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The deformation problem of a circular elastic flexible plate under variable load

V. B. Rystygulova
Kazakh National Pedagogical University named after Abai, avenue Dostyk 13, 050010 Almaty, Kazakhstan, E-mail: RystygulovaV@mail.ru

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1. Introduction

It is well known that for the solutions of equations with variable coefficients the methods of nonlinear mathematics are applied. These methods allow to solve analytically significant amount of problems given in the classic monographs of the leading experts in the mechanics of deformable bodies, geomechanics and geotechnics, theoretical mechanics, thermoelasticity, heat conduction theory and other areas. In many cases these remarkable monographs contain rather general nonlinear or with variable coefficients differential equation processes [1-7]. Solutions in such publications are being built for a large extent simplified equations or differential equations set. The present work gives analytical solutions and graphics assigned problems method of partial discretization of nonlinear differential equation.

In the work [8] obtained an analytic solution of the problem bending of a thin elastic component axisymmetric inhomogeneous plate with a hole, each component of which has a variable stiffness and working in a non-uniform temperature field. In the work [9] investigated the stress-strain state the cylindrical shell of variable and constant in thickness due to distributed the load. In both works solving problems were obtained by method partial discretization nonlinear the differential equations, since this method allows you to get solve the problem without any simplifications.

In this paper presents the analytical solutions and graphs problem about deformation round elastic, flexible plate loaded by a variable load by method the partial discretization of nonlinear differential equations.

Round and ring plates as elements of machine-building objects meet quite often. These constructions can be attributed turbine disks, the bottom of the combustion chambers of jet engines, gas turbines, boilers, tanks, reservoirs, etc.

2. Formulation of the problem

Let us suppose that an alternating load acts on the circular plate. The plate is warmed irregularly along the radius and thickness (Fig. 1).

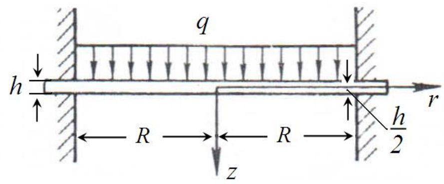

Fig. 1 Round plate with pinched contour

In the present work is obtained an analytic solution to the axisymmetric bending of a thin plate with variable mechanical characteristics, differential equations, which in general have the form [10]:

$$\frac{d}{dr}\left(\nabla^2\Phi\right)=\frac{1}{D_0}\frac{dD_0}{dr}\left(\frac{d^2\Phi}{dr^2}-\frac{\mu}{r}\frac{d\Phi}{dr}\right)+\frac{1}{D_0}\frac{dD_0}{dr}(1-\mu)N_T-(1-\mu)\frac{dN_T}{dr};\tag{1}$$

$$D_2\nabla^2\nabla^2\omega=q_z(r)-\nabla^2M_T-2\frac{dD_2}{dr}\frac{d^3\omega}{dr^3}-\left[\nabla^2D_2+\frac{1}{r}\frac{dD_2}{dr}+\frac{\mu}{r}\frac{dD_2}{dr}\right]\frac{d^2\omega}{dr^2}-\left[\frac{\mu}{r}\frac{d^2D_2}{dr^2}-\frac{1}{r^2}\frac{dD_2}{dr}\right]\frac{d\omega}{dr};\tag{2}$$

where

$$\left.\begin{aligned}&D_0(r)=\frac{1}{1-\mu^2}\int_{-h/2}^{h/2}E(r,z)dz; && N_T(r)=\frac{1}{1-\mu}\int_{-h/2}^{h/2}E(r,z)\alpha(r,z)T(r,z)dz;\\&D_2(r)=\frac{1}{1-\mu^2}\int_{-h/2}^{h/2}E(r,z)z^2dz; && M_T(r)=\frac{1}{1-\mu}\int_{-h/2}^{h/2}E(r,z)\alpha(r,z)T(r,z)\,zdz,\end{aligned}\right\}\tag{3}$$

$q_z(r)=q_0+q_1r$, Φ is function of voltage, μ is Poisson's ratio, q_z is variable lateral load, ω is flexure of an arbitrary point median surface, $D_0(r)$ and $D_2(r)$ are variables stiffness., r is the radius of the plate, h is plate thickness. The quantities $N_T(r)$ and $M_T(r)$ are determine efforts of the bending moment due to heat exposure.

In these equations ∇^2 and $\nabla^2\nabla^2$ have the following meanings:

$$\nabla^2 = \frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right) = \frac{d^2}{dr^2} + \frac{1}{r}\frac{d}{dr}; \qquad \nabla^2\nabla^2 = \frac{1}{r}\frac{d}{dr}\left\{r\frac{d}{dr}\left[\frac{1}{r}\frac{d}{dr}\left(r\frac{d}{dr}\right)\right]\right\} = \frac{d^4}{dr^4} + \frac{2}{r}\frac{d^3}{dr^3} - \frac{1}{r^2}\frac{d^2}{dr^2} + \frac{1}{r^3}\frac{d}{dr}. \quad (4)$$

3. The general solution of the problem

For the illustration obtaining a particular solution of the deformation problem of circular elastic flexible plate we assume that the modulus of elasticity, coefficient of thermal expansion and temperature vary according to the laws:

$$E(r,z) = E_0 r + E_1 z; \quad \alpha(r,z) = \alpha_0 rz; \quad T(r,z) = t_0 rz, \quad (5)$$

where $E(r,z)$ is modulus of elasticity, $\alpha(r,z)$ is coefficient of thermal expansion, $T(r,z)$ is temperature, variables are that depend on the the radius and the plate thickness. E_0, E_1, α_0, t_0 are coefficients, which determine physiccal characteristics of the plate. Using the changes of laws parameter (4) the differential Eqs. (1) and (2) can be written as:

$$\frac{d^3\Phi}{dr^3} + \frac{\mu-1}{r^2}\frac{d\Phi}{dr} = -\frac{1}{6}E_0\alpha_0 t_0 h^3 r^2; \quad (6)$$

$$\frac{d^4\omega}{dr^4} + \frac{4}{r}\frac{d^3\omega}{dr^3} + \frac{1+\mu}{r^2}\frac{d^2\omega}{dr^2} = \frac{12(1-\mu^2)}{E_0 h^3}\left(q_0 - \frac{E_1\alpha_0 t_0 h^5}{20(1-\mu)}\right)\frac{1}{r} + \frac{12(1-\mu^2)}{E_0 h^3}q_1. \quad (7)$$

Discretizing of the second term of Eq. (6) and the third term of the Eq. (7) we will obtain:

$$\frac{d^3\Phi}{dr^3} = \Omega r^2 - \frac{\