Investigation of the steady state operation conditions of rotational vibroactuators

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1. Introduction

With the development of new integrated systems in the majority of engineering fields the tendency to substitute conventional mechanisms by the ones operating on new concepts and principles prevails. One group of such mechanisms (vibroactuators or piezoelectric actuators) is based on smart material application [1-3]. Smart material based sensors and actuators are especially convenient for the use as components of mechatronic systems because they are of simple structure and easily electronically controlled components.

In recent years already traditionally mechanical units with the application of active material exciters (e.g. piezoelectric exciters) became attractive for the mentioned above design approach as allowing achieving simple structures with sufficient performance characteristics, e.g. piezoelectric drives (motors) able to transform electrical energy to continuous motion of the output link (rotor) with no intermediate links appear to be ideal choice to serve as actuators for different systems of precision engineering [4, 5]. The new achievements in micro-scale engineering systems is hardly possible without the development of micro scale design solutions allowing high positioning accuracy together with high resolution and other high dynamic characteristics of the units.

There can be found great number of research publications, patents presenting the theoretical analysis, modeling, experimental research of various aspects of such type piezoactuator development – structural optimization, excitation loading regime analysis, control system development [6-9]. Nevertheless the main principal operational solution which proved its effectiveness in practical applications is the generation of high frequency travelling wave in ring shaped resonator with the help of piezoelectric elements and the resonator being in frictional contact with the rotor (output link) forces it to motion [10-14]. Such principal can be applied both in 1 DOF rotary actuators and in multi DOF actuators. As stator – rotor interaction in these motors is of frictional nature the main challenge still remains to ensure its stability for certain period of time together with the selection of the necessary characteristics of the contacting bodies enabling good output characteristics of the rotor. E.g. in [15] the insertion of the third frictional layer between the stator and the rotor is reported, in [16] special attempts to optimize mechanical characteristics of contacting surfaces is presented.

In the present paper the structural solution of a travelling wave vibroactuator based on ring shaped piezoceramic active element (stator) which is elastically suspended and contacts the output link (rotor) by its inner surface – i.e. concave surface what ensures stability of its contact characteristics is analyzed. Single input link or multiple input link structures can be developed.

As the operational principle of wave vibroactuator is based on frictional interaction of an input link in which a high frequency travelling wave oscillations are excited with an output link the characteristics of motion of the output link are determined by vibration parameters of the contact zone and surface mechanical characteristics of both the input and output links. In the paper the input - output link interaction is presented in the case of kinematic excitation i.e. under the assumption of the optimal vibration parameters of the contact point (elliptical motion) with the defined amplitude ratios along two perpendicular axis and the corresponding phase shift ensured by the geometrical parameters, excitation zones pattern and multiphase excitation scheme, and the focus is made on the investigation of surface characteristics of the output link (rotor).

Vibrations of the mechanical system in which dry friction is present are described by nonlinear differential equation. The feature of motion of such systems is that due to nonlinear influence of dry friction, the motion of the links can be separated into different phases. The motion of the phases is described by different easily integrated differential equation. The differential equation describing the motion of nonlinear systems are mainly used for the determination of transition instant from one phase to another.

The feature of the steady state is that in each phase there are two instants in the time domain when the velocity becomes equal zero. If in time the steady motion approaches some limit motion, the later with a great probability can be considered as a asymptotically stable motion.

2. Structure and operation principle of the vibroactuator

Two principle schemes of the vibroactuators (Fig. 1) with elastically suspended input link *1* are presented. The first one (Fig. 1, a) consists of a rotor (output link) *2* and the elastically suspended stator (input link) *1* while the second one (Fig. 1, b) has three stators *1* which are elastically suspended. In both schemes travelling wave of high frequency elastic oscillations is generated in a piezoceramic ring resulting in elliptic motion of the contact zone. Nevertheless the arrangement of the stators with regular angular displacement of 120° with respect to each other ensures higher stability of their contact zone what allows higher reliability of the vibroactuator.

The concave – convex shape nature of piezoceramic ring – rotor contact zone and the three point contact of the stator-rotor due to the mentioned above angular arrangement of the rings is the key factor to achieve the stability of the stator-rotor contact.
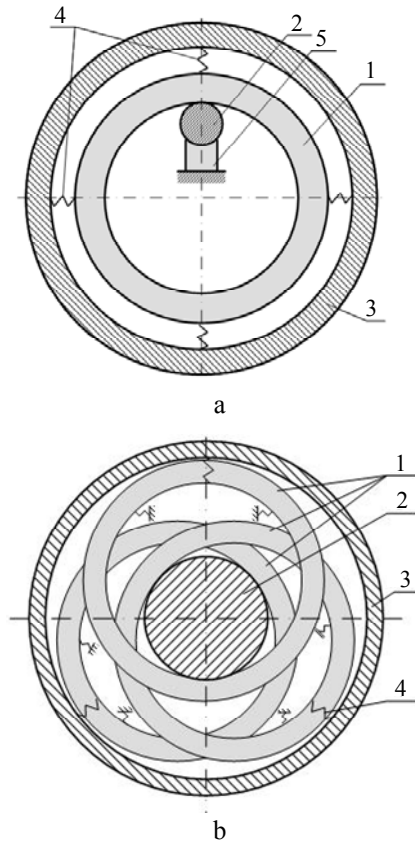
Fig. 1 Structure of vibroactuators: a – with one point contact and b – with three points contact of the stator and the rotor: *1* - piezoceramic ring; *2* - rotor; *3* - frame of the unit; *4* - elastic suspension elements; *5* - supports of rotor

