ISSN 1392–1207. MECHANIKA. 2014 Volume 20(3): 259–265

Statistical strength criterion for materials with hexagonal close-packed crystal lattice

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crossref http://dx.doi.org/10.5755/j01.mech.20.3.4946

1. Introduction

At present in the design and operation of critical products from non-conventional metal materials their strength and plasticity resources evaluation is actual. In this context, statistical strength and plasticity criteria that allow taking into account the structure and peculiarities of intercrystalline interaction for polycrystalline materials under deformation and fracture have large opportunities from our point of view. It is not the least of the factors that directly in them relation between micro- and macrocharacteristics of the processes analyzed is set via the corresponding averaging methods. This kind of criteria takes an intermediate place between phenomenological and physical approaches. Phenomenological criteria connect strength with various invariants of a tensor of stresses or deformations. The physical approach allows to estimate features of elastic-plastic deformation depending on operating mechanisms of deformation (sliding, doubling) demand enough difficult procedures of calculation for construction of macroscopically characteristics (a curve of deformation and others) [1-3]. Statistical criterion considered in given article, possess simplicity of use in engineering practice and allow to estimate strength precisely enough.

Development of the statistical approach was made in works of Weibull, Frenkel, Fisher and Hollomon, Afanasev, Bolotin, Novozhilov and others [4]. The statistical approach to material strength evaluation proposed by Volkov S.D. [5] is based on the hypothesis that microstresses are distributed by the normal law while fracture occurs when critical probability of exceeding by microstresses responsible for fracture is reached. It was assumed that incipient microcracks appear in the planes with orientation close to the basic area corresponding to the principal macroscopic stress σ_1 while local criterion of fracture is $\xi_{11} \geq \xi_c$. Here ξ_{11} is microstress ($\vec{\xi}_{11} \| \vec{\sigma}_1$) where mathematical expectation is $\langle \xi_{11} \rangle = \sigma_1$ and local strength is $\xi_c = const$. The hypothesis acceptance that critical probability does not depend on stress state kind results in the fact that generalized condition of strength will be expressed as follows:

$$z_c = \frac{\xi_c - \sigma_1}{S(\xi_{11})} = const , \tag{1}$$

that is prescribed integration limit of density function corresponding to the critical probability. The exact criterion kind depends on the function $S(\xi_{11}) = \Phi(\sigma_1, \sigma_2, \sigma_3)$ determining dependence of root-mean-square deviation on macrostress state. Volkov S.D. supposed that dispersion:

$$D(\xi_{11}) = S^2(\xi_{11}) = KW , \tag{2}$$

where W is potential energy of deformation. Having determined $K \cdot z_c$ and ξ_c by tests with two different kinds of stress state, he got two-parameter criterion of strength.

Pisarenko G.S. and Lebedev A.A. proposed to use not the whole of energy but distortion strain energy only having got as well the two-parameter criterion which for the main part of materials describes well the strength for limited range of stress states which are managed to be imitated in laboratory conditions. However, for a number of materials such as columbium alloys, the Pisarenko-Lebedev criterion describes experimental results poor even for the state of plane stress [6].

In the work [6] the statistical approach has been developed due to consideration of various mechanisms of discontinuance (oriented and non-oriented fracture) and use of local strength criteria taking into account the strength anisotropy of grains. Only for materials with low strength anisotropy the crystals of which do not have any cleavage planes or have many families of cleavage planes $\xi_{11} \geq \xi_c$ can be used as local criterion of fracture as they have low strength anisotropy (oriented fracture). To such materials refractory of 5a and 6a groups of Mendeleev periodic system (niobium, vanadium, tantalum, chromium, molybdenum) with cubic crystalline lattice can be related. However, for such materials or rather like for all polycrystals the hypothesis Eq. (2) is not acceptable. We can show this for the case of elastic deformation.

