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Dynamic characteristics of magnetic fluid based sliding bearings

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1. Introduction

Slider bearings are often designed for sustaining transverse burdens in engineering sciences. For the application to fluid film lubrication, the steady inclined slider bearings lubricated with a conventional viscous fluid (CVF) have been evaluated under different flow conditions [1]. Owing to the development of modern engineering sciences, the increasing use of magnetic fluids as lubricants has been highlighted. Many applications of magnetic fluids are widely observed in dampers, seals, sensors, loudspeakers, gauges, steppers and coating systems [2]. In the area of thin film lubrication, the steady-state performances of porous inclined slider bearings with magnetic fluids in the presence of an externally applied magnetic field have been investigated by Agrawal [3], Ram and Verma [4] and Shah and Bhat [5]. According to their results, the load capacity of inclined slider bearings increases with increasing magnetization of the magnetic fluid when comparing with the case of CVF. All the above investigators [3-5] have used the Jenkins model [6] for the lubricant flow. On the other hand, Shliomis [7, 8] proposed a ferrofluid flow model, in which the effects of rotation of magnetic particles and their magnetic moments are included. Using this Shliomis model, the ferrofluid-based squeeze film characteristics of curved annular plates are examined by Shah and Bhat [9]. It is shown that the volume concentration of magnetic fluids and the strength of applied magnetic fields provide an increase in the load capacity and the approaching time of squeeze films. Recently, further inclusions of inertia force effects on the circular plates have been considered by Lin [10]. Comparing with the non-inertia non-ferrofluid situation, the effects of fluid inertia forces result in a higher load capacity and a longer approaching time for the magnetic fluid-based circular squeeze film. For the slider bearings, the steady performances of plane bearings are analyzed by Shukla and Kumar [11] based on the Shliomis model. According to their results, the effects of rotation of particles and applied magnetic fields increase the load capacity of curved annular plates and inclined slider bearings, respectively. However, all the above investigations [3-5, 9] have contributed the results of magnetic-fluid-based bearings operating under the steady state, where the transverse squeeze motion is neglected. Since the study of dynamic stiffness and damping characteristics is also important in bearing selection, a further study is of interest.

Taking the effects of the squeeze motion and the rotation of magnetic particles into account, the aim of the present study is assess the dynamic characteristics of inclined slider bearings lubricated with magnetic fluids in the presence of transversely uniform magnetic fields. Expressions for the load capacity, stiffness and damping coefficients are derived. Comparing with the case of CVF, the effects of magnetic fields and rotation of magnetic particles on the dynamic characteristics are presented through the variation of the Langevin parameter and the volume concentration parameter.

2. Ferrohydrodynamic lubrication

According to the ferrohydrodynamic flow model by Shliomis [7, 8], the electromagnetic field equations are:

$$\nabla \times \vec{H} = 0 ; \tag{1}$$

$$\nabla \left(\vec{H} + \vec{M} \right) = 0 , \tag{2}$$

where $\vec{H}$ is the applied magnetic field and $\vec{M}$ is the magnetization vector. The incompressible continuity equation is:

$$\nabla \ \vec{v} = 0 , \tag{3}$$

where $\vec{v}$ is the fluid velocity vector. The equations of motion of the magnetic fluid with internal rotation are:

$$\rho \frac{D\vec{v}}{Dt} = -\nabla p + \eta \nabla^2 \vec{v} + \mu_0 \left(\vec{M} \ \nabla \right) \vec{H} + + \frac{1}{2\tau_S} \nabla \times \left(\vec{S} - I\vec{\Omega} \right), \tag{4}$$

where ρ is the fluid density, t is the time, p is the pressure, η is the viscosity of the suspension, μ_0 is the free space permeability, τ_s is the magnetic moment relaxation time, $\vec{S}$ is the internal angular moment and I is the sum of moments of inertia of the particles per unit volume. In addition:

$$\vec{\Omega} = (1/2) \nabla \times \vec{v} ; \tag{5}$$

$$\vec{M} = M_0 \frac{\vec{H}}{H_0} + \frac{\tau_B}{I} \left(\vec{S} \times \vec{M} \right); \tag{6}$$

$$\vec{S} = I\vec{\Omega} + \mu_0 \tau_S \left(\vec{M} \times \vec{H} \right), \tag{7}$$

where M_0 is the equilibrium magnetization and τ_B is the Brownian relaxation time.

Fig. 1 shows the physical configuration of a wide slider bearing lubricated with a magnetic fluid in the presence of a transversely magnetic field, where the upper part has the transverse squeeze motion with a velocity $V = \partial h / \partial t$. It is assumed that the thin film lubrication theory is applicable for the present analysis. In this situation, the above field equations can be simplified.