3. Theoretical investigation

Performance characteristics of the vibroactuator are predefined by the interaction processes between input link and the output link. These interaction processes are of frictional nature, that's why to ensure stability of the output characteristics is still a challenge.

For theoretical investigation of the vibroactuator a simplified dynamical model of the applied vibratory system is used (Fig. 2). In it the input link *1* is represented by a contact point – an absolutely rigid particle moving according the defined in advance law. The input link *1* is in contact (pressed to) with the output link *2* which can perform continuous motion.
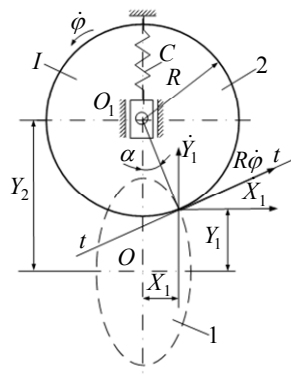

Fig. 2 Dynamical model of vibroactuator: *1* - input link; *2* - output link

The case under investigation is when the output link *2* performs rotational motion.

When exicited the input link *1* (particle) moves along elliptical trajectory the coordinates of which are expressed as follows

$$\left. \begin{array}{l} X_1 = K \sin\left(\omega t + \psi_0\right) \\ Y_1 = L \cos \omega t \end{array} \right\} \tag{1}$$

where X_1 and Y_1 are coordinates of the contact point, K and L are amplitudes of vibrations of the point, ω is angular frequency of vibrations, ψ_0 is phase shift.

The output link starts rotational motion under the effect of dry friction force moment

$$M_R = R f_0 N \; sign \, v_{21} \tag{2}$$

where $v_{21} = v_2 - v_1$ is velocity difference of the points of input and output links at contact.

Normal reaction force at contact point between the links is describe as follows

$$N = \left[N\ddot{Y}_2 + C\left(Y_2 + \Delta\right)\right]\frac{1}{\cos \alpha} \tag{3}$$

where *M* is mass of the rotational output link, $C\left(Y_2 + \Delta\right)$ is elasticity force, Δ is initial deformation of elastic element,

The tangential velocities of links are expressed as follows

$$\left. \begin{array}{l} v_1 = \dot{X}_1 \cos \alpha + \dot{Y}_1 \sin \alpha \\ v_2 = R\dot{\varphi} + \dot{Y}_2 \sin \alpha \end{array} \right\} \tag{4}$$

where $sin\,\alpha = \dfrac{X_1}{R}$ and $cos\,\alpha = \dfrac{Y_2 - Y_1}{R}$.

Taking into account the expressions given above equations of the output link can be written in the following form

$$I\ddot{\varphi} + H\dot{\varphi} + H_0 sign\dot{\varphi} + Rf_0\left[M\ddot{Y}_2 + C\left(Y_2 + \Delta\right)\right]\times$$
$$\times \frac{1}{\cos \alpha} sign\left[R\dot{\varphi} + \left(\dot{Y}_2 - \dot{Y}_1\right)sin\,\alpha - \dot{X}_1 \cos \alpha\right] = 0 \tag{5}$$

Or in dimensionless form

$$\varphi'' + h\varphi' + rf_0\left(\mu y_2'' + cy_2 + f\right)\frac{1}{\cos \alpha}\times$$
$$\times sign\left[r\varphi' + \left(y_2' - y_1'\right)sin\,\alpha - x_1' \cos \alpha\right] = 0 \tag{6}$$

where $x_1 = \dfrac{X_1}{K}$; $y_1 = \dfrac{Y_1}{K}$; $y_2 = \dfrac{Y_2}{K}$; $r = \dfrac{R}{K}$; $h_0 = \dfrac{H_0}{I\omega^2}$;

$h = \dfrac{H}{I\omega}$; $l = \dfrac{L}{K}$; $\mu = \dfrac{MK^2}{I}$; $c = \dfrac{CK^2}{I\omega^2}$; $f = \dfrac{K}{I\omega^2}$;

$\tau = \omega t$.