2. Theoretical

Microstresses in the random point of nonuniform micro-sized medium with elastic deformation can be found as the sum:

$$\xi_{11} = \sigma_1 \bar{\xi}_{11}^{(1)} + \sigma_2 \bar{\xi}_{11}^{(2)} + \sigma_3 \bar{\xi}_{11}^{(3)} ,$$

where $\bar{\xi}_{11}^{(k)}$ is microstress caused by single principal macrostress σ_k , then dispersion $D(\xi_{11})$ can be found as sum of three dependent random quantities:

$$D(\xi_{11}) = \sigma_1^2 B_{11}^{11} + \sigma_2^2 B_{11}^{22} + \sigma_3^2 B_{11}^{33} + 2\left(\sigma_1 \sigma_2 B_{11}^{12} + \sigma_1 \sigma_3 B_{11}^{13} + \sigma_2 \sigma_3 B_{11}^{23}\right), \tag{3}$$

where $B_{11}^{k} \equiv B_{11}^{kk} = D\left(\overline{\xi}_{11}^{(k)}\right)$ is dispersions and $B_{11}^{km} = cov\left(\overline{\xi}_{11}^{(k)}\overline{\xi}_{11}^{(m)}\right) = \left\langle \overline{\xi}_{11}^{(k)}\overline{\xi}_{11}^{(m)} \right\rangle - \left\langle \overline{\xi}_{11}^{(k)} \right\rangle \left\langle \overline{\xi}_{11}^{(m)} \right\rangle$ is covariances (correlation moments) of microstresses caused by single macrostresses $\overline{\sigma}_k$ and $\overline{\sigma}_m$. Here, angle brackets mean averaging. In the work [6] by the model of polycrystal based on the deformations homogeneity hypothesis (Voigt hypothesis) and by the finite element model it is shown that for quasi-isotropic crystal:

$$B_{11}^{11} \neq B_{11}^{22} = B_{11}^{33},\ B_{11}^{12} = B_{11}^{13} \neq B_{11}^{23}. \quad (4)$$

Last equation shows the inconsistency of hypothesis (2) because its confirmation requires performance of the following conditions for dispersions $B_{11}^{11} = B_{11}^{22} = B_{11}^{33}$ and covariances $B_{11}^{12} = B_{11}^{13} = B_{11}^{23}$. Moreover $B_{11}^{ij} / B_{11}^{ii} = \nu$, where ν is Poisson coefficient. It should be noted that $\left\langle \overline{\xi}_{11}^{(1)} \right\rangle = 1, \left\langle \overline{\xi}_{11}^{(2)} \right\rangle = \left\langle \overline{\xi}_{11}^{(3)} \right\rangle = 0$.

Dispersions B_{11}^{k} and microstresses covariances B_{11}^{km} arising from the elastic anisotropic grains interaction are defined analytically with use of a hypothesis of strains uniformity, and also numerically for polycrystal model by means of finite elements solution for elastic plane problem for cubic crystals [6-8].

Calculation by a method of final elements of models taking into account anisotropy of properties of grains is made with the most various purposes [9-11]. Here the results received at the decision of a volume problem for materials with close-packed hexagonal space lattice are discussed. For implementation of this approach the finite element model of the polycrystal was used in the form of a thin plate consisting of one layer of grains in the shape of hexagonal prisms (39 hexahedrons and fragments forming a rectangular plate). Grain thickness was equal to diameter of the circle around the hexahedron. Each grain was split into 1193 elements in the form of tetrahedron containing four nodes with three nodal displacements. Anisotropic element type was used for which the elastic properties $C_{i,j}$ are given by the 6x6 matrix. Components of the matrix of elastic properties were determined by conversion of the fourth-rank elastic tensor for different orientations of crystallographic axes of the grain given by Eulerian angles. An angle change pitch was $\pi / 24$ in the range from 0 to $\pi / 2$. Total number of considered orientations of grains was 2197. Orientations of grains of the model were chosen from the resultant aggregation by means of the random number generator. For each material 5 different combinations of orientations of 39 grains were considered. For fastening 7 connections in four points were given. In one angle point of the base $u_1 = u_2 = u_3 = 0$, in the most remote from it base point $u_2 = u_3 = 0$, in two remaining angle points $u_2 = 0$. Such fastening ensures no constraints due to availability of connections and during extension along any axis in the elastic isotropic model monoaxial extension occurs in all elements. During calculation of the polycrystal model complex stress state occurs in the model volume caused by interaction of anisotropic grains. For each orientation two kinds of monoaxial extensions were considered along x_1 axis and along x_2 axis caused by single macroscopic stresses on lateral surfaces $\overline{\sigma}_1$ and $\overline{\sigma}_2$. For every kind of extension six components of microstresses $\overline{\xi_{ij}^{(1)}}$ and $\overline{\xi_{ij}^{(2)}}$ and deformations caused by single macrostresses were determined. For evaluation of representativeness of samplings normal modules of elasticity and Poisson ratios of the model in x_1, x_2 directions were determined. Spread of values of elastic constants of the model depends on the rate of elastic anisotropy of crystals. For majority of materials it was in the range ±5%. The exceptions were cadmium, zinc and graphite (mentioned in ascending order of degree of anisotropy) for which deviations were two or three times more.

