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Thermal radiation and chemical reaction effects on MHD mixed convection heat and mass transfer in a micropolar fluid

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Nomenclature

A - characteristic velocity, B - reciprocal of characteristic length, B_0 - magnetic field coefficient, Kg/(s^{2}A); C - concentration, mol/m^3; C_p - specific heat at constant pressure, J/(kg K); C_∞ - ambient concentration, mol/m^3; C_f - skin-friction coefficient, D - mass diffusivity, m^2/s; E - reciprocal of the product of characteristic length and time, f - dimensionless stream function, g - dimensionless microrotation, g^* - acceleration due to gravity, m/s^2; j - dimensional micro-inertia density, kg/m^3; ϑ - dimensionless micro-inertia density, k - thermal conductivity, W/(m K); k_1 - mean absorption coefficient, 1/m; L - buoyancy parameter, m_w - wall couple stress, Pa/m; M - magnetic parameter, M_1 - the ratio (temperature/length), Km^{-1}; M_w - non-dimensional couple stress on the wall, N - coupling parameter, N_1 - ratio (concentration/length), mol/m^4; Nu - dimensionless Nusselt number, Pr - Prandtl number, q_r - radiative heat flux, W/m^2; R - thermal radiation parameter, W; R^* - rate of chemical reaction, mol/(ms); Sc - Schmidt number, Sh - Sherwood number, T - temperature of the fluid, K; T_∞ - ambient temperature, K; $U(x)$ - ambient velocity, m/s; u, v - velocity components in the x - and y - directions respectively, m/s; x, y - cartesian coordinates along the plate and normal to it, m; α - thermal diffusivity, m^2/s; β_T - coefficient of thermal expansion, K^{-1}; β_C - coefficient of concentration expansion, K^{-1}; γ - spin-gradient viscosity, m^2/s; δ - non-dimensional chemical reaction parameter, η - similarity variable, m; θ - dimensionless temperature, κ -thermal conductivity of the fluid, W/(mK); λ - non-dimensional spin-gradient viscosity, μ - viscosity of the fluid, kg/(ms); υ - kinematic viscosity, m^2/s; ρ - density of the fluid, kg/m^3; σ - electrical conductivity, s^{3}A^2/(kgm^3.); σ^* - Stefan-Boltzmann constant, W/(m^{2}K^4); τ_w - wall shear stress, Pa; ϕ - dimensionless concentration, ψ - stream function, ω - microrotation component, kg.m^2/s; w - condition at wall, ∞ - condition at infinity, ' - differentiation with respect to η.

1. Introduction

Mixed convection flows are of great interest because of their various engineering, scientific, and industrial applications in heat and mass transfer. Mixed convection of heat and mass transfer occurs simultaneously in the fields of design of chemical processing equipment, formation and dispersion of fog, distributions of temperature, moisture over agricultural fields, groves of fruit trees, and damage of crops due to freezing and pollution of the environment. Extensive studies of mixed convection heat and mass transfer of a non-isothermal vertical surface under boundary layer approximation have been undertaken by several authors. The majority of these studies dealt with the traditional Newtonian fluids. It is well known that most fluids which are encountered in chemical and allied processing applications do not satisfy the classical Newton's law and are accordingly known as non-Newtonian fluids. Due to the important applications of non-Newtonian fluids in biology, physiology, technology, and industry, considerable efforts have been directed towards the analysis and understanding of such fluids. A number of mathematical models have been proposed to explain the rheological behavior of non-Newtonian fluids. Among these, the fluid model introduced by Eringen [1] exhibits some microscopic effects arising from the local structure and micro motion of the fluid elements. Further, they can sustain couple stresses and include classical Newtonian fluid as a special case. The model of micropolar fluid represents fluids consisting of rigid, randomly oriented (or spherical) particles suspended in a viscous medium where the deformation of the particles is ignored. Micropolar fluids have been shown to accurately simulate the flow characteristics of polymeric additives, geomorphologic sediments, colloidal suspensions, haematological suspensions, liquid crystals, lubricants etc. The mathematical theory of equations of micropolar fluids and applications of these fluids in the theory of lubrication and in the theory of porous media are presented by Lukaszewicz [2]. The heat and mass transfer in micropolar fluids is also important in the context of chemical engineering, aerospace engineering and also industrial manufacturing processes. The problem of mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by a number of investigators [3-7].