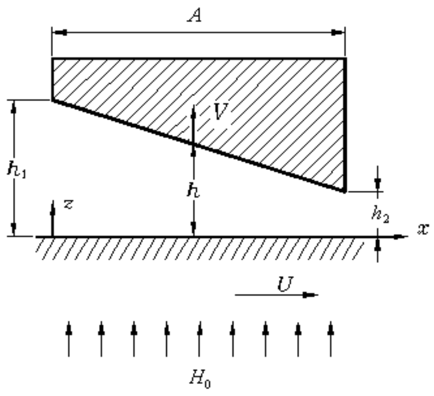

Fig. 1 Configuration of the dynamic slider bearing

According to the reduction and derivation in the squeeze film problems between curved annular plates by Shah and Bhat [9], the x–motion equation gives the form:

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta\left(1+\mu_0 M_0 H_0 \bar{\tau} / 4\eta\right)} \frac{\partial p}{\partial x}, \tag{8}$$

where

$$\bar{\tau} = \frac{\tau_B}{1+\mu_0 \tau_B \tau_s M_0 H_0 / I}. \tag{9}$$

According to the Langevin relationship, the magnetization depends upon the strength of magnetic fields for a system of spherical particles.

$$M_0 = nm\left(\coth\alpha - 1/\alpha\right); \tag{10}$$

$$\alpha = \frac{\mu_0 m H_0}{k_B T}; \tag{11}$$

$$I = 6\eta\phi\tau_s; \tag{12}$$

$$\tau_B = \frac{3\eta\phi}{n k_B T}, \tag{13}$$

where n is the number of particles per unit volume, m is the magnetic moment of a particle, α is the Langevin parameter, k_B is the Boltzmann constant, T is the temperature, and ϕ is the volume concentration of particles. Using Eqs. (10)-(13), the x–motion equation can be expressed in the form:

$$\frac{\partial^2 u}{\partial z^2} = \frac{1}{\eta+\eta_r} \frac{\partial p}{\partial x}, \tag{14}$$

where

$$\eta_r = \frac{3}{2}\eta\phi \frac{\alpha - \tanh\alpha}{\alpha + \tanh\alpha}. \tag{15}$$

The non-slip boundary conditions for the velocity component are: $u = U$ at $z = 0$ and $u = 0$ at $z = h$. Integrating Eq. (14), the expression of U can be obtained:

$$u = \frac{1}{2\left(\eta+\eta_r\right)} \frac{\partial p}{\partial x}\left(z^2 - hz\right) + U\left(1-\frac{z}{h}\right). \tag{16}$$

The integral form of the continuity Eq. (3) for the present one-dimensional slider bearing is:

$$\int_{z=0}^{h} \frac{\partial u}{\partial x} dz = -\frac{\partial h}{\partial t}. \tag{17}$$

Using the expression of u, the modified Reynolds equation can be derived after performing the integration.

$$\frac{\partial}{\partial x}\left\{\frac{h^3}{\eta+\eta_r}\frac{\partial p}{\partial x}\right\} = 6U\frac{\partial h}{\partial x} + 12\frac{\partial h}{\partial t}. \tag{18}$$

According to the ferrohydrodynamic flow model by Shliomis [7, 8], the viscosity of the suspension η is approximated by the Einstein formula:

$$\eta = \eta_0\left(1+\frac{5}{2}\phi\right), \tag{19}$$

where η_0 denotes the viscosity of the carrier liquid. Using Eqs. (15) and (19), the modified Reynolds equation is then rewritten in the form:

$$\frac{\partial}{\partial x}\left\{\frac{h^3}{\eta_0\left(1+\frac{5}{2}\phi\right)\left(1+\frac{3}{2}\phi\frac{\alpha-\tanh\alpha}{\alpha+\tanh\alpha}\right)}\frac{\partial p}{\partial x}\right\} = 6U\frac{\partial h}{\partial x} + 12\frac{\partial h}{\partial t}. \tag{20}$$

This modified Reynolds equation can be applied to the study of ferrohydynamic slider bearings taking into account the effects of the transverse squeeze motion and the rotation of magnetic particles, in which the film shape is $h = h\left(x,t\right)$.