For averaging and determination of mathematical expectations $< \overline{\xi}_{ij}^{(k)} >$, dispersions and covariances, difference of volumes of finite elements with consideration of FEM grid topology was taken into account. Here, it was established that $< \overline{\xi}_{ij} > = \overline{\sigma}_{ij}$ as required by boundary conditions. Fig. 1 shows microstress fields $\overline{\xi}_{22}^{(2)}$ for titanium polycrystal in the middle plane of the plate with monoaxial extension $\overline{\sigma}_2$.

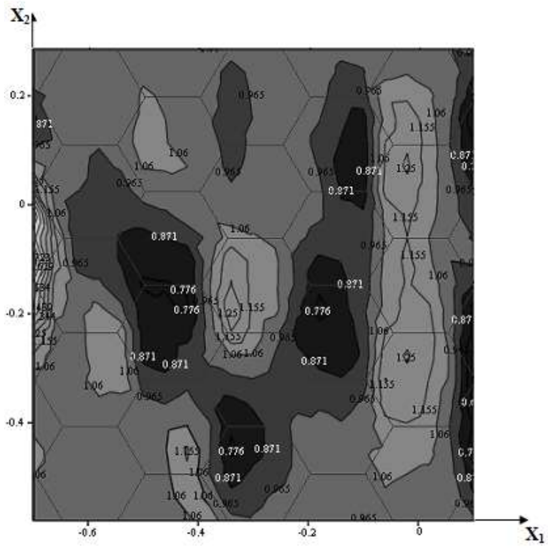

Fig. 1 Change of microstresses $\overline{\xi}_{22}^{(2)}$ in the cross section passing through the middle of the plate; monoaxial extension is; material is titanium

Taking into account the quasi-isotropic conditions (4) determination of average values of dispersions $\left\langle B_{ii}^{ii} \right\rangle$, $\left\langle B_{ii}^{kk} \right\rangle (k \neq i)$ was conducted by averaging of ten values of the corresponding dispersions and for $\left\langle B_{ii}^{ik} \right\rangle (k \neq i)$ of twenty values of covariances for each material got by solving five variants with different orientations of crystallographic axes. It was not possible to determine covariances of B_{11}^{23} using only two kinds of extensions by single macrostresses $\overline{\sigma}_{11} = \overline{\sigma}_{22} = 1$ therefore they were found in assumption that $cov_{23}\left(\bar{\xi}_{11}^{(2)}\bar{\xi}_{11}^{(3)}\right) = cov_{12}\left(\bar{\xi}_{33}^{(1)}\bar{\xi}_{33}^{(2)}\right)$ by averaging 10 values $cov_{12}\left(\bar{\xi}_{33}^{(1)}\bar{\xi}_{33}^{(2)}\right)$.