The Eq. (6) is differentiated with respect to the new dimensionless variable τ and $\varphi' > 0$.

For the determination of conditions and parame-

ters necessary for steady state motion regime two types of nonsliding and two types of sliding motion regimes were analysed together taking into account the condition of contact existence

$$X_1^2 + \left(Y_2 - Y_1\right)^2 = R^2 \tag{7}$$

where Y_2 is distance between the centers of input and output links; R is radius of the output link and

$$y_2 = l\cos\tau \pm \sqrt{r^2 - sin^2\,\tau} \tag{8}$$

Solving the equation the square root $\sqrt{r^2 - sin^2\tau}$ is positive.

The rotation motion is the first type-contact nonsliding motion, when

$$\tau = \tau_i;\ v_{21} = 0 \tag{9}$$

In this case the motion of output link 2 is described by the following equation

$$\varphi'' - \frac{\cos\tau\,sin^2\,\tau}{r^2\sqrt{r^2 - sin^2\,\tau}} - \frac{1}{r}\cos\tau\sqrt{r^2 - \cos\tau} \tag{10}$$

The angular coordinate φ and velocity φ' of the output link 2 are determined from the following equation

$$\begin{aligned}\varphi &= arcsin\,sin\,\tau + c \\ \varphi' &= \frac{1}{r^2}\cos\tau\left(\frac{sin\,\tau}{\sqrt{r^2 - sin^2\,\tau}} + \sqrt{r^2 - sin^2\,\tau}\right)\end{aligned}\right\} \tag{11}$$

Constant of integration c is determined when condition $v_{21} = 0$ is satisfied.

The second type motion is contact sliding motion, when

$$\tau \in \tau_i,\ \tau_{i+1};\ v_{21} < 0 \tag{12}$$

In this case the input link 1 surpasses the output link 2.

The angular coordinate φ and velocity φ' of the output link 2 are determined from the equation

$$\varphi'' + h\varphi' = rf_0\mu\left[-l\cos\tau - \frac{r^2 - 2r^2\,sin^2\,\tau + sin^4\,\tau}{\sqrt{r^2 - sin^2\,\tau}\left(r^2\,sin^2\,\tau\right)}\right] + $$
$$+ f_0 rc\left(l\cos\tau + \sqrt{r^2 - sin^2\,\tau}\right) - \left(h_0 - rf_0 f\right) \tag{13}$$

$$\varphi = \frac{l(b - a)}{h^2 + 1}\left(\cos\tau + sin\,\tau\right) - ar^2\int exp\left(-h\tau\right)\left[\int\frac{exp\left(-h\tau\right)\cos^2\,\tau}{\sqrt{r^2 - sin^2\,\tau}\left(r^2\,sin^2\,\tau\right)}d\tau\right]d\tau + a\int exp\left(-h\tau\right)\times$$

$$\times\left[\int\frac{exp\left(-h\tau\right)sin^2\,\tau}{\sqrt{r^2 - sin^2\,\tau}}d\tau\right]d\tau + b\int exp\left(h\tau\right)\int exp\left(h\tau\right)\sqrt{r^2 - sin^2\,\tau}d\tau\frac{m}{h}\tau + c_2\tau + c_1 \tag{14}$$

where $a = f_0 r\mu$, $b = f_0 rc$, $m = h_0 - f_0 rf$

In general case constants c_1 and c_2 are determined from initial conditions which are coordinates of the previous motion phase.

If motion of the first phase takes place when $\tau = \tau_i$ it is obvious that coordinates (characteristics) during this motion are

$$\begin{aligned}\varphi\left(\tau_i\right) &= \varphi_i \\ \varphi'\left(\tau_i\right) &= \varphi_i'\end{aligned}\right\} \tag{15}$$

and constants c_1 and c_2 are expressed as follows

$$c_1 = \varphi_i - \frac{l(b - a)}{h^2 + 1}\left(-sin\,\tau_i + \cos\tau_i\right) + ar^2\int exp\left(-h\tau_i\right)\left[\int\frac{exp\left(h\tau_i\right)\cos^2\,\tau_i}{\sqrt{r^2 - sin^2\,\tau_i}\left(r^2 - sin^2\,\tau_i\right)}d\tau_i\right]d\tau_i - a\int exp\left(-h\tau_i\right)\times$$

$$\times\left[\int\frac{exp\left(h\tau_i\right)sin^2\,\tau_i}{\sqrt{r^2 - sin^2\,\tau_i}}d\tau_i\right]d\tau_i - b\int exp\left(-h\tau_i\right)\left[\int exp\left(-h\tau_i\right)\sqrt{r^2 - sin^2\,\tau_i}d\tau_i\right]d\tau_i + \frac{m}{h}\tau_i - c_2\tau_i$$