3. Inclined slider bearing characteristics

For the one-dimensional inclined slider bearings displayed in Fig. 1, the film thickness can be expressed as:

$$h\left(x,t\right) = \left(h_{10} - h_{20}\right)\left(1-\frac{x}{A}\right) + h_2\left(t\right). \tag{19}$$

Introduce the non-dimensional variables and parameters:

$$x^* = \frac{x}{A};\; h_2^* = \frac{h_2}{h_{20}};\; h^* = \frac{h}{h_{20}};\; r = \frac{h_{10}}{h_{20}}; \tag{20}$$

$$p^* = \frac{p h_{20}^{\,2}}{\eta_0 U A};\; t^* = \frac{Ut}{A};\; V^* = \frac{dh_2^{\,*}}{dt^*}. \tag{21}$$

Then the modified Reynolds equation and film thickness can be expressed in a non-dimensional form:

$$\frac{\partial}{\partial x^*}\left\{h^{*3}\frac{\partial p^*}{\partial x^*}\right\} = -6\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)\left(r-1\right) + 12\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)V^*; \tag{22}$$

$$h^*\left(x^*,t^*\right)=\left(r-1\right)\left(1-x^*\right)+h_2^*\left(t^*\right), \quad (23)$$

where

$$\beta=\frac{3}{2}\phi\frac{\alpha-tanh\,\alpha}{\alpha+tanh\,\alpha}. \quad (24)$$

The boundary conditions for the film pressure are: $p^*=0$ at $x^*=0$ and $x^*=1$. Integrating the modified Reynolds equation, one can obtain:

$$p^*=\left\{-6\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)\left[\left(r-1\right)-2V^*\right]\right\}\times$$
$$\times g_1\left(x^*,h_2^*\right)+k\;g_2\left(x^*,h_2^*\right); \quad (25)$$

where

$$g_1\left(x^*,h_2^*\right)=\int_{x^*=0}^{x^*=x^*}\frac{x^*}{h^{*3}}dx^*; \quad (26, a)$$

$$g_2\left(x^*,h_2^*\right)=\int_{x^*=0}^{x^*=x^*}\frac{1}{h^{*3}}dx^*; \quad (26, b)$$

$$k\left(h_2^*,v^*\right)=6\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)\times$$
$$\times\left[\left(r-1\right)-2V^*\right]\frac{g_{11}\left(h_2^*\right)}{g_{21}\left(h_2^*\right)}; \quad (26, c)$$

$$g_{11}\left(h_2^*\right)=g_1\left(x^*=1,h_2^*\right); \quad (26, d)$$

$$g_{21}\left(h_2^*\right)=g_2\left(x^*=1,h_2^*\right). \quad (26, e)$$

The ferrohydrodynamic film force is obtained by integrating the film pressure:

$$f=\int_{x=0}^{A}p\,Bdx, \quad (27)$$

where B denotes the width of the bearing. After performing the integration, the ferrohydrodynamic film force can be expressed in a non-dimensional form:

$$f^*=\frac{f\,h_{20}^{\;2}}{\eta_0UA^2B}=-6\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)\times$$
$$\times\left[\left(r-1\right)-2V^*\right]G_1\left(h_2^*\right)+k\;G_2\left(h_2^*\right), \quad (28)$$

where

$$G_1\left(h_2^*\right)=\int_{x^*=0}^{x^*=1}g_1\left(x^*,h_2^*\right)dx^*; \quad (29, a)$$

$$G_2\left(h_2^*\right)=\int_{x^*=0}^{x^*=1}g_2\left(x^*,h_2^*\right)dx^*. \quad (29, b)$$

The steady load is obtained by taking the film force under the steady state “0”: $h_{20}^{\;*}=const$, $V_0^{\;*}=0$.

$$f_0^*=-6\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)\left(r-1\right)\left(G_1\right)_0+\left(k\right)_0\left(G_2\right)_0. \quad (30)$$

The stiffness coefficient S_d and the damping coefficient B_d are obtained by performing the partial derivatives of f with h_2 and V, respectively, and then taking their results under the steady state. Expressing in a non-dimensional form gives:

$$S_d^{\;*}=\frac{S_dh_{20}^{\;3}}{\eta_0UA^2B}=6\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)\left(r-1\right)\times$$
$$\times\left(\frac{\partial G_1}{\partial h_2^*}\right)_0-\left(\frac{\partial k}{\partial h_2^*}\right)_0\left(G_2\right)_0-\left(k\right)_0\left(\frac{\partial G_2}{\partial h_2^*}\right)_0; \quad (31)$$

$$B_d^{\;*}=\frac{B_dh_{20}^{\;3}}{\eta_0A^3B}=-12\left(1+\frac{5}{2}\phi\right)\left(1+\beta\right)\left(G_1\right)_0-$$
$$-\left(\frac{\partial k}{\partial v^*}\right)_0\left(G_2\right)_0. \quad (32)$$

4. Results and discussion

According to the above derivation, two parameters are observed to influence the steady load capacity and dynamic characteristics of the inclined slider bearings. First the concentration parameter ϕ indicates the amount of volume concentration of particles ϕ. For $\phi=0$, the case of a non-magnetic fluid or a CVF is recovered. Second, the Langevin parameter α measures the strength of an applied magnetic field. For $\alpha=0$, the case without magnetic fields is obtained. To present the bearing characteristics, the results are illustrated for the bearing operating under the steady inlet and outlet film ratio, $r=h_{10}/h_{20}=2$.