For each material confidence intervals were determined for mathematical expectation of the corresponding sampling of dispersions and covariances on the assumption that these values are not correlated between themselves. For this purpose dispersions of dispersions $D\left(B_{11}^{11}\right)$, $D\left(B_{11}^{22}\right)$ and dispersion of covariances $D\left(B_{11}^{12}\right)$, $D\left(B_{11}^{23}\right)$ were calculated. Then with probability of 0.95 the confidence interval $b=\sqrt{\left(Z^2 D\left(B_{11}^{km}\right)\right)/N}$ of possible spread of average values of dispersions and covariances was determined. Here Z is quantile of normalized normal law of distribution corresponding to the given probability while N is a number of considered cases (10 for B_{ii}^{ii}, B_{ii}^{jj}, B_{mm}^{ij} $(i, j = 1,2;\ m = 3)$ and 20 for B_{ii}^{kj}.

As it can be seen from the table, the values of *P*, *Q*, *F* parameters for the studied HCP materials differ greatly. From here it appears the necessity of more detailed study of effect of these parameters on strength with complex stress state with application of experimental data.

As can be seen from Fig. 2, microstresses concentration ξ_{11} strongly depends on the stress-strain state which is influenced by the ratios of principal macroscopic stresses σ_2/σ_1 and σ_3/σ_1. Therefore there are strong reasons for the creation of a strength criterion considering influence of a type of stress-strain state on concentration of microstresses. Taking into account Eq. (4), expressing in formulae (1) z_c and ξ_c during two test types with various stress states in the work [6] the statistical criterion of oriented fracture was got:

$$\chi\sqrt{P\sigma_1^2+\sigma_2^2+\sigma_3^2+2Q\left(\sigma_1\sigma_2+\sigma_1\sigma_3\right)+2F\sigma_2\sigma_3}+\left(1-\chi\sqrt{P}\right)\sigma_1=\sigma_p \qquad (5)$$

where $\chi=\dfrac{\sigma_p}{\sigma_c}$, a σ_p, σ_c are true fracture stresses at tension and compression, $P=\dfrac{D\left(\bar{\xi}_{11}^{(1)}\right)}{D\left(\bar{\xi}_{11}^{(2)}\right)}$; $Q=\dfrac{cov_{12}\left(\bar{\xi}_{11}\right)}{\bar{D}\left(\bar{\xi}_{11}^{(2)}\right)}$; $F=\dfrac{cov_{23}\left(\bar{\xi}_{11}\right)}{\bar{D}\left(\bar{\xi}_{11}^{(2)}\right)}$. Taking into account the quasi-isotropic conditions (4) one can see that *F* is correlation coefficient of $\rho_{23}\left(\bar{\xi}_{11}^{(2)}\bar{\xi}_{11}^{(3)}\right)$. It can be shown that $Q/\sqrt{F}$ is correlation coefficient of $\rho_{12}\left(\bar{\xi}_{11}^{(1)}\bar{\xi}_{11}^{(2)}\right)$.

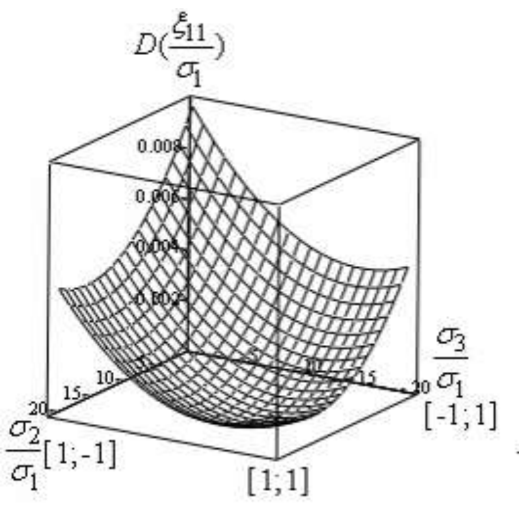

Fig. 2 Normal microstresses dispersion variation in a polycrystal of the titan as a principal stresses function. Calculated on a formula (3) where B_{11}^{ij} values are found from finite elements solution

In the work [6, 7] with the use of analytical and numerical models of the polycrystal *P*, *Q*, *F* parameter values were got for a large number of metals with cubic crystal lattice. Taking into account that for these materials the parameter values got by the finite element method calculation are close to the values *P*=16/9, *Q*=-8/9, *F*=-7/63 got by the model of the polycrystal with the use of the deformation homogeneity hypothesis, for materials with the cubic crystal lattice in the work [7] two-parameter strength criterion was proposed which describes the strength of columbium alloys well.

