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Research on Error Compensation of CNC Machine Tool Based on Multibody System Kinematics

Lei YU
School of Mechanical and Electrical Engineer, Changchun Institute of Technology, Changchun, Jilin 130012, China, E-mail: jd_yl@ccit.edu.cn

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1. Introduction

With the rapid development of the manufacturing industry, CNC machining has placed higher demands on production efficiency and quality, leading to increased attention on research regarding the improvement of machine tool accuracy, particularly in the model of spatial geometric errors of CNC machine tools [1, 2]. In recent years, due to its ability to provide highly abstract descriptions and detailed generalizations of complex engineering problems, multibody system theory has received widespread attention compensation for CNC machine tools [3], and has gradually become an important analytical method in the field of precision manufacturing of CNC machine tools.

The core of multibody system theory lies in abstracting the complex machine tool structure into a topological network composed of kinematic pairs and links, and precisely description relative pose relationships of moving components during continuous motion through homogeneous coordinate transformation matrices, thereby systematically constructing the spatial geometric error model of the machine tool from drive input to the end effector [4-6]. Based on this model, it is possible to extract error mapping functions from complex error sources and explicitly reveal the generation mechanisms, transmission patterns, accumulation effects, and coupling mechanism among various error sources.

Although existing research has utilized multibody system theory to construct various CNC machine tool error models, there are still deficiencies in terms of generality: most models are customized for specific machine tool structures, lacking a universal error model framework capable of uniformly describing the dual-branch kinematic chains of "workpiece-tool" and "tool-machine", and there is limited discussion on the systematic compensation conditions for tool pose errors. To this end, based on the kinematics theory of Multibody systems, this paper constructs a universal geometric error model applicable to different topologies, explicitly providing unified expressions for position and orientation errors as well as error compensation equations for achieving precision machining. The validity and engineering applicability of the proposed model were verified by applying it to the VMC-500 CNC machine tool and conducting Z-axis error measurement and compensation experiments using the Renishaw ML10 dual-frequency laser interferometer. This study provides systematic theoretical support and an extendable technical path for improving the accuracy of CNC machine tools.

2. Multibody System Kinematics Theory Analysis

In the research on the kinematics theory of Multibody systems, error compensation usually follows the technical route of model-identification-compensation: 1. First, establish a comprehensive error model containing all geometric error terms based on multibody system theory; 2. Secondly, identify and verify the key error parameters in the model using measuring instruments such as laser interferometers and ball bars; 3. Finally, the identified error model is embedded into the CNC system in the form of reverse compensation values to correct the commanded trajectory between the tool and the workpiece real-time, thereby effectively improving the actual machining accuracy of the machine tool without changing the hardware.

2.1. Geometric description of multibody system theory

In multibody system kinematics, the geometric description of the relative position and orientation relationship between typical bodies and adjacent bodies is the core foundation for constructing kinematic model.

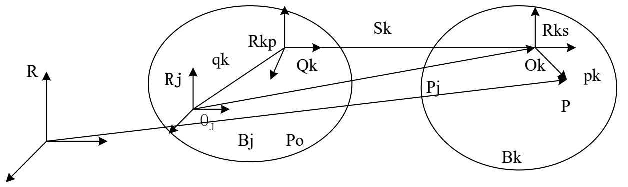

Fig. 1 Geometric description of typical body B_j and adjacent body B_i

As shown in Fig. 1, a geometric relationship is established between each typical body (such as B_j) and its adjacent lower-order body (such as B_i) is tem through a fixed linkage coordinate system (such as O_0-$x_0y_0z_0$ and O_j-$x_jy_jz_j$).Among them, the change in the position vector of point O_i relative to point O_j characterizes the translational motion between the two bodies, while the attitude change of the moving coordinate system's basis vectors (n_{j1}, n_{j2}, n_{j3}) relative to (n_{i1}, n_{i2}, n_{i3}) characterizes the rotational relationship between the two bodies. This method of equating the relative motion between entities to the relative transformation between coordinate systems allows the kinematic study of complex mechanical systems to be transformed into a mathematical description of the pose relationships between coordinate systems, greatly simplifying the system model process.

To quantitatively express these transformations, transformation matrices are typically used to realize the mapping between coordinate systems; the general 4×4 homogeneous transformation matrix can simultaneously handle translation and rotation by representing spatial transformations as linear operations through the introduction of homogeneous coordinates [7, 8]. In the specific parameterization process, various methods such as Euler angles and Cardan angles can be used to determine the rotation transformation; this paper homogeneous matrix based on Euler angles as the descriptive tool.