Table

Comparison of the steady load capacity $f_0^{\;*}$ with the CVF results by Taylor and Dowson [1]

At r = 2: CVF [1]				
0.1587				
At r = 2: present study				
	$\alpha=0$	$\alpha=5$	$\alpha=15$	$\alpha=25$
$\phi=0$	0.1589	0.1589	0.1589	0.1589
$\phi=0.1$	0.1986	0.2185	0.2247	0.2261

Table shows the comparison of the load capacity of the present study with the results of a CVF. For $\phi=0$, the load of the present result (0.1589) is closed to the load (0.1587) by Taylor and Dowson [1]. By the use of a magnetic fluid with $\phi=0.1$ as the lubricant without applied magnetic fields ($\alpha=0$), an increase of the load (0.1986) for the inclined slider bearing is predicted. When the uniform magnetic fields ($\alpha=5$, 15, 25) are applied to the bearing, further increments of the load are obtained.

Fig. 2 presents the stiffness coefficient $S_d^{\;*}$ versus ϕ for various α. The result for a VCF is also included. Without magnetic fields ($\alpha=0$), the bearing stiffness increases with the value of ϕ. In the presence of transversely magnetic fields ($\alpha=5$, 10, 15, 20, 25), higher stiffness coefficients are observed for the bearing.

Fig. 3 displays the damping coefficient $B_d^{\;*}$ versus α for various values of ϕ. Comparing with case of a CVF, the bearing with a magnetic fluid $\phi=0.1$ is found to result in a higher damping coefficient. It is also observed that better damping characteristics are achieved for the magnetic-fluid-based inclined slider bearing with larger values of the Langevin parameter α and the volume concentration parameter ϕ.

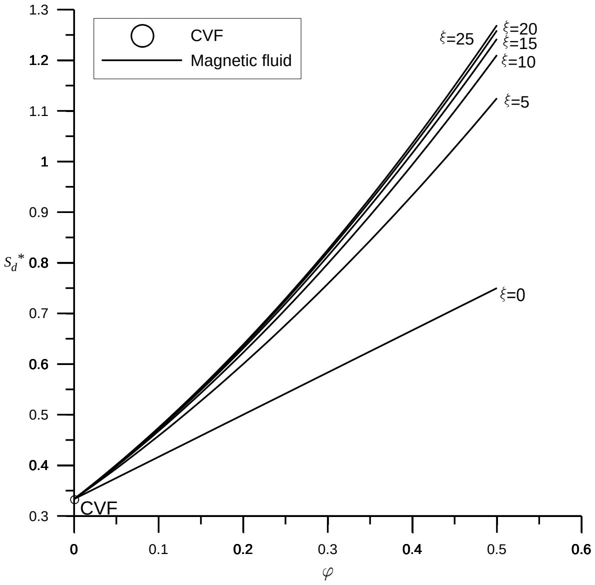

Fig. 2 Non-dimensional stiffness coefficient S_d^* for various values of α

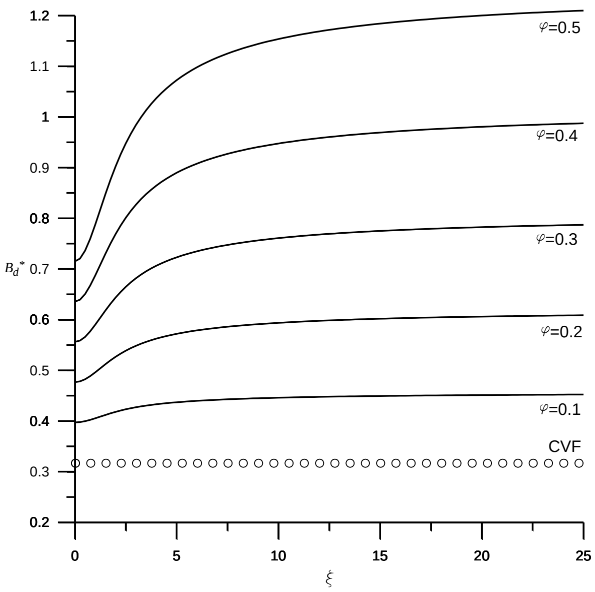

Fig. 3 Non-dimensional damping coefficient B_d^* for various values of ϕ

5. Conclusions

A magnetic-fluid-lubricated Reynolds equation for slider bearings have been derived in the present paper. This derived lubrication can applied to the study of dynamic characteristics of ferrofluid-based slider bearings, where the general film shape is $h = h(x,t)$.