However, during the tests of polycrystals with the hexagonal close-packed space lattice it was established that these parameter values greatly depend on the test material. This was established both by the model with the use of the deformations homogeneity hypothesis that allows getting the analytical solution [8] and by the method of the finite elements [13]. This is obviously related to the less symmetry of HCP crystals compared with the cubic ones. Therefore, if there is no data on *P*, *Q*, *F* values for materials, one should use the five-parameters criterion in the form Eq. (5) that makes it inconvenient for application.

Earlier in the work [2] specified in our article, we have considered process of change of dispersions of micro stresses and deformations at elastic-plastic deformation. It has been shown that level of concentration of the micro stresses, arising from interaction it is elastic the anisotropic grains, characterised in the variation factor, in α to iron at an input in plastic deformation decreases a little, and then is stabilised and remains at the further deformation.

The fact of preservation of stability of a picture of micronon-uniform deformation at the big plastic deformations is confirmed in a great number of experimental works (Gur'ev A. V., Romanov A. H. and others). In them it is shown an invariance of factors of concentration of the local deformations measured on bases several times smaller, than the size of grain, in a wide range of plastic deformations.

It should be noticed that the statistical criterion includes not value of dispersions, but relative parameters *P*, *Q*, *F*. By working out of criterion of strength it was supposed that process of plastic deformation poorly influences these relative parameters. For this reason it is proposed in a present work to use for the first approximation parameters *P*, *Q* and *F* found from the numerical finite elements calculation of a microtresses field for a polycrystal model for an elastic problem. Results can be specified with use of experimental data on a fracture of cylindrical specimens with cuts of various sharpness which will provide various stress-strain states in the fracture zone. Such approach is often used for an estimation of fracture toughness [14,15], influences of speed of deformation [16] and destruction mechanisms [17] and another.

3. Experimental

Test methods were used that can be easily applied in laboratory conditions namely, tension of cylindrical samples with various sharp circumferential notches. For analysis of stress state in the stress concentration zone the well-known Bridgeman and Davidenkov approaches were used. Comparative analysis of these approaches required building of deformation curves in the coordinates $S_1 = f(e_i)$. Here $S_1 = F/A$ is averaged true stress determined by cross section area A corresponding to the load F; $e_i = 2\ell n(d_0/d_i)$ is true deformation in the stress concentration zone determined by initial d_0 and current diameter of minimum cross section. For tests 5V titanium alloy samples were taken: smooth and with notch radius 2.3; 1.5; 0.85; 0.5 mm with ratio of diameters d/D = 0.707. In addition to this, the test results with six fold resharpening of the sample that were conducted after the neck started forming and Δe_i = 5% plastic deformation was reached that allowed getting the characteristics without considerable influence of the neck shape. The technique, described to work [18] has been used.

Table

Results of calculation of criterion parameters and confidence intervals *b*, ordered by ratio of extreme modulus of crystal elasticity

Parameters	Be	Co	Mg	Ti	Zr	Cd	Zn	Graphite
E_{max}/E_{min}	1.16	1.16	1.17	1.37	1.4	2.7	3.5	6.98
P	2.49	1.12	0.92	0.57	1.55	3.448	2.70	1.89
b_P	0.266	0.219	0.099	0.096	0.210	0.449	0.255	0.882
Q	-1.03	-0.38	-0.282	-0.48	-0.386	-0.83	-0.69	-0.43
b_Q	0.095	0.113	0.065	0.105	0.089	0.125	0.101	0.191
F	-0.12	-0.024	0.11	0.095	0.202	0.175	0.033	-0.28
b_F	0.028	0.077	0.087	0.065	0.039	0.029	0.054	0.12