To meet the requirements of error model, ideal feature matrices (describing theoretical error-free motion), error feature matrices (describing the deviation between actual and ideal and angular error feature matrices (specifically describing orientation errors) are distinguished, thereby laying a geometric and mathematical foundation for subsequent comprehensive spatial error model, analysis, and compensation research machine tools.

To quantitatively describe the actual relative pose between adjacent body B_i and B_j, the following three types of 4×4 homogeneous transformation matrices are defined:

1. Ideal Feature Matrix

Describe the theoretical transformation relationship from coordinate system O_j-$x_jy_jz_j$ to O_i-$x_iy_iz_i$ under error-free conditions. For a prismatic joint, its form is:

$$T_{ij}^{ideal}=\begin{bmatrix}1&0&0&x_{ij}\\0&1&0&y_{ij}\\0&0&1&z_{ij}\\0&0&0&1\end{bmatrix}. \tag{1}$$

2. Error Characteristic Matrix

Describes the additional transformation caused by geometric errors in actual motion, containing six error components and three rotational errors, and is approximate the small error assumption as:

$$\Delta T_{ij}=\begin{bmatrix}1&-\varepsilon_z&\varepsilon_y&\delta_x\\\varepsilon_z&1&-\varepsilon_x&\delta_y\\-\varepsilon_y&\varepsilon_x&1&\delta_z\\0&0&0&1\end{bmatrix}. \tag{2}$$

3. Angular Error Characteristic Matrix

A matrix describing the pose error, consisting only of the upper-left 3×3 submatrix related to rotation from the aforementioned error characteristic matrix, used for the independent analysis of tool pose deviation

$$T_{ij}^{actual}=T_{ij}^{ideal}\cdot\Delta T_{ij}. \tag{3}$$

2.2. Coordinate system and vector representation

2.2.1. Representation of the coordinate system of a Multibody

In a Multibody system, establish a three-dimensional right-handed Cartesian coordinate system associated with the inertial element B_0 and all moving elements B_j, referred to as generalized coordinate system; the coordinate systems of adjacent bodies are called sub-coordinate systems, and the coordinate systems of moving bodies are called moving coordinate systems, with the three axes of each coordinate system defined as the X, Y, and Z axes, respectively.

2.2.2. Coordinate representation of points and vectors

For any point P in the space, add 1 in the three coordinates to make it a four-dimensional column vector:

$$p=\left(p_1,p_2,p_3,1\right)^T. \tag{4}$$

Then, the Eq. (4) is called the homogeneous coordinates of the point p. The parameters p_1, p_2, p_3 are determined the point's coordinates.

According to the representation method of homogeneous coordinates of points, the homogeneous coordinates of the origin of the coordinate system are in the form of:

$$o=\left(0,0,0,1\right)^T. \tag{5}$$

For any vector in space, it is stipulated that the projections on the three coordinate axes are added with 0 to form a four-dimensional column vector:

$$\vec{v}=\left(v_1,v_2,v_3,0\right)^T, \tag{6}$$

where in v_1, v_2, v_3 determine the orientation of the vector.

Using homogeneous coordinates to represent points and vectors transforms the transformation relationships between sub-coordinate systems in a generalized coordinate system into general matrix operations, providing convenience for motion model.

3. Kinematic Analysis of Multibody Systems

The kinematic equations of multibody systems are generally classified into: zero-order kinematic equations used to describe pose, first-order kinematic equations used to describe angular velocity and velocity, second-order kinematic equations used to describe angular acceleration and acceleration, and higher-order kinematic equations used to describe angular jerk and jerk [9]. In the precision analysis and model of machine tools with error motions, the primary focus is on the analysis and description of pose variations in both dynamic and static state; therefore, the general zero-order kinematic equations are most commonly adopted.

In the following derivation, j_{rp} denotes the homogeneous coordinate column vector of point P in coordinate system O_j-$x_jy_jz_j$, and j_v denotes the homogeneous projection column vector of vector v in this coordinate system; T_{ij} is the ideal homogeneous transformation matrix from coordinate system j to coordinate system i, and ΔT_{ij} is the corresponding error characteristic matrix.