Comparing with the conventional viscous fluid case, the magnetic fluid-based inclined slider bearing provides an improvement in the load carrying capacity, and the dynamic stiffness and damping characteristics. When the the volume concentration parameter is equal to zero, the steady load of the present result agrees closely with the result that predicted by Taylor and Dowson [1]. This close agreement provides a support of the present study.

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References

1 **Taylor, C.M.; Dowson D.** 1974. Turbulent lubrication theory – application to design, J. Lubr. Tech. 96: 36-46. http://dx.doi.org/10.1115/1.3451905.
2 **Raj, K.; Moskowitz, B.; Casciari, R.** 1995. Advances in ferrofluid technology. J. Magn. Magn. Mater. 149: 174-180. http://dx.doi.org/10.1016/0304-8853(95)00365-7.
3 **Agrawal, V.K.** 1991. Magnetic-fluid-based porous inclined slider bearing, Wear 151: 123-128. http://dx.doi.org/10.1016/0043-1648(91)90352-U.
4 **Ram, P.; Verma, P.D.S.** 1990. Ferrofliuid lubrication in porous inclined slider bearing, Indian Journal of Pure and Applied Mathematics 30: 1273-1281.
5 **Shah, R.C.; Bhat, M.V.** 2002. Ferrofliuid lubrication in porous inclined slider bearing with velocity, Int. J. Mech. Sci. 44: 2495-2502. http://dx.doi.org/10.1016/S0020-7403(02)00187-X.
6 **Jenkins, J.T.** 1972. A theory of magnetic fluids, Arch. Rational Mech. Anal. 46: 42-60. http://dx.doi.org/10.1007/BF00251867.
7 **Shliomis, M.I.** 1972. Effective viscosity of magnetic suspensions, Soviet Physics Jetp 34: 1291-1294.
8 **Shliomis, M.I.** 1974. Magnetic fluids, Soviet Physics Usp 17: 153-169. http://dx.doi.org/10.1070/PU1974v017n02ABEH004332.
9 **Shah, R.C.; Bhat, M.V.** 2005. Ferrofluid squeeze film between curved annular plates including rotation of magnetic particles, J. Eng. Math. 51: 317-324. http://dx.doi/10.1007s10665-004-1770-9.
10 **Lin, J.R.** 2012. Derivation of ferrofluid lubrication equation of cylindrical squeeze films with convective fluid inertia forces and application to circular disks, Tribology International 49: 110-115. http://dx.doi.org/10.1016/j.triboint.2011.11.006.
11 **Shukla, J.B.; Kumar, D.** 1987. A theory for ferromagnetic lubrication, J. Magn. Magn. Mater. 65: 375-378. http://dx.doi.org/10.1016/0304-8853(87)90076-8.

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SLYDIMO GUOLIŲ SU MAGNETINIU SKYSČIU DINAMINĖS CHARAKTERISTIKOS

R e z i u m ė

Šiame straipsnyje pateikti magnetiniu skysčiu tepamų slydimo guolių dinamikos tyrimų rezultatai. Slydimo guoliui su pasvirusiu plėvelės sluoksniu tepti parinktas magnetinis skystis, garantuojantis didesnes apkrovas ir standumo bei slopinimo koeficientus, palyginti su tradiciniais klampiaisiais skysčiais. Dinaminėms slydimo guolių charakteristikoms pagerinti tepant magnetiniu skysčiu kartu panaudojamas ir magnetinis laukas.

Jaw-Ren Lin

DYNAMIC CHARACTERISTICS OF MAGNETIC FLUID BASED SLIDING BEARINGS

S u m m a r y

The dynamic behaviors of magnetic fluid-lubricated slider bearings have been investigated in this paper. For the slider bearing with an inclined film profile, the use of magnetic fluids as lubricants is found to provide an increase in the load carrying capacity, and the stiffness and damping coefficients as compared to the case of a conventional viscous fluid. The dynamic characteristics of slider bearings are improved by using magnetic fluid lubricants together with the application of magnetic fields.

Keywords: ferrofluids, sliding bearings.

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