For approximation of deformation curve of the smooth sample with sixfold resharpening the power law hardening equation $s_i = \sigma_y + K \cdot e_i^n$ was used, where K = 225.16 MPa, n = 0,21395 – hardening parameters; $\sigma_y = 665$ MPa – limit of elasticity. Approximation error is 1.5%. For justification of the calculation method for stress intensity s_i by S_1 the deformation curves for the samples with necks and notches were rebuilt in the coordinates of stress intensity s_i and intensity of deformations $s_i = f(e_i)$. For determination of s_i the Bridgeman and Davidenkov solutions were used. Coefficients of “hardening” k_σ taking into account curvature of the neck causing inhomogeneous stress state in the minimum cross section area for the smooth sample do not differ greatly.

However, it was established that for small radius of curvature in circumferential notches Bridgeman correction ensures better conformity to the single deformation curve in the coordinates $s_i - e_i$. Therefore it is used in further calculations to determine stress intensity:

$$s_i = S_1 / k_\sigma; \quad k_\sigma = (1 + 4R/d)\,\ell n(1 + d/4R), \tag{6}$$

here d, R are diameter and radius of curvature in the neck or in the notch. For circumferential notches diameters and radii were determined both before deformation and after fracture.

For determination of P, Q, F parameters the criterion Eq. (5) is represented in the form convenient for further calculations:

$$s_i^c = s_p^C \Big/ \left(\chi_q \sqrt{P\left(n_1^2 + n_2^2 + n_3^2\right) + 2Q\left(n_1 n_2 + n_2 n_3\right) + 2F\ n_2 n_3} + \left(1 - \chi_q \sqrt{P}\right) n_1 \right), \tag{7}$$

here s_i^c is stress intensity corresponding to fracture in such stress state, $\chi_q = s_p^C / s_s^C$, s_p^C, s_s^C are true fracture stresses at tension and compression; $n_1 = \sigma_1 / s_i$; $n_2 = \sigma_2 / s_i$; $n_3 = \sigma_3 / s_i$ are relative parameters characterizing stress state in the fracture point; $\sigma_1, \sigma_2, \sigma_3$ design stresses according to Bridgeman and Davidenkov; n_1, n_2, n_3 are expressed via rigidity index in stress state $N = (\sigma_1 + \sigma_2 + \sigma_3)/s_i$ and Lode parameter μ_σ characterizing the kind of deviator:

$$n_1 = \left(N + (3 - \mu_\sigma) / \sqrt{3 + \mu_\sigma^2} \right) / 3;$$

$$n_2 = \left(N + 2\mu_\sigma / \sqrt{3 + \mu_\sigma^2} \right) / 3; \; n_3 = \left(N - (3 + \mu_\sigma) / \sqrt{3 + \mu_\sigma^2} \right) / 3.$$

According to Bridgeman for the central part of the cross section in the neck where fracture starts:

$$N = 1 + 3\ell n(1 + d/(4R)), \;\; \mu_\sigma = -1.$$

Experimental and design values of fracture stress S_1 were compared. Satisfactory fit was reached. For more precise definition of parameters variation of P, Q, F, χ_r was made for minimization of the sum of squared difference of theoretical and experimental values of fracture stresses for each type of curvature radii:

$$\sum_{m=1}^{n} Re\left(k_\sigma s_i^c\left(P,\, Q,\, F,\, \chi_r\right) - S_1^{exp}\right)_m^2 = min\,, \tag{8}$$

where stress intensity $s_i^c(P,Q,F,\chi_r)$ is calculated by the criterion Eq. (7) in which N and n_1, n_2 respectively are determined by the value of radius in the notch measured after fracture, n – number of sample types; $Re()$ – means that only real part of the complex number is taken which can be theoretical value of stresses and deformations in the process of iterations when no imposed below given restrictions are provided resulted from the study of deviator and meridian fracture surface cross section. Taking into account that the second derivative sign $d^2F/d\sigma_0^2$ in the meridian cross section will not change in the whole interval of spherical tensor change $[-\infty \ldots \sigma_0^c]$, where σ_0^c is strength with triaxial uniform tension, general solution was got for the restriction in χ_q. The most strict is the restriction for $\mu_\sigma = -1$,

$$0 \le \chi_r < \frac{\sqrt{2}\cdot\sqrt{F+2P-4Q+1}-2\sqrt{P}}{F-4Q+1}\,, \tag{9}$$

that was used in calculations. From the condition of reality of values $d^2F/d\sigma_0^2$ the restriction $F > -\dfrac{P-2Q^2}{P}$ follows.