$$c_2 = \varphi_i' - \frac{l(b - a)}{h^2 + 1}\left(-sin\,\tau_i + \cos\tau_i\right) + ar^2\,exp\left(-h\tau_i\right)\int\frac{exp\left(h\tau_i\right)\cos^2\,\tau_i}{\sqrt{r^2 - sin^2\,\tau_i}\left(r^2 - sin^2\,\tau_i\right)}d\tau_i - a\,exp\left(-h\tau_i\right)\times$$

$$\times\int\frac{exp\left(h\tau_i\right)sin^2\,\tau_i}{\sqrt{r^2 - sin^2\,\tau_i}}d\tau_i - b\,exp\left(-h\tau_i\right)\int exp\left(-h\tau_i\right)\sqrt{r^2 - sin^2\,\tau_i}d\tau_i + \frac{m}{h} \tag{16}$$

The third type of motion is analogous the first one i.e.

$$\tau = \tau_{i+1};\ v_{21} = 0 \tag{17}$$

The fourth type of motion is contact sliding motion when

$$\tau \in \left(\tau_{i+1},\ \tau_{i+2}\right);\ v_{21} > 0 \tag{18}$$

The angular coordinate φ and velocity of the output link 2 φ' for this type of motion are determined from the following equation

$$\varphi'' + h\varphi' = -a\left(l\cos\tau + \frac{r^2 - 2r^2\sin^2\tau + \sin^4\tau}{\sqrt{r^2 - \sin^2\tau}\left(r^2 - \sin^2\tau\right)}\right) - b\left(l\cos\tau + \sqrt{r^2 - \sin^2\tau}\right) - m_1 \qquad (19)$$

$$\varphi = \frac{l(a-b)}{h^2+1}\left(-\cos\tau + \sin\tau\right) + ar^2\int\exp\left(-h\tau\right)\left[\int\frac{\exp\left(h\tau\right)\cos^2\tau}{\sqrt{r^2 - \sin^2\tau}\left(r^2 - \sin^2\tau\right)}d\tau\right]d\tau - a\int\exp\left(-h\tau\right)\times$$

$$\times\left[\int\frac{\exp\left(h\tau\right)\sin^2\tau}{\sqrt{r^2-\sin^2\tau}}d\tau\right]d\tau - b\int\exp\left(-h\tau\right)\left[\int\exp\left(h\tau\right)\sqrt{r^2-\sin^2\tau}d\tau\right]d\tau - \frac{m_1}{h}\tau + c_{21}\tau + c_{11} \qquad (20)$$

where $m_1 = h_0 + f_0 fr$.

In this case the velocity of the output link 2 is greater than the velocity of the input link 1.

$$\varphi\left(\tau_{i+1}\right) = arcsin\frac{\sin\tau_{i+1}}{r} = \varphi_{0i} \qquad (21)$$

$$\varphi'\left(\tau_{i+1}\right) = \frac{1}{r^2}\cos\tau_{i+1}\left(\frac{\sin\tau_{i+1}}{\sqrt{r^2 - \sin^2\tau_{i+1}}} + \sqrt{r^2 - \sin^2\tau_{i+1}}\right) \qquad (22)$$

here φ_{0i} is constant of the previous motion stage.

Then constants c_{21} and c_{11} are expressed as follows

$$c_{21} = \frac{l(a-b)}{h^2+1}\left(\sin\tau_{i+1} + \cos\tau_{i+1}\right) - ar^2\exp\left(-h\tau_{i+1}\right)\int\frac{\exp\left(h\tau_{i+1}\right)\cos^2\tau_{i+1}}{\sqrt{r^2-\sin^2\tau_{i+1}}\left(r^2-\sin^2\tau_{i+1}\right)}d\tau_{i+1} +$$

$$+ a\exp\left(-h\tau_{i+1}\right)\int\frac{\exp\left(h\tau_{i+1}\right)\sin^2\tau_{i+1}}{\sqrt{r^2-\sin^2\tau_{i+1}}}d\tau_{i+1} + b\exp\left(-h\tau_{i+1}\right)\int\exp\left(-h\tau_{i+1}\right)\sqrt{r^2-\sin^2\tau_{i+1}}d\tau_{i+1} +$$

$$+ \frac{m_1}{h} + \frac{1}{r^2}\cos\tau_{i+1}\left(\frac{\sin\tau_{i+1}}{\sqrt{r^2-\sin^2\tau_{i+1}}} + \sqrt{r^2-\sin^2\tau_{i+1}}\right) \qquad (23)$$

$$c_{11} = arcsin\frac{\sin\tau_{i+1}}{r} + \varphi_{0i} - \frac{l(a-b)}{h^2+1}\left(\sin\tau_{i+1} - \cos\tau_{i+1}\right) - ar^2\int\exp\left(-h\tau_{i+1}\right)\times$$