In recent years, several simple boundary layer flow problems have received new attention within the more general context of magnetohydrodynamics (MHD). Several investigators have extended many of the available boundary layer solutions to include the effects of magnetic fields for those cases when the fluid is electrically conducting. The study of MHD flow for an electrically conducting fluid past a heated surface has important applications in many engineering problems such as plasma studies, petroleum industries, MHD power generators, cooling of nuclear reactors, the boundary layer control in aerodynamics, and crystal growth. In addition, there has been a renewed interest in studying MHD flow and heat transfer in porous media due to the effect of magnetic fields on flow control and on the performance of many systems using electrically conducting fluids. The problem of MHD mixed convection heat and mass transfer in the boundary layer flow along a vertical surface submerged in a micropolar fluid has been studied by several investigators. Kim and Fedorov [8] considered the case of mixed convection flow of a micropolar fluid past a semi-infinite moving vertical porous plate with varying suction velocity normal to the plate in the presence of radiation. Seddeek et al. [9] investigated the analytical solution for the effect of radiation on flow of a magneto-micropolar fluid past a continuously moving plate with suction and blowing. Mahmoud et al. [10] analyzed the effects of slip and heat generation/absorption on MHD mixed convection flow of a micropolar fluid over a heated stretching surface. Hayat et al. [11] studied the effects of heat and mass transfer on the mixed convection flow of a MHD micropolar fluid bounded by a stretching surface using Homotopy analysis method. Das [12] considered the effects of partial slip on steady boundary layer stagnation point flow of an electrically conducting micropolar fluid impinging normally towards a shrinking sheet in the presence of a uniform transverse magnetic field.

Radiation effects on convective heat transfer and MHD flow problems have assumed an increasing importance in electrical power generation, astrophysical flows, solar power technology, space vehicle re-entry and other industrial areas. Since the solution for convection and radiation equation is quite complicated, are few studies about simultaneous effect of convection and radiation for internal flows. Ghaly [13] discussed the effect of radiation on heat and mass transfer over a stretching sheet in the presence of a magnetic field. Raptis et al. [14] studied the effect of radiation on two-dimensional steady MHD optically thin gray gas flow along an infinite vertical plate taking into account the induced magnetic field.

Chemical reaction effects on heat and mass transfer are of considerable importance in hydrometallurgical industries and chemical technology. Research on combined heat and mass transfer with chemical reaction and thermophoresis effect can help to design for chemical processing equipment, formation and dispersion of fog, distribution of temperature and moisture over agricultural fields as well as groves of fruit trees, damage of crops due to freezing, food processing, cooling towers, chemically-reactive vapour deposition boundary layers in optical materials processing, catalytic combustion boundary layers, chemical diffusion in disk electrode modelling and carbon monoxide reactions in metallurgical mass transfer and kinetics. Deka et al. [15] have examined the effect of homogeneous first-order chemical reaction on the flow past an impulsively started infinite vertical plate with uniform heat flux and mass transfer. Chamkha [16] have analyzed the MHD flow of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. The problem of chemically reactive species of non-Newtonian fluid in a porous medium over a stretching sheet was investigated by Akyildiz et al. [17]. Bakr [18] considered the steady and unsteady MHD micropolar flow and mass transfers flow with constant heat source in a rotating frame of reference in the presence of the first-order chemical reaction, taking an oscillatory plate velocity and a constant suction velocity at the plate.

Motivated by the investigations mentioned above, the aim of this investigation is to consider the effects of transverse magnetic field, coupling number, radiation and first order chemical reaction on the mixed convection heat and mass transfer along a vertical plate with variable wall temperature and concentration conditions embedded in a micropolar fluid. The governing system of partial differential equations is transformed into a system of non-linear ordinary differential equations using similarity transformations. This system of nonlinear ordinary differential equations is solved numerically using Keller - box method given in Cebeci and Bradshaw [19]. The effects of various parameters on the skin friction coefficient, wall couple stress are given in the form of a table and the effects of various parameters on the heat and mass transfer rates are presented graphically.

2. Mathematical formulation

Consider a steady, laminar, incompressible, two-dimensional mixed convective heat and mass transfer along a semi infinite vertical plate embedded in a free stream of electrically conducting micropolar fluid with velocity $U(x)$, temperature T_∞ and concentration C_∞. Choose the co - ordinate system such that x - axis is along the vertical plate and y - axis normal to the plate. The physical model and coordinate system are shown in Fig. 1. The plate is maintained at temperature $T_w(x)$ and concentration $C_w(x)$. These values are assumed to be greater than the ambient temperature T_∞ and concentration C_∞ at any arbitrary reference point in the medium (inside the boundary layer). A uniform magnetic field of magnitude B_0 is applied normal to the plate. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The fluid has constant properties except the density in the buoyancy term of the balance of momentum equation. The fluid is considered to be a gray, absorbing emitting radiation but non-scattering medium and the Rosseland approximation [20] is used to describe the radiative heat flux in the energy equation. The radiative heat flux in the x - direction is considered negligible in comparison to the y - direction. The viscous dissipation and the Joule heating effects are assumed to be negligible in the energy equation. Also, it is assumed that there exists a homogenous chemical reaction of first-order with rate constant R^* between the diffusing species and the fluid.