The other restriction is the expression $P + 4Q + 2F + 2 \ge 0$ that follows from the condition of non-negativity of dispersions of microstresses in any stress state.

After minimization of the expression (8) with the use of the above mentioned restrictions it was got: $\chi_r = 0.45$, P=0.59, Q= - 0.49, F= - 0.17. The values of P and Q parameters are inside of the confidence interval got by calculation of FEM, only F value is beyond the limits got by finite element modeling that is connected with imperfectness of the polycrystal model (one layer of grains). So there are grounds to convert the criterion Eq. (5) in three-parameter one if the calculated P and Q parameters are used.

Accuracy of description of fracture was compared with other strength criteria that are particular cases of the criterion Eq. (5). With the values of the parameters $P = 1$, $Q = F = -0.5$ the criterion Eq. (5) corresponds to the Pisarenko-Lebedev criterion. With $P = 1$, $Q = F = -\nu$, where Poisson ratio ν equal for titanium to 0.32, the criterion Eq. (5) corresponds to the Volkov criterion. With these fixed values of P, Q, F parameters minimization of the expression (8) was performed to find χ parameter for Pisarenko-Lebedev and Volkov criteria. The criteria χ = 0.63 and 1.72 were got, respectively. According to the Volkov criterion tensile strength is considerably higher than compression strength that is not proved by the experiment. Fig. 3 shows meridian cross sections corresponding to $\mu_\sigma = -1$ for the Pisarenko-Lebedev criterion (straight line *1*), for the Volkov criterion with χ =1.72 (curve *2*) and for χ =0.42 (curve *3*) corresponding to the maximum value of restriction Eq. (9). The criterion of oriented fracture Eq. (5) is represented by curve *4* (Fig. 3).

The range of stress states ensured by the samples with circumferential notches from $N = 1$ (smooth sample with resharpenings) to $N = 4.3$ is marked with arrows. It is seen that within it as well as for compression spherical tensors the criterion of oriented fracture and the Pisarenko-Lebedev criterion give approximately the same results. However, in the area of the most dangerous stress states close to uniform tension the Pisarenko-Lebedev criterion enhances strength two point five times more compared with the criterion Eq. (5). So, for this area of stress states there are grounds to use alternative three-parameter criterion that allows higher reliability of calculations.

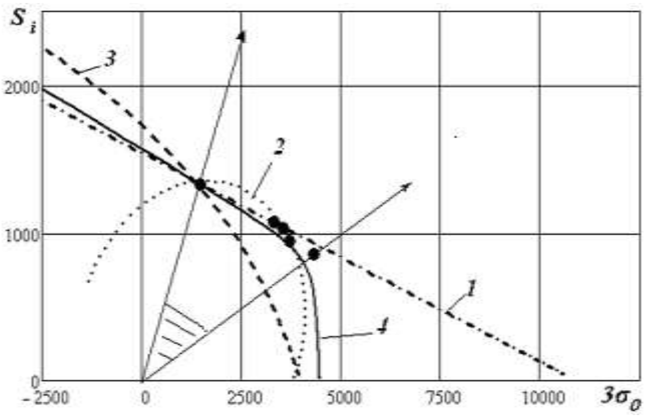

Fig. 3 Meridian cross sections of fracture surface corresponding to $\mu_\sigma = -1$ (tension with accuracy to spherical tensor) for various strength criteria (please find clarifications in the text)