$$\times\left[\int\frac{\exp\left(-h\tau_{i+1}\right)\cos^2\tau_{i+1}}{\sqrt{r^2-\sin^2\tau_{i+1}}\left(r^2-\sin^2\tau_{i+1}\right)}d\tau_{i+1}\right]d\tau_{i+1} + a\int\exp\left(-h\tau_{i+1}\right)\left[\int\frac{\exp\left(-h\tau_{i+1}\right)\sin^2\tau_{i+1}}{\sqrt{r^2-\sin^2\tau_{i+1}}}d\tau_{i+1}\right]d\tau_{i+1} +$$

$$+ b\int\exp\left(-h\tau_{i+1}\right)\left[\int\exp\left(-h\tau_{i+1}\right)\sqrt{r^2-\sin^2\tau_{i+1}}d\tau_{i+1}\right]d\tau_{i+1} + \frac{m_1}{h}\tau_{i+1} + c_{21}\tau_{i+1} \qquad (24)$$

The junction points of the solution for continuous rotational motion in cases of sliding existence and when there is no sliding are determined in the following way: if there is the transition of the rotational motion from its state with sliding i.e. when velocity of the output link is lower than the velocity of the input link, to the state with no sliding, this can occur if the condition $v_{21} < 0$ becomes not valid any more and the condition $v_{21} = 0$ is satisfied.

Solution of the Eq. (9) with respect to τ indicates the junction point of the mentioned two types of rotational motion.

If there is the transition of the rotational motion from its state with sliding i.e. when velocity of the output link exceeds the velocity of the input link, to the state with no sliding, this can occur if the condition $v_{21} > 0$ becomes not valid any more.

In this case the junction point is determined from the condition analogous to Eq. (9).

After a number of motion stages of the types as described above the system performs steady state motion or approaches this motion regime, i.e. the system performs periodical motion the pattern of which consists of the above mentioned motion phases, while the output link moves by rotational motion. If in time the steady state motion approaches certain limit type motion, the later can be called asymptotically stable motion. To find out the steady state motion parameters so called junction method according which the parameters of the end of one motion phase serve as initial conditions for the following motion phase is used. Thus the parameters ensuring periodical sequence of motion regimes can be determined.

It is considered that the steady state motion is determined by the following parameters

$$\left.\begin{array}{l}\tau_i = \theta;\ \tau_{i+1} = \theta + \delta;\ \tau_{i+2} = 2\pi + \theta\\ \varphi_i = 0;\ \varphi_i = \varphi'_{i+2} = v;\ \varphi_{i+2} = s\end{array}\right\} \qquad (25)$$

The parameters of steady state motion θ, δ, v, s are obtained using the solutions of the four type equations

of motion. Taking into account (23) Eqs. (7-22) will become

$$v = \frac{1}{r^2}\cos\theta\left(\frac{\sin\theta}{\sqrt{r^2-\sin^2\theta}}+\sqrt{r^2-\sin^2\theta}\right)$$

$$\frac{l(b-a)}{h^2+1}\left[\sin(\theta+\delta)+\cos(\theta+\delta)\right]-ar^2\exp\left[-h(\theta+\delta)\right]\int\frac{\exp\left[h(\theta+\delta)\right]\cos^2(\theta+\delta)}{\sqrt{r^2-\sin^2(\theta+\delta)}\left[r^2-\sin^2(\theta+\delta)\right]}$$

$$\times d(\theta+\delta)+a\exp\left[-h(\theta+\delta)\right]\int\frac{\exp\left[h(\theta+\delta)\right]\sin^2(\theta+\delta)}{\sqrt{r^2-\sin^2(\theta+\delta)}}d(\theta+\delta)+b\exp\left[-h(\theta+\delta)\right]\times$$

$$\times\int\exp\left[h(\theta+\delta)\right]\sqrt{r^2-\sin^2(\theta+\delta)}d(\theta+\delta)-\frac{m}{h}+c_2-\frac{1}{r^2}\cos(\theta+\delta)\times$$

$$\times\left[\frac{\sin\theta+\delta}{\sqrt{r^2-\sin^2\theta+\delta}}+\sqrt{r^2-\sin^2\theta+\delta}\right]=0$$

$$\frac{l(b-a)}{h^2+1}\left[\sin\theta+\cos\theta\right]+ar^2\exp\left[-h(2\pi+\theta)\right]\int\frac{\exp\left[h(2\pi+\theta)\right]\cos^2\theta}{\sqrt{r^2-\sin^2\theta}\left(r^2-\sin^2\theta\right)}d(2\pi+\theta)-$$

$$-a\exp\left[-h(2\pi+\theta)\right]\times\int\frac{\exp\left[h(2\pi+\theta)\right]}{\sqrt{r^2-\sin^2\theta}}\sin^2\theta d(2\pi+\theta)-b\exp\left[-h(2\pi+\theta)\right]+$$