Using the Boussinesq and boundary layer approimations, the governing equations for the micropolar fluid are given by [21, 3-7]:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0\,; \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U(x)\frac{d}{dx}U(x)+\frac{\mu+\kappa}{\rho}\frac{\partial^2 u}{\partial y^2}+\frac{\kappa}{\rho}\frac{\partial \omega}{\partial y}+ \\ +g^*\left(\beta_T\left(T-T_\infty\right)+\beta_C\left(C-C_\infty\right)\right)+\frac{\sigma B_0^2}{\rho}\left(U(x)-u\right); \tag{2}$$

$$\rho j\left(u\frac{\partial \omega}{\partial x}+v\frac{\partial \omega}{\partial y}\right)=\gamma\frac{\partial^2 \omega}{\partial y^2}-\kappa\left(2\omega+\frac{\partial u}{\partial y}\right); \tag{3}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho C_p}\frac{\partial q_r}{\partial y}; \tag{4}$$

$$u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-R^*\left(C-C_\infty\right). \tag{5}$$

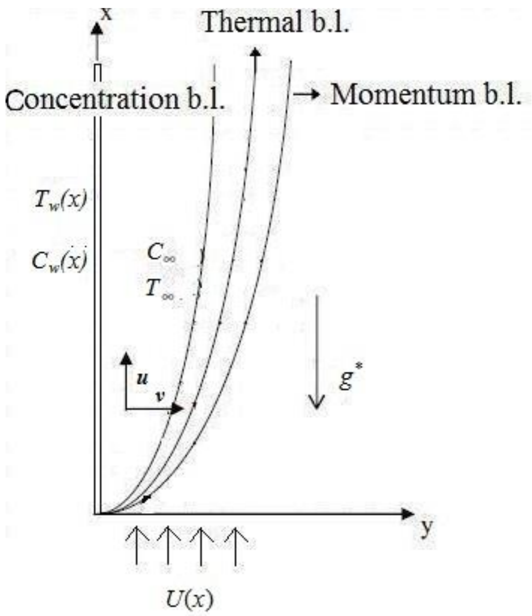


Fig. 1 Physical model and coordinate system

where u and v are the components of velocity along x - and y - directions respectively, p is the pressure, ω is the component of microrotation whose direction of rotation lies in the xy - plane, g^* is the gravitational acceleration, T is the temperature, C is the concentration, β_T is the coefficient of thermal expansion, β_C is the coefficient of solutal expansion, C_p is the specific heat capacity, B_0 is the coefficient of the magnetic field, μ is the dynamic coefficient of viscosity of the fluid, q_r is the radiative heat flux, ρ is the density, κ is the vortex viscosity, j is the micro-inertia density, γ is the spin-gradient viscosity, σ is the magnetic permeability of the fluid, α is the thermal diffusivity, D is the molecular diffusivity and R^* is rate of chemical reaction.

The boundary conditions are:

$$u=v=\omega=0,\ T=T_w\left(x\right),\ C=C_w\left(x\right)\quad \text{at}\quad y=0; \tag{6a}$$

$$u\to U\left(x\right),\ \omega\to 0,\ T\to T_\infty,\ C\to C_\infty\quad \text{as}\quad y\to\infty, \tag{6b}$$

where the subscripts w and ∞ indicates the conditions at wall and at the outer edge of the boundary layer, respectively. The boundary condition $\omega = 0$ in Eq. (6a), represents the case of concentrated particle flows in which the micro elements close to the wall are not able to rotate, due to the no - slip condition.

The radiative heat flux q_r is described by the Rosseland approximation such that:

$$q_r=-\frac{4\sigma^*}{3k_1}\frac{\partial T^4}{\partial y}, \tag{7}$$

where σ^* and k_1 are the Stefan - Boltzmann constant and the mean absorption coefficient respectively. We assume that the differences of the temperature within the flow are sufficiently small such that T^4 may be expressed as a linear function of the temperature. This is accomplished by expanding in a Taylor series about T_∞ and neglecting higher - order terms, thus:

$$T^4\cong 4T_\infty^3 T-3T_\infty^4. \tag{8}$$

Using Eqs. (7) and (8) in the last term of Eq. (4), we obtain:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}+\frac{16\sigma^* T_\infty^3}{3\rho C_p k_1}\frac{\partial^2 T}{\partial y^2}. \tag{9}$$

The continuity Eq. (1) is satisfied by introducing the stream function ψ such that:

$$u=\frac{\partial\psi}{\partial y};\quad v=-\frac{\partial\psi}{\partial x}. \tag{10}$$

In order to explore the possibility for the existance of similarity, we assume:

$$\left.\begin{aligned}&\psi=Ax^a f\left(\eta\right);\eta=Bx^b y;\omega=Ex^e g\left(\eta\right);\\&\frac{T-T_\infty}{T_w\left(x\right)-T_\infty}=\theta\left(\eta\right);T_w\left(x\right)-T_\infty=M_1Bx^l;\\&\frac{C-C_\infty}{C_w\left(x\right)-C_\infty}=\varphi\left(\eta\right);C_w\left(x\right)-C_\infty=N_1Bx^m,\end{aligned}\right\} \tag{11}$$

where $U(x) = ABx^n$, a, b, e, l, m and n are constants, A, B, E, M_1, N_1, are, yet unknown, constants. Substituting (10) and (11) in (2), (3), (4) and (9), we obtain:

$$A^2B^2x^{2a+2b-1}\left[\left(a+b\right)^2\left(f'\right)^2-aff''\right]=A^2B^2nx^{2n-1}+\frac{\mu+\kappa}{\rho}AB^3x^{a+3b}f'''+$$
$$+\frac{\kappa}{\rho}BEx^{b+e}g'+g^*\left[\beta_T M_1x^l\theta+\beta_C N_1x^m\varphi\right]-\frac{\sigma B_0^2}{\rho}\left(ABx^n-ABx^{a+b}f'\right); \tag{12}$$

$$ABEx^{a+b+e-1}\left[ef'g-afg'\right]=\frac{\gamma}{\rho j}B^2Ex^{2b+e}g''-\frac{\kappa}{\rho j}\left(2Ex^e g+AB^2x^{a+2b}f''\right); \tag{13}$$

$$ABM_1x^{a+b+l-1}\left[lf'\theta-af\theta'\right]=M_1B^2x^{2b+l}\left[\alpha+\frac{16\sigma T_\infty^3}{3\rho C_p k_1}\right]\theta''; \tag{14}$$

$$ABN_1x^{a+b+m-1}\left[mf'\phi-af\phi'\right]=DN_1B^2x^{2b+m}\phi''-R^*N_1x^m\phi, \tag{15}$$

where the prime denotes the differentiation with respect to η. It is found that similarity exists only if:

$$\left.\begin{aligned}&2a+2b-1=2n-1=a+3b=b+e=l=\\&=m=n=a+b;\\&a+b+e-1=2b+e=e=a+2b;\\&a+b+l-1=2b+l;\\&a+b+m-1=2b+m=m.\end{aligned}\right\}\quad(16)$$

These relations give:

$$b=0,\ a=e=l=m=n=1. \quad (17)$$

Hence, appropriate similarity transformations are:

$$\left.\begin{aligned}&\psi=Ax\,f(\eta);\ \eta=By;\ \omega=Exg(\eta);\\&\frac{T-T_\infty}{T_w(x)-T_\infty}=\theta(\eta);\ T_w(x)-T_\infty=M_1Bx;\\&\frac{C-C_\infty}{C_w(x)-C_\infty}=\varphi(\eta);\ C_w(x)-C_\infty=N_1Bx.\end{aligned}\right\}\quad(18)$$

Making use of the dimensional analysis, the constants A, B, E, M_1 and N_1 have, respectively, the dimensions of velocity, reciprocal of length, the reciprocal of the product of length and time, the ratio of (temperature/length) and of the ratio (concentration/length) and are defined as below:

$$\left.\begin{aligned}&A=\left(\upsilon^2M_1g^*\beta_T\right)^{\frac14};\ B=\left(\frac{M_1g^*\beta_T}{\upsilon^2}\right)^{\frac14};\\&E=\left(\frac{M_1g^*\beta_T}{\upsilon^2}\right)^{\frac14}.\end{aligned}\right\}\quad(19)$$

In view of (17) and (19) substituting (18) into Eqs. (12) – (15), we obtain:

$$\left(\frac{1}{1-N}\right)f'''+\left(\frac{N}{1-N}\right)g'-(f')^2+\theta+L\varphi+ +ff''+M(1-f')+1=0; \quad (20)$$

$$\lambda g''-\left(\frac{N}{1-N}\right)\vartheta\,(2g+f''')-f'g+fg'=0; \quad (21)$$

$$\frac{1}{3Pr}(3+4R)\theta''+f\theta'-f'\theta=0; \quad (22)$$

$$\frac{1}{Sc}\phi''-\delta\phi+f\phi'-f'\phi=0, \quad (23)$$

where $Pr=\frac{\nu}{\alpha}$ is the Prandtl number, $Sc=\frac{\nu}{D}$ is the Schmidt number, $\lambda=\frac{\gamma}{j\rho\nu}$ is the spin-gradient viscosity, $N=\frac{\kappa}{\kappa+\mu}$, $(0\le N<1)$ is the Coupling number, $L=\frac{\beta_cN_1}{\beta_TM_1}$ is the buoyancy parameter, $M=\frac{\sigma B_0^2}{\mu B^2}$ is the magnetic field parameter, $\vartheta=\frac{1}{jB^2}$ is the micro-inertia density, $R=\frac{4\sigma^*T_\infty^3}{k\,k_1}$ is the radiation parameter and $\delta=\frac{R^*}{\nu B^2}$ is the chemical reaction parameter. The primes denote differentiation with respect to similarity variable η.

The boundary conditions (6) in terms of f, g, θ and ϕ becomes:

$$\left.\begin{aligned}&f(0)=0;\ f'(0)=0;\ g(0)=0;\ \theta(0)=1;\\&\phi(0)=1\ \text{at}\ \eta=0;\end{aligned}\right\}\quad(24a)$$

$$\left.\begin{aligned}&f'(\infty)\to1;\ g(\infty)\to0;\ \theta(\infty)\to0;\\&\phi(\infty)\to0\ \ as\ \ \eta\to\infty.\end{aligned}\right\}\quad(24b)$$

3. Skin friction and wall couple stress

The wall shear stress and wall couple stress:

$$\tau_w=\left[(\mu+\kappa)\frac{\partial u}{\partial y}+\kappa\omega\right]_{y=0},\ m_w=\gamma\left[\frac{\partial\omega}{\partial y}\right]_{y=0}. \quad (25)$$

The dimensionless wall shear stress $C_f=\frac{2\tau_w}{\rho A^2}$, wall couple stress $M_w=\frac{Bm_w}{\rho A^2}$, where A is the characteristic velocity, are given by:

$$C_f=\left(\frac{2}{1-N}\right)f''(0)\bar{x}\ \text{ and }\ M_w=\left(\frac{\lambda}{\vartheta}\right)g'(0)\bar{x}, \quad (26)$$

where $\bar{x}=Bx$.