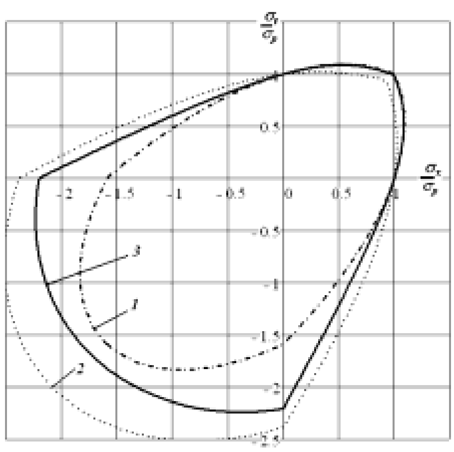

Fig. 4 Fracture contours for the plane stress state for titanium alloy 5B by the Pisarenko-Lebedev criterion (curve *1*), by the Volkov criterion with normalized χ=0.42 (curve *2*) and oriented fracture (curve *3*)

Fig. 4 shows fracture contours in the plane stress state for the criteria considered. As is obvious, strength differences in the area of biaxial tension are not great. Maximum difference observed with $\sigma_1 = \sigma_2$ is less 7%. However, tests of samples with circumferential notches when the conditions of volumetric stress state are implemented, give more 30% differences in strength evaluation by the Volkov criterion and criterion Eq. (5). This points out that for determination of *F* parameters one should use tests not in the plane but in the volumetric stress state that was implemented in the procedure proposed.

4. Conclusions

For polycrystals with hexagonal crystal lattice the statistical strength criterion was proposed that is more reliable in forecasting of strength in any stress state. The procedure of determination of criterion parameters was developed when two of these parameters are determined by FEM calculation on the polycrystal model while three other parameters were determined experimentally on the samples with circumferential notches.

Acknowledgments

The reported study was supported by RFBR, research project № 14-08-00837, and within a basic part of state task of Ministry of Education and Science of the Russian Federation, research project № 2014/16.

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STATISTINIS STIPRUMO KRITERIJUS MEDŽIAGOMS SU SUTANKINTA ŠEŠIAKAMPE KRISTALINE GARDELE

R e z i u m ė

Ištirtos normalinių mikro įtempių koncentracijos priklausomybės kylančios dėl tampriųjų anizotropinių grūdelių poveikio kombinuotam įtempių –deformacijų būviui. Panaudojant gautas priklausomybes polikristalams, kurių grūdeliai neturi skilimo plokštumų, pasiūlyti statistiniai orientuoto lūžio stiprumo kriterijai. Titano eksperimentinių duomenų analizė parodė, kad pasiūlytas kriterijus numato patikimesnę stiprumo prognozę kieto įtempio būviui lyginant su žinomais fenomenologiniais kriterijais. Du pasiūlytų kriterijų parametrai stačiakampio gretasienio centruotos kristalinės gardelės medžiagoms nustatyti polikristaliniam modeliui naudojant baigtinių elementų analizę. Trys kiti parametrai pasiūlyti nustatyti cilindrinių bandinių su įvairių aštrumų antgalių įpjovomis eksperimentiniais tempimo bandymais.

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STATISTICAL STRENGTH CRITERION FOR MATERIALS WITH HEXAGONAL CLOSE-PACKED CRYSTAL LATTICE

S u m m a r y

Dependences of concentration of the normal microstresses arising from interaction of elastic anisotropic grains, on a type of a combined stress-strain state are investigated. With use of the received dependences, for polycrystals which grains have no cleavage planes, the statistical strength criterion of the oriented fracture is suggested. The experimental data analysis for titan has shown that the criterion suggested provides more reliable strength prognosis for rigid stress states in comparison with known phenomenological criteria. Two parameters of a suggested criterion for hexagonal close-packed crystal lattice materials are determined for polycrystal model using the finite element analysis. Three others are suggested to be defined experimentally by means of a tension test performed on cylindrical specimens with notches of a various tip sharpness.

Keywords: statistical strength criterion, microstresses, close-paced hexagonal space lattice.

Accepted August 20, 2013
Received May 08, 2014