$$\int\exp\left[h(2\pi+\theta)\right]\sqrt{r^2-\sin^2\theta}d(2\pi+\theta)-\frac{m_1}{h}+c_{21}-\frac{1}{r^2}\cos\theta\left(\frac{\sin\theta}{\sqrt{r^2-\sin^2\theta}}+\sqrt{r^2-\sin^2\theta}\right)=0$$

$$s = \frac{l(b-a)}{h^2+1}\left(\left[\sin\theta-\cos\theta\right]\right)+ar^2\int\exp\left[-h(2\pi+\theta)\right]\left[\int\frac{\exp\left[h(2\pi+\theta)\right]\cos^2\theta}{\sqrt{r^2-\sin^2\theta}\left(r^2-\sin^2\theta\right)}d(2\pi+\theta)\right]-$$

$$-a\int\exp\left[-h(2\pi+\theta)\right]\times\left[\int\frac{\exp\left[h(2\pi+\theta)\right]\sin^2\theta}{\sqrt{r^2-\sin^2\theta}}d(2\pi+\theta)\right]d(2\pi+\theta)-b\int\exp\left[-h(2\pi+\theta)\right]\times$$

$$\times\left[\int\exp\left[h(2\pi+\theta)\right]\sqrt{r^2-\sin^2\theta}d(2\pi+\theta)\right]d(2\pi+\theta)-\frac{m_1}{h}(2\pi+\theta)+c_{21}(2\pi+\theta)+c_{11}$$

(26)

Constants c_2, c_{11} and c_{21} are determined from Eqs. (17), (23) and (24) taking into account conditions as in (26).

Not all regimes determined from Eq. (26) can be applied and not all of them are stable. The condition of the steady state regimes existence is the solution possibility of Eq. (26).

In order to determine which of the steady state regimes satisfying conditions of existence are stable it is necessary to solve the Eq. (26) making variations of the parameters in the zones close a steady state regime. If the solutions of differential Eq. (26) obtained after parameter variations decay, such regime is stable, in the opposite case unstable.

$$\left.\begin{array}{l}\tau_i = \theta+\Delta_\theta;\ \tau_{i+1}=\theta+\delta+\Delta_\delta;\ \tau_{i+1}-\tau_i=\delta-\Delta_\delta-\Delta_\theta\\[4pt]\tau_{i+2}=2\pi+\theta+\rho\Delta_\theta;\ \tau_{i+2}-\tau_{i+1}=2\pi+\rho\Delta_\theta-\delta-\Delta_\delta\\[4pt]\varphi_i'=v+\Delta_v;\ \varphi_{i+2}'=V+\rho\Delta_v;\ \varphi_i'=\Delta_\varphi\end{array}\right\}\ \ (27)$$

here Δ_θ, Δ_δ, Δ_v, Δ_φ is variations, ρ is characteristic parameter.

If $|\rho|<1$ the analysed regime is stable, if $|\rho|>1$ – unstable.

Criteria according which the quality of rotational vibroactuator can be determined are average velocity $\dot\varphi$ of the output link, nonuniformity coefficient of motion δ, efficiency η. These criteria are determined when the vibroactuator motion is in steady state.

Dynamic characteristics of the vibroactuator when its motion is in steady state are shown in Figs. 3-7.
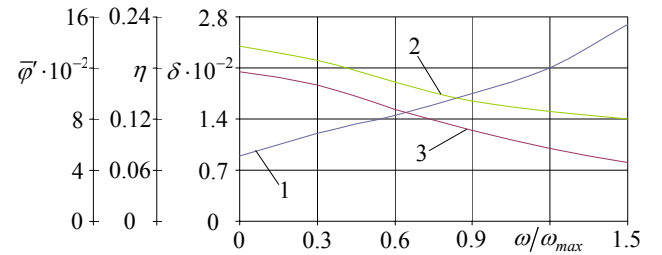
Fig. 3 Dependencies of average velocity $\overline{\varphi}'$, efficiency η and nonuniformity coefficient of motion δ versus frequency (ω/ω_{max}): 1 – $\overline{\varphi}'$; 2 – η; 3 – δ, $\psi_0=0.1$, $f_0=0.1$, $f=9\cdot10^{-5}$, $h_0=0.1$, $h=0.06$

Increase of the angular frequency and the excitation phase shift allows to increase the average velocity of motion and decreases not uniformity of motion.
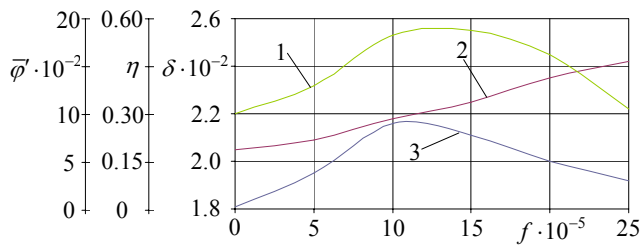

Fig. 4 Dependencies of average velocity $\overline{\varphi}'$, efficiency η and nonuniformity coefficient of motion δ versus pressing force f: $1 - \overline{\varphi}'$; $2 - \eta$; $3 - \delta$, $\psi_0 = 0.1$, $f_0 = 0.1$, $\omega/\omega_{max} = 0.5$, $h_0 = 0.1$, $h = 0.06$
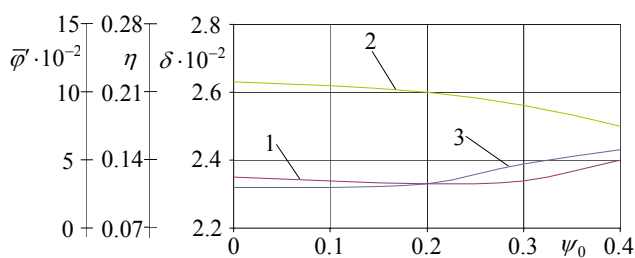

Fig. 5 Dependencies of average velocity $\overline{\varphi}'$, efficiency η and nonuniformity coefficient of motion δ on phase shift ψ_0: $1 - \overline{\varphi}'$; $2 - \eta$; $3 - \delta$, $f_0 = 0.1$, $f = 9 \cdot 10^{-5}$, $h_0 = 0.1$, $h = 0.06$
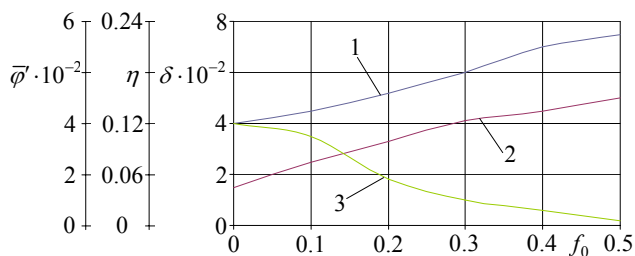

Fig. 6 Dependencies of average velocity $\overline{\varphi}'$, efficiency η and nonuniformity coefficient of motion δ on dry friction coefficient f_0: $1 - \overline{\varphi}'$; $2 - \eta$; $3 - \delta$, $\psi_0 = 0.1$, $\omega/\omega_{max} = 0.5$, $f = 9 \cdot 10^{-5}$, $h_0 = 0.1$, $h = 0.06$
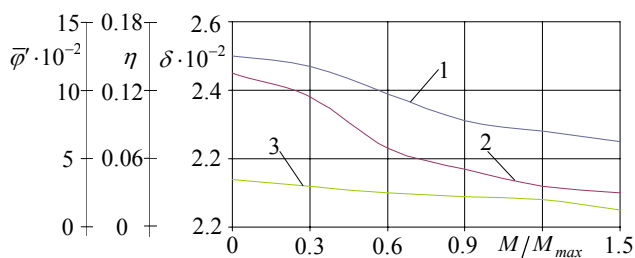

Fig. 7 Dependencies of average velocity $\overline{\varphi}'$, efficiency η and nonuniformity coefficient of motion δ versus the output link mass M/M_{max}: $1 - \overline{\varphi}'$; $2 - \eta$; $3 - \delta$, $\psi_0 = 0.1$, $f_0 = 0.1$, $f = 9 \cdot 10^{-5}$, $h_0 = 0.1$, $h = 0.06$, $\omega/\omega_{max} = 0.5$

Coefficient of dry friction, pressing force and mass of the moving link has the significant influence on dynamic parameters of the output link. In case of dry friction coefficient increase, angular velocity increases and in case of the increase of moved mass it decreases.

4. Conclusions

The wave rotational vibroactuators are presented as a simplified dynamical model of a nonlinear oscillating system. Analytical expressions for the description of a steady state motion are obtained when the links do not rebound from each other. The conditions of the steady state motion existence are indicated.

Performing the analysis of differential Eq. of the links motion the main characteristics of the wave rotational vibroactuator are determined.

The obtained results of the theoretical analysis indicate the proposed structure vibroactuator to have sufficient performance characteristics for the application in precise low power mechanisms.

References

1. **Bansevičius, R**. 2002. Piezoelectric Actuators. The Mechatronics. -Boca Raton: CRS Press LLC: 51-62.
2. **Bansevičius, R.; Blechertas, V.** 2008. Multi-degree-of-freedom ultrasonic motors for mass-cosumer devices, Journal of Electroceramics 20 (3-4): 221-224.
3. **Mažeika, D.; Bansevičius, R.; Kulvietis, G.** 2009. Investigation of contact point trajectories of the beam type piezoelectric actuator with two preloaded masses, Journal of Vibroengineering 11(4): 643-650.
4. **Bansevičius, R.; Barauskas, R.; Kulvietis, G.; Ragulskis, K.** 1988. Vibromotors for Precision Microrobots. Hemisphere Publishing Corp, USA. 310 p.
5. **Mažeika, D.; Bansevičius, R.** 2009. Study of resonant vibrations shapes of the beam type piezoelectric with preloaded mass, Mechanika 2(76): 33-37.
6. **Bakšys, B.; Liutkauskienė, K.** 2010. Manipulation of controlled dry friction, Mechanika 3(83): 28-33.
7. **Jurėnas, V.; Bansevičius, R.; Navickaitė, V.** 2010. Piezoelectric bimorphs for laser shutter systems: optimization of dynamic characteristics, Mechanika 5(85): 44-47.
8. **Mažeika, D.; Vasiljev, P.; Kulvietis, G.; Vaičiulienė, S.** 2009. New linear piezoelectric actuator based on traveling wave, Journal of Vibroengineering 11(1): 68-77.
9. **Beneš, L.; Kaloč, R.; Minar, L.** 2010. New approach to the analysis of the contact surfaces of rolling kinematic couple, Transport 25(4): 382-386.
10. **Kurila R.** 2006. Dynamic of wave vibrational motors, Mechanika 1(57): 33-36.
11. **Chen, Y.; Liu, Q.I.; Zhou, T.Y.** 2006. A traveling wave ultrasonic motor of high torque, Elsevier, Ultrasonic 44: 581-584.
12. **Toyama, Sh.; Kure, Sh.; Yoshida, A.** 2009. Development of piezoelectric actuators with rotational and translational motions (TR motor), Journal of Vibroengineering 11(3): 374-378.
13. **Storck, H.; Littman, W.; at all.** 2002. The effect of friction reduction in presence of ultrasonic vibration and as relevance to travelling wave ultrasonic motors,

Elsevier, Ultrasonic 40: 379-383.

14. **Bar-Cohen, Y.; Bao, X.; Grandia, W.** 1998. Rotary ultrasonic motors actuated by traveling flexural waves, Proceedings of the SPIE International Smart Materials and Structures Conference, SPIE Paper No. 3329-82, San Diego, CA, 1-6 March 1998.

15. **Qu, J.; Sun, F.; Zhao, Ch.** 2006. Perfomance evaluation of traveling wave ultrasonic motor based with visco-elastic friction layr on stator, Ultrasonics 45: 22-31.

16. **United States Patent** No.: 5596240. 1997. Ultrasonic motor.

17. **Shigematsu, T.; Kurasawa, M. K.** 2008. Friction drive of an SAW motor. Part IV: Physics of contact. IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control 55(10), 2277-2287.

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VIBRACINIŲ PAVARŲ NUOSTOVIOJO JUDĖJIMO REŽIMO SĄLYGŲ TYRIMAS

R e z i u m ė

Straipsnyje pateikti banginės vibracinės pavaros su žiediniu žadintuvu, kontaktuojančiu su išėjimo grandimi tašku, nuostoviojo judėjimo režimo sąlygų teorinių tyrimų rezultatai. Banginės vibracinės pavaros dinamika ištirta pagal sudarytą supaprastintą netiesinės virpesių sistemos modelį. Ištirta vibracinės pavaros įėjimo ir išėjimo grandžių sąveika, esant kinematiniam žadinimui. Įvertinta kontaktinio taško virpesių optimalūs parametrai (kontaktinio taško judėjimo elipsine trajektorija su užduotu virpesių amplitudžių santykiu išilgai dviejų statmenų koordinačių ašių ir fazių poslinkiu parametrai); žadinimo zonų išdėstymas, esant daugiafaziniam žadinimui ir rotoriaus paviršiaus parametrai. Ištirti vibracinės pavaros keturių tipų judesiai: du kontaktiniai neslystamieji ir du kontaktiniai slystamieji. Gauta lygčių sistema nuostoviojo judėjimo režimo parametrams nustatyti lygčių sistema.

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INVESTIGATION OF THE STEADY STATE OPERATION CONDITIONS OF ROTATIONAL VIBROACTUATORS

S u m m a r y

The paper presents theoretical investigation results of steady state motion regimes of rotational wave vibroactuator performed on the basis of simplified model of nonlinear vibrating system. The point contact type input – output link interaction of the vibroactuator is analyzed in case of kinematic excitation i.e. under the assumption of optimal vibration parameters of the contact point (elliptical motion with the defined amplitude ratios along two perpendicular axes and the corresponding phase shift ensured by geometrical parameters, excitation zones pattern and multiphase excitation scheme) with the focus made on the investigation of surface characteristics of the rotor. Four types of motion of rotational vibroactuator are analyzed: two contact types with no slipping and two contact types with the phenomena of slipping. The system of equations for the determination of the parameters of steady state type of motion is obtained.