights from Dynamic Core Flow Experiments, *Energies* 18(17): 4770. <https://doi.org/10.3390/EN18174770>.
10. **Lu, N.; Zhang, H. Q.; Dong, C. M.; Li, H. T.; Chen, Y.** 2025. NIGWO-iCaps NN: A Method for the Fault Diagnosis of Fiber Optic Gyroscopes Based on Capsule Neural Networks, *Micromachines* 16(1): 73. <https://doi.org/10.3390/MI16010073>.
11. **Zhang, X.; Yuan, Z. Y.; Zhu, M. H.** 2026. Byzantine-resilient federated online learning for Gaussian process regression, *Automatica* 183: 112554. <https://doi.org/10.1016/j.automatica.2025.112554>.
12. **An, Y. G.** 2024. Research on CNC Machine Tool Motion Control System Based on Image Processing, *Journal of Engineering System* 2(4): 111-114. <https://doi.org/10.62517/JES.202402419>.
13. **Zhang, S.; Fan, C.; Su, L.; Zhang, J.; Fang, T.** 2025.  $H_\infty$  Synchronization for Hidden Markov Jump Complex Dynamical Networks Based on Sampled Data, *International Journal of Control, Automation and Systems* 23(4): 1026-1034. <https://doi.org/10.1007/S12555-024-0946-5>.
14. **Hu, D. K.; Gao, W. D.; Ang, K. K.; Hu, M. J.; Huang, R.; Chuai, G.; Li, X. Y.** 2025. CHMMConvScaleNet: a hybrid convolutional neural network and continuous hidden Markov model with multi-scale features for sleep posture detection, *Scientific Reports* 15: 12206. <https://doi.org/10.1038/S41598-025-93541-0>.

Y. Hu, B. Yu

## THE RESEARCH ON HORIZONTAL MOTION AXIS CONTROL SYSTEM ERROR PREDICTION OF THE GVM CENTER

### S u m m a r y

As the cornerstone of high-end equipment manufacturing industry, the GVM Center breaks through the limitations of traditional processing through intelligent control technology, and has become a core equipment to promote the upgrading of precision manufacturing, which plays an irreplaceable role in enhancing the competitiveness of national high-end manufacturing. However, in the actual machining process of the GVM Center, due to the superposition of various factors, it is inevitable to produce complex machining errors. In order further improve the machining quality of the GVM Center, it is necessary to carry out error compensation technology research. HMM, as an important content in the field pattern recognition, has further developed its application scope with the rapid change of information data in recent years. Based on the data characteristics of HMM, the research on error compensation of the GVM Center is an innovative attempt. This research combines the theory of HMM dual stochastic process, deeply analyzes the motion error form of the horizontal moving axis X Y axis of the GVM Center, integrates the error lookahead prediction idea, describes the basic theory of HMM in detail, and embeds it into the g vertical machining center. It constructs a control error compensation model based on HMM, and analyzes the specific steps of HMM to implement error compensation. Based on the actualizing content, the error compensation effect of HMM with different training data is compared and analyzed, and the effectiveness of the research content is proved through data analysis before and after error. The research results provide theoretical support and engineering examples for the intelligent error control of high-precision CNC machine tools, and have important application value in the fields of aerospace-wall parts and precision mold processing.

**Keywords:** gantry-type vertical machining center, control error compensation, HMM, bivariate stochastic process, error modelling.

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