4. Heat and mass transfer rates

The heat and mass transfers from the plate respectively are given by:

$$q_w=-k\left(\frac{\partial T}{\partial y}\right)_{y=0}-\frac{4\sigma^*}{3k_1}\left(\frac{\partial T^4}{\partial y}\right)_{y=0}\ \text{ and}$$

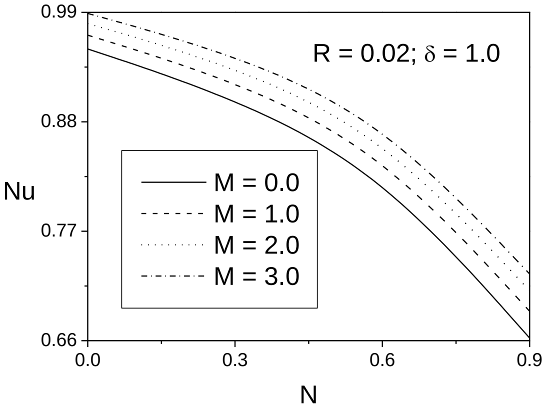
$$q_m=-D\left(\frac{\partial C}{\partial y}\right)_{y=0}. \quad (27)$$

The non-dimensional rate of heat transfer, called the Nusselt number $Nu=\frac{q_w}{Bk(T_w-T_\infty)}$ and rate of mass transfer, called the Sherwood number $Sh_x=\frac{q_m}{BD(C_w-C_\infty)}$ are given by:

$$Nu=-\theta'(0)\left(1+\frac{4R}{3}\right)\ \text{ and }\ Sh=-\phi'(0). \quad (28)$$

5. Results and discussion

The flow Eqs. (20) and (21) which are coupled, together with the energy and concentration Eqs. (22) and (23), constitute non-linear non-homogeneous differential equations for which closed-form solutions cannot be obtained. Hence, the governing Eqs. (20) to (23) are solved numerically using the Keller-box implicit method [19]. This method has been proven to be adequate and give accurate results for boundary layer equations. In the present study, the boundary conditions for η at ∞ are replaced by a sufficiently large value of η where the velocity, microrotation, temperature and concentration approach zero. The value of η_∞ is taken as 6 and a grid size of η as 0.01. In order to study the effects of the coupling number N, magnetic field parameter M, thermal radiation parameter R, chemical reaction parameter δ, Prandtl number Pr and Schmidt number Sc on the physical quantities of the flow, the remaining parameters are fixed as $L = 1$, $\lambda = 1$ and $\vartheta = 0.1$. The values of micropolar parameters λ and ϑ are chosen so as to satisfy the thermodynamic restrictions on the material parameters given by Eringen [1].



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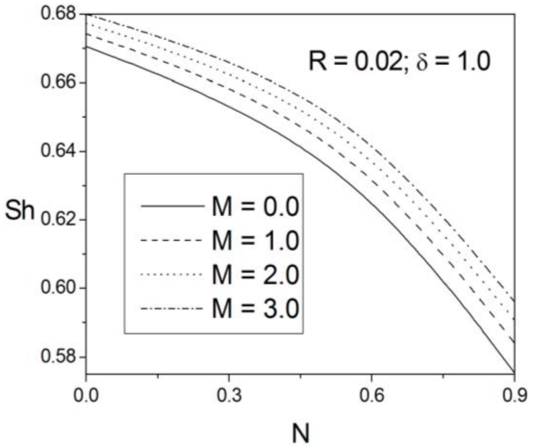
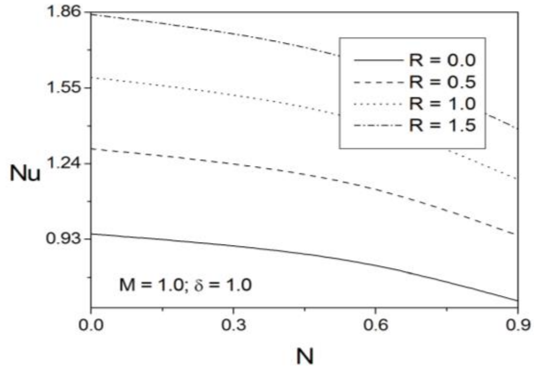


b

Fig. 2 Effect of Magnetic parameter on a) heat transfer rate; b) mass transfer rate

Fig. 2 depict the variation of heat and mass transfer rates (Nusselt number Nu and Sherwood number Sh) with coupling number N for different values of magnetic parameter M. The coupling number N characterizes the coupling of linear and rotational motion arising from the micromotion of the fluid molecules. Hence, N signifies the coupling between the Newtonian and rotational viscosities. As $N \rightarrow 1$, the effect of microstructure becomes significant, whereas with a small value of N the individuality of the substructure is much less pronounced. As $\kappa \rightarrow 0$ i.e., $N \rightarrow 0$, the micropolarity is lost and the fluid behaves as nonpolar fluid. Hence, $N \rightarrow 0$ corresponds to viscous fluid. It is observed from Figs. 2, a and 2, b that the both Nusselt number and Sherwood number decrease as coupling number increases. It is noticed that the heat and mass transfer rates are more in case of viscous fluids. Therefore, the presence of microscopic effects arising from the local structure and micromotion of the fluid elements reduce the heat and mass transfer rates. Further, it is seen that both the Nusselt number and Sherwood number are increasing as the magnetic parameter is increasing. This is due to the motive force created by traverse magnetic field which tends to accelerate the flow.



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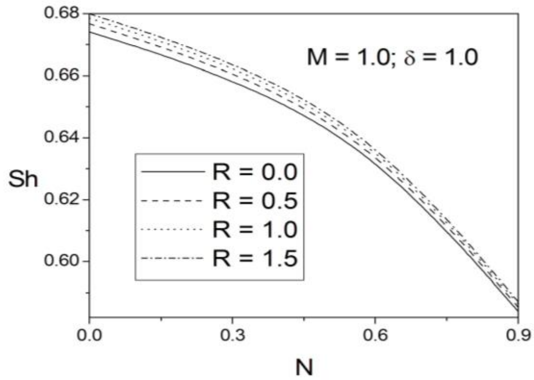
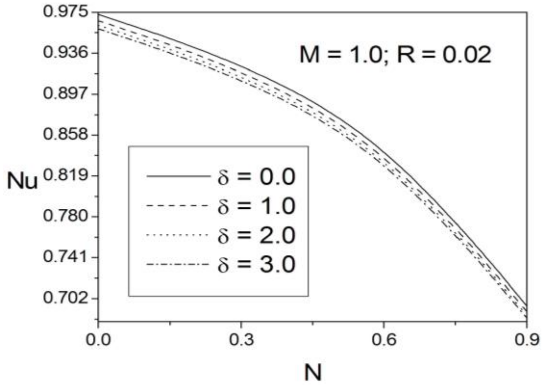


b

Fig. 3 Effect of Radiation parameter on a) heat transfer rate; b) on mass transfer rate

The effect of radiation parameter on heat and mass transfer coefficient is displayed in Figs. 3, a and 3, b. It is noticed from these figures that both the Nusselt number and Sherwood number increase with the increase in the radiation parameter. Higher values of radiation parameter R imply higher values of wall temperature. Consequently, the temperature gradient and hence Nusselt number and Sherwood number increase.

The variation of heat and mass transfer coefficients with coupling number for different values of chemical reaction parameter δ is depicted in Figs. 4, a and 4, b. It is clear from these figures that an increase in the chemical reaction parameter δ, leads to a decrease in the Nusselt number and an increase in the Sherwood number. Increase in the values of chemical reaction parameter δ implies more interaction of species concentration with the momentum boundary layer and less interaction with the thermal boundary layer. Hence, chemical reaction parameter has more significant effect on Sherwood number than it does on Nusselt number.



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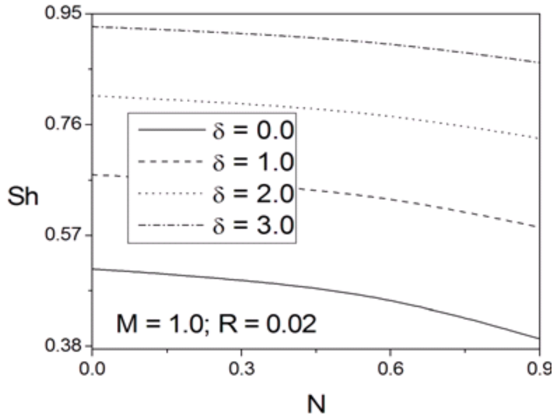
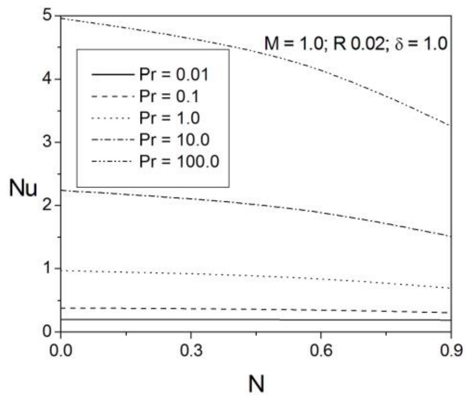


b

Fig. 4 Effect of Chemical reaction parameter on a) heat transfer rate; b) on mass transfer rate

Figs. 5, a and 5, b show the variation of heat and mass transfer coefficients with Prandtl number *Pr*. It is interesting to note that the Nusselt number is increasing whereas the Sherwood number is decreasing as Prandtl number increases. The reason is that smaller values of *Pr* are equivalent to increasing the thermal conductivities, and therefore heat is able to diffuse away from the heated plate more rapidly than for higher values of *Pr*. Hence in the case of smaller Prandtl numbers as the boundary layer is thicker and the rate of heat transfer is reduced.

The effect of Schmidt number on the heat and mass transfer coefficients is plotted in Figs. 6, a and 6, b. It is clear that the Nusselt number is decreasing while the Sherwood number is increasing with increasing values of *Sc*. The Schmidt number embodies the ratio of the momentum to the mass diffusivity. The Schmidt number therefore quantifies the relative effectiveness of momentum and mass transport by diffusion in the hydrodynamic (velocity) and concentration (species) boundary layers. Its effect on the species concentration has similarities to the Prandtl number effect on the temperature. That is increase in the values of Sc cause the velocity and species concentration and its boundary layer thickness to decrease significantly.



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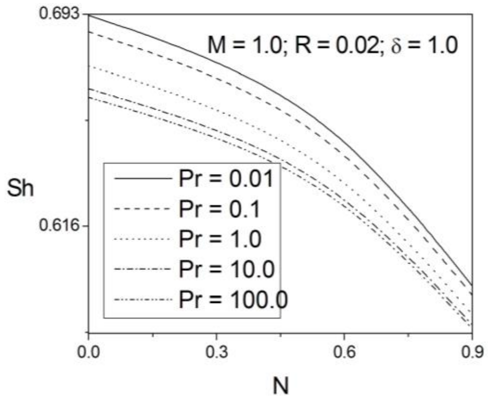

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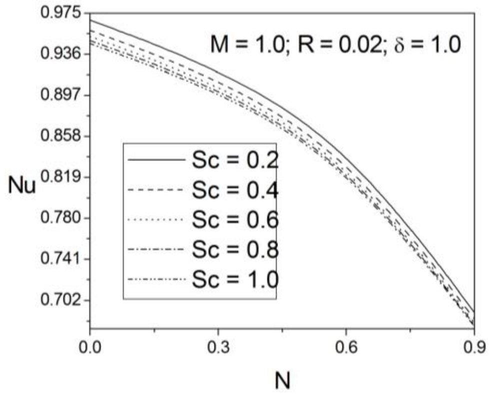
Fig. 5 Effect of Prandtl number on a) heat transfer rate; b) mass transfer rate



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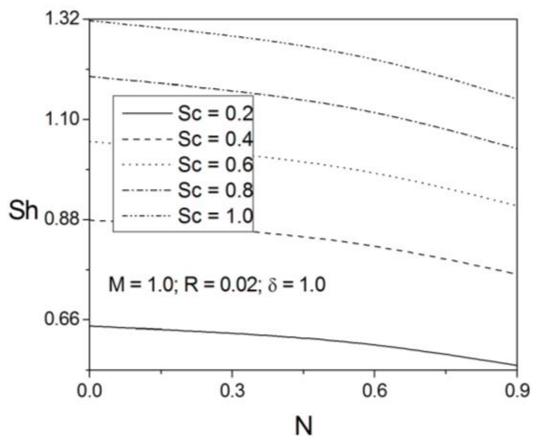

b

Fig. 6 Effect of Schmidt number on a) heat transfer rate; b) mass transfer rate

Table 1 shows the effects of the coupling number *N*, Prandtl number *Pr*, Schmidt number *Sc*, the magnetic parameter *M*, radiation parameter *R* and chemical reaction parameter δ on the non - dimensional skin friction C_f and the dimensionless wall couple stress M_w. It is seen from this table that both the skin friction and the wall couple stress decrease with increasing coupling number *N*. For increasing value of *N*, the effect of microstructure becomes significant; hence the wall couple stress decreases. The skin friction coefficient decreases and the wall couple stress increases with increasing Prandtl number and Schmidt number. Also, the effect of magnetic parameter is to increase the skin friction coefficient and decrease the wall couple stress. Further, it is observed that the skin friction coefficient is increasing and wall couple stress is decreasing with increase in the value of radiation parameter. The increase in chemical reaction parameter decreases the skin friction coefficient and increases the wall couple stress.

Table 1

Effect of skin friction and wall couple stress for various values of *N*, *Pr*, *Sc* and *M*

N	*Pr*	*Sc*	*M*	*R*	δ	$f''(0)$	$-g'(0)$
0.0	1.0	0.2	1.0	0.02	1.0	2.39348	0.00000
0.3	1.0	0.2	1.0	0.02	1.0	1.96198	0.03005
0.6	1.0	0.2	1.0	0.02	1.0	1.42737	0.09077
0.9	1.0	0.2	1.0	0.02	1.0	0.61107	0.28577
0.5	0.01	0.2	1.0	0.02	1.0	1.76858	0.07088
0.5	0.1	0.2	1.0	0.02	1.0	1.72412	0.06863
0.5	1.0	0.2	1.0	0.02	1.0	1.62226	0.06451
0.5	10	0.2	1.0	0.02	1.0	1.52651	0.06221
0.5	100	0.2	1.0	0.02	1.0	1.46377	0.06140
0.5	1.0	0.2	1.0	0.02	1.0	1.62685	0.06470
0.5	1.0	0.4	1.0	0.02	1.0	1.59272	0.06340
0.5	1.0	0.6	1.0	0.02	1.0	1.57242	0.06272
0.5	1.0	0.8	1.0	0.02	1.0	1.55810	0.06228
0.5	1.0	1.0	1.0	0.02	1.0	1.54712	0.06197
0.5	1.0	0.2	0.0	0.02	1.0	1.45336	0.06276
0.5	1.0	0.2	1.0	0.02	1.0	1.62685	0.06470
0.5	1.0	0.2	2.0	0.02	1.0	1.78148	0.06624
0.5	1.0	0.2	3.0	0.02	1.0	1.92248	0.06752
0.5	1.0	0.2	1.0	0.0	1.0	1.62563	0.06466
0.5	1.0	0.2	1.0	0.5	1.0	1.64952	0.06546
0.5	1.0	0.2	1.0	1.0	1.0	1.66530	0.06604
0.5	1.0	0.2	1.0	1.5	1.0	1.67698	0.06650
0.5	1.0	0.2	1.0	0.0	0.0	1.64977	0.06557
0.5	1.0	0.2	1.0	0.5	1.0	1.62685	0.06470
0.5	1.0	0.2	1.0	1.0	2.0	1.60994	0.06409
0.5	1.0	0.2	1.0	1.5	3.0	1.59666	0.06363

6. Conclusions

In this paper, mixed convection heat and mass transfer in an electrically conducting micropolar fluid over a vertical plate with wall temperature and concentration conditions in the presence of a first order chemical reaction and radiation is considered. A uniform magnetic field is applied normal to the plate. Using the similarity variables, the governing equations are transformed into a set of ordinary differential equations and numerical solution for these equations has been presented. The higher values of the coupling number *N* (i.e., the effect of microrotation becomes significant) result in lower Nusselt number and Sherwood number compared to the Newtonian fluid case. The numerical results indicate that the skin friction coefficients as well as rate of heat and mass transfers in the micropolar fluid are lower compared to that of the Newtonian fluid.

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ŠILUMINĖS SPINDULIUOTĖS IR CHEMINĖS REAKCIJOS POVEIKIS MIŠRIAI MAGNETOHIDRODINAMINEI ŠILUMOS KONVEKCIJAI IR MASĖS PERNEŠIMUI MIKROPOLINIAME SKYSTYJE

R e z i u m ė

Šiame straipsnyje tiriamos mišrios konvekcijos šilumos srauto ir masės pernešimo charakteristikos vertikalioje plokštelėje su kintančia sienelės temperatūra ir koncentracija mikropoliniame skystyje esant pirmos eilės cheminei reakcijai ir šiluminei spinduliuotei. Pastovaus dydžio magnetinis laukas yra nukreiptas statmenai į plokštelę. Pagrindinės netiesinės dalinių išvestinių diferencialinės lygtys ir jų ribinės sąlygos yra transformuotos į susietų netiesinių paprastų diferencialinių lygčių sistemą naudojant panašumo transformacijas, o po to skaitmeniškai išspręstos Kelerio dėžutės metodu. Šilumos perdavimo greitis Nuselto skaičiaus terminais ir konvekcinis su difuzine mase perdavimo santykis Šervudo skaičiaus terminais prie plokštelės yra pavaizduoti grafiškai. Plėvelės trinties koeficientas, sienelės sąryšio įtempiai pateikti lentelės pavidalu esant skirtingoms magnetinių parametrų reikšmėms, sąryšio skaičiui, spinduliuotės bei cheminės reakcijos parametrams, Prandtlio bei Šmidto skaičiams.

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THERMAL RADIATION AND CHEMICAL REACTION EFFECTS ON MHD MIXED CONVECTION HEAT AND MASS TRANSFER IN A MICROPOLAR FLUID

S u m m a r y

This paper analyzes the flow heat and mass transfer characteristics of the mixed convection on a vertical plate with variable wall temperature and concentration in a micropolar fluid in the presence of a first order chemical reaction and radiation. A uniform magnetic field of magnitude is applied normal to the plate. The governing nonlinear partial differential equations and their associated boundary conditions are transformed into a system of coupled nonlinear ordinary differential equations using similarity transformations and then solved numerically using the Keller - box method. The rate of heat transfer in terms of Nusselt number and the ratio of convective to diffusive mass transport in terms of Sherwood number at the plate are presented graphically. The skin-friction coefficient, the wall couple stress are shown in a tabular form for different values of magnetic parameter, coupling number, radiation parameter, chemical reaction parameter, Prandtl number and Schmidt number.

Keywords: mixed convection, micropolar fluid, MHD, radiation, chemical reaction, heat mass transfer.

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