Let the homogeneous coordinates of an arbitrary point P in space in its sub-coordinate system O_j-$x_jy_jz_j$ is:

$$P_j=\left[P_{xj},P_{yj},P_{zj},1\right]^T. \tag{7}$$

Therefore, the homogeneous coordinate form of the ideal zero-order motion position equation of point P in the static coordinate system is:

$$P_{0ideal}=\left[P_{x0},P_{y0},P_{z0},1\right]^{T}=$$
$$=\left[\prod_{t=n,L^{n}(j)=0}^{t=1}T_{L^{t}(j)L^{t-1}(j)p}T_{L^{t}(j)L^{t-1}(j)s}\right]P_{j} \quad (8)$$

and the homogeneous coordinate form of point P under the ideal zero-order motion position equation in any sub-coordinate system O_k-$x_ky_kz_k$ is:

$$P_{kideal}=\left[P_{xk},P_{yk},P_{zk},1\right]^{T}=$$
$$=\left[\prod_{u=n,L^{n}(k)=0}^{u=1}T_{L^{u}(k)L^{u-1}(k)p}T_{L^{u}(k)L^{u-1}(k)s}\right]^{-1}\times$$
$$\times p\left[\prod_{t=n,L^{n}(j)=0}^{t=1}T_{L^{t}(j)L^{t-1}(j)p}T_{L^{t}(j)L^{t-1}(j)s}\right]P_{j}. \quad (9)$$

Let the homogeneous projection of a vector in space in the coordinate system O_j-$x_jy_jz_j$ is:

$$\vec{v}_{j}=\left[v_{xj},v_{yj},v_{zj},0\right]^{T}. \quad (10)$$

Therefore, the homogeneous projection form of the ideal zeroth-order motion attitude equation in a static coordinate system is:

$$v_{0ideal}=\left[v_{x0},v_{y0},v_{z0},0\right]^{T}=$$
$$=\left[\prod_{t=n,L^{n}(j)=0}^{t=1}T_{L^{t}(j)L^{t-1}(j)p}T_{L^{t}(j)L^{t-1}(j)s}\right]v_{j}. \quad (11)$$

In any moving coordinate system O_k-$x_ky_kz_k$, the homogeneous projection form of the ideal zeroth-order motion attitude equation is:

$$v_{kideal}=\left[v_{xk},v_{yk},v_{zk},0\right]^{T}=$$
$$=\left[\prod_{u=n,L^{n}(k)=0}^{u=1}T_{L^{u}(k)L^{u-1}(k)p}(R)T_{L^{u}(k)L^{u-1}(k)s}(R)\right]^{-1}\times$$
$$\times\left[\prod_{t=n,L^{n}(j)=0}^{t=1}T_{L^{t}(j)L^{t-1}(j)p}(R)T_{L^{t}(j)L^{t-1}(j)s}\right]\vec{v_{j}}. \quad (12)$$

In Eqs. (7) to (12), the first three terms of the point equation are the coordinates of the X, Y, and Z axes, while the three terms of the vector equation are the projections onto the X, Y, and Z axes; subscripts are used to denote the number of the sub-coordinate system, thereby indicate coordinate system the projections or coordinates belong.

4. Numerical Control Machine Tool Error Model Analysis

There are many different combinations of the branches where the components of CNC machine tools are located and the relative motion modes between the components; the general error solving mown in Fig. 2 can be calculated using a Multibody system topological structure model. The figure shows two branches, each consisting of five moving bodies with six degrees of freedom between adjacent bodies. When applied to the VMC-500 CNC machine tool, the redundant parameters in the system can be set to zero, thereby accurately describing the VMC-500 CNC machine tool.

As shown in Fig. 2, the "workpiece-frame" kinematic chain B-W consists of the chain B(O)-W_1-W_2-W_3-W_4-W_5-W, where W_1, W_2, W_3, W_4, W_5 are five moving bodies, and the motion between adjacent bodies is either rotation or translation; The "tool-spindle" kinematic chain B-T is composed of the chain B(O)-T_1-T_2-T_3-T_4-T_5-T,where T_1, T_2, T_3, T_4, T_5 are five moving bodies, and the motion between adjacent bodies is either rotational or translational.

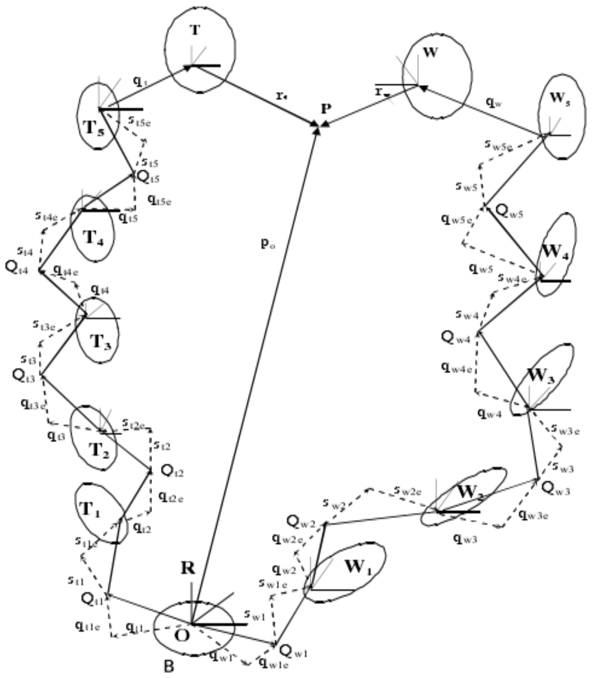

Fig. 2 CNC machine error model

Regarding the VMC-500 vertical CNC machine tool used in the experiments of this paper, its specific topology is as follows:

1. Kinematic Chain Composition

The VMC-500 is a three-axis vertical milling machine with the following motion configuration: X-axis (transverse table movement), Y-axis (longitudinal table movement), and Z-axis (vertical spindle head movement). According to Multibody system theory, its "workpiece-frame" kinematic chain is: Base (O) - X-axis - Y-axis - Workpiece (W); the "tool-frame" kinematic chain is: Base (O) - Z-axis - Tool (T). The two kinematic chains intersect at the base, forming a closed loop.

2. Axis sequence and type of kinematic pair

X-axis: Prismatic joint, motion direction is the transverse direction of the table (left-right); Y-axis: Prismatic joint, motion direction is the longitudinal action of the table (front-back); Z-axis: Prismatic joint, motion direction is the vertical direction of the spindle head (up-down). All joints are arranged orthogonal right angles, with no rotary axes.

3. Retained Geometric Error Terms

In the error characteristic matrices ΔT_{ij} of the general model, the following error terms are retained or the VMC-500 machine tool: positional errors of each linear axis:

positioning errors δ_x, δ_y, δ_z; straightness errors of each linear axis: horizontal direction straightness $\delta(x)_y$, $\delta(x)_z$, etc.; angular errors of each linear axis: pitch, yaw, and roll errors ε_x, ε_y, ε_z; perpendicular errors: perpendicular deviations between the X and Y axes, the X and Z axes, and the Y and Z axes.

Setting the redundant parameters in the general model to zero yields a simplified error model applicable to the VMC-500 machine tool.

Establish orthogonal Cartesian coordinate systems $R_i(O_i\text{-}x_iy_iz_i)$ on each moving body (where i = 1,2,3,4,5), the workpiece coordinate system $R_w(O_w\text{-}x_wy_wz_w)$, the tool coordinate system $R_t(O_t\text{-}x_ty_tz_t)$, and the machine bed coordinate system $R_o(O_o\text{-}x_oy_oz_o)$. In addition, let the column vector of the workpiece's machining point in R_w $(O_w\text{-}x_wy_wz_w)$ is r_w, and the column vector of the tool point in $R_t(O_t\text{-}x_ty_tz_t)$ is r_t.

During the machining process, the tool posture (the tool feed angle) must also be considered. Let the tool axis vector be expressed in the tool axis is:

$$v_t = \left[v_{tx}, v_{ty}, v_{tz}, 0 \right]^T . \quad (13)$$

Its actual expression in the inertial coordinate system R is:

$$\overrightarrow{v_{Rt}} = \left[AOT \right] \overrightarrow{v_t} = \prod_{j=0}^{m} \left[AT_j T_{j+1} \right] \overrightarrow{v_t} . \quad (14)$$

The superscript in Eq. (14), $[AT_jT_{j+1}]$ represents the transformation matrix between adjacent bodies.

Let the theoretical tool pose for the workpiece being machined is:

$$\overrightarrow{v_w} = \left[v_{wx}, v_{wy}, v_{wz}, 0 \right]^T . \quad (15)$$

The actual expression in the inertial coordinate system R is:

$$\overrightarrow{v_{Rw}} = \left[AOW \right] \overrightarrow{v_w} = \prod_{i=0}^{m} \left[AT_i T_{i+1} \right] \overrightarrow{v_w} . \quad (16)$$

During actual machining on the machine tool, assuming there is a deviation between the actual and theoretical tool poses, the deviation formula is:

$$\begin{aligned} \vec{E} &= \left[e_{vx}, e_{vy}, e_{vz}, 0 \right]^T = \\ &= \overrightarrow{v_{Rt}} - \overrightarrow{v_{Rw}} = \\ &= \prod_{j=0}^{m} \left[AT_j T_{j+1} \right] \overrightarrow{v_t} - \prod_{i=0}^{m} \left[AT_i T_{i+1} \right] \overrightarrow{v_w} . \end{aligned} \quad (17)$$

To achieve precision machining, the actual tool posture must coincide with the theoretical tool posture, i.e., the tool deviation must be equal to zero, and the equation is:

$$\overrightarrow{v_{Rw}} = \left[AOW \right] \overrightarrow{v_w} = \prod_{i=0}^{m} \left[AT_i T_{i+1} \right] \overrightarrow{v_w} . \quad (18)$$

Eq. (18) is the error expression for universal CNC machine tools during the machining process, reflecting the deviation between the actual tool posture and the theoretical posture at the optimal cutting angle. When this model is applied to a specific machine tool model, the error calculation equation for that machine can be obtained by simply determining the values of n and m according to the machine model.

In Eq. (18), n and m represent the number of moving bodies in the tool and workpiece kinematic chains, respectively; T_{AOT} is the actual total transformation matrix of the tool side, and T_{AOW} is the actual total transformation matrix of the workpiece side; (j) is the motion characteristic matrix of the j-kinematic pair.

5. Error Compensation Effectiveness Analysis

At present, the detection methods for geometric errors of CNC machine tools generally include the following: ball bar measurement method [10, 11], one-dimensional ball array measurement method [12], orthogonal grating measurement method [13], and laser interferometer measurement method [14]. Among them, the use of laser interferometers for geometric error testing is the most widespread. The laboratory uses the ML10 model dual-frequency laser interferometer produced by Renishaw Company including the following components: ML10 laser head, tripod, PCM10 display/control interface card, a set of optical devices (reflector, beam splitter etc.), data analysis application software package, etc., as shown in Fig. 3.

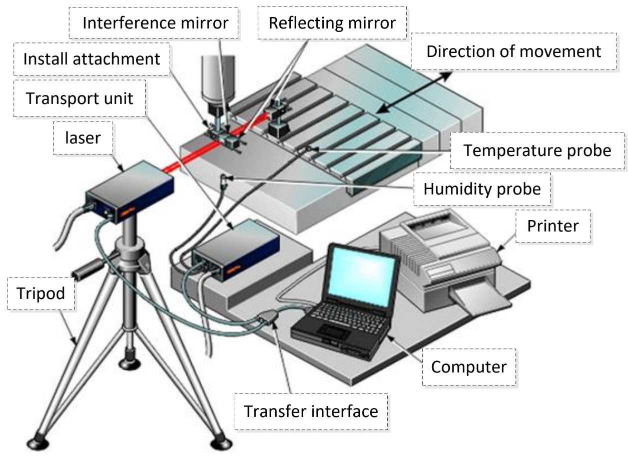

Fig. 3 Laser interferometer

The error compensation strategy adopted in this paper is an offline compensation method. Taking the VMC-500 CNC machine tool as an example, the processing curve $x^2+y^2 = 5625$ is taken as the research subject, with a processing diameter of D = 80 mm. The machining curve was divided into 20 segments, the machining program was compiled, and the data was input into the machining software. The number of passes was set to 100, and the error value of every machining point in each pass was detected in real-time. Taking the Z-axis as the experimental object, the following experimental results were obtained, as shown in Table 1.

Data1 and Data2 are the Z-axis error data obtained from two repeated measurement trials, respectively.

The ct is the counting unit (count) for laser interferometer measurement, and after calibration, 1 ct corresponds to 0.01 μm; if conversion is required, multiply each value in the table by 0.01.

Before measurement, the laser head is preheated for 30 minutes, with the ambient temperature controlled at and humidity ≤ 65%. Each measurement point is measured 3 times to obtain the average value, resulting in a Z-axis positioning uncertainty $U = 0.15$ μm ($k = 2$). The compensation algorithm employs a reverse superposition method.

Table 1

Z-axis error measured record

Point	Error data (unit: ct)		
	Before compensation		Compensated
	Data1	Data2	
1	18.881	19.397	16.887
2	11.625	10.220	-1.587
3	1.697	1.288	3.891
4	-2.634	-1.266	-0.589
5	3.919	7.730	-2.364
6	-3.581	2.802	3.161
7	-1.954	1.234	0.694
8	-3.541	1.852	3.385
9	-2.100	1.109	0.957
10	-2.210	4.138	5.916
11	3.467	-2.720	4.040
12	3.216	-2.453	5.243
13	7.378	1.080	6.786
14	4.256	4.939	1.843
15	4.200	1.288	4.256
16	-6.431	-2.720	3.058
17	-2.409	-2.453	2.104
18	2.766	1.080	3.069
19	7.097	4.939	5.296
20	-5.081	-4.413	3.059
Variance	6.4534	5.8352	3.061

It should be noted that the error at Node 1 remains close to the pre-compensation level after compensation, which may be related to the proximity to the travel end point, backlash, or local geometric distortion.

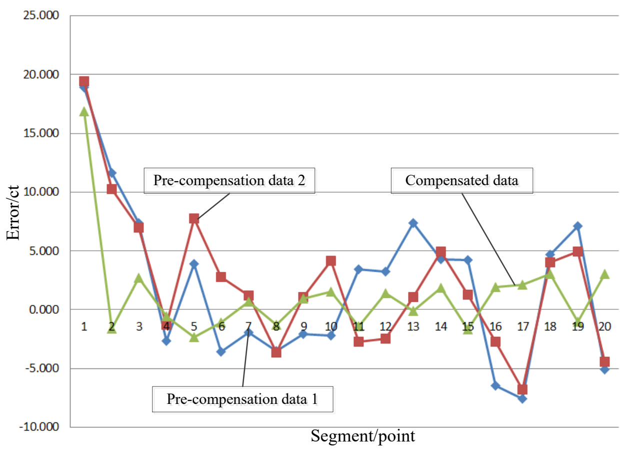

Fig. 4 Comparison of error curves before and after compensation

Fig. 4 shows the error data curves before and after compensation, which illustrates that: 1. The error data of the CNC machine tool at the test points is relatively stable, indicating that the machine tool possesses stable machining performance; 2. Compared to the error data 1 and error data 2 before compensation, the error magnitude of the CNC machine tool was significantly reduced after implementing error compensation; 3. Compared with the variance analysis of errors before compensation on CNC machine tools, the variance after implementing error compensation is smaller, which further demonstrates the effectiveness of the error compensation.

6. Conclusions

1. Based on the kinematics theory of Multibody systems, this study constructs a geometric error model comprising dual branches of "workpiece-tool" and "tool-machine tool," derives mathematical expressions for positional and pose errors, and provides error compensation condition equations for precision machining. Preliminary experimental results using the Z-axis VMC-500 CNC machine tool indicate that the error variance decreased from 6.4534 to 3.061 after compensation, and the error curve tends to stabilize, validating the effectiveness of the established model under specific conditions.

2. The present study is subject to the following limitations: the experiment was conducted only on the single Z-axis of a VMC-500 ml, excluding multi-axis linkage conditions; error compensation considered only static geometric errors, without accounting for the effects of thermal, dynamic, and servo errors; and measurement uncertainty analysis was not performed. Although the proposed universal error model is theoretically applicable to different types of CNC machine tools, its wider validity requires further verification through experiments involving more machine models, multiple axes, and actual machine in conditions. Future research will focus on addressing these limitations.

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L. Yu

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S u m m a r y

Based on the kinematics theory of multibody systems, this paper investigates the error compensation problem of CNC machine tools. By abstracting the true of the CNC machine tool as a multibody system composed of kinematic pairs and components, a complete spatial error model from drive input to tool execution is constructed using homogeneous coordinate transformation MCS, and the generation, accumulation, and coupling mechanisms of various geometric errors are analyzed. The paper details the establishment of coordinate systems in multibody systems, the homogeneous coordinate method for and vectors, and the construction of zero-order motion equations. On this basis, a universal geometric error model with dual branches of "workpiece-tool" and "tool-machine tool" is established, expressions for position and orientation errors are derived, and error compensation condition equations for achieving precision machining are proposed. In preliminary experimental verification, the errors of the Z-axis of a VMC-500 machine tool were measured and compensated using a Renishaw ML10 dual-frequency laser interferometer. The results indicate that the compensated error variance decreased from 6.4534 and 5.8352 to 3.061, the fluctuation range of the error was narrowed, and the method demonstrated potential effectiveness and engineering applicability.

Keywords: kinematics of multibody systems, error compensation of CNC machine tools, homogeneous coordinate transformation, geometric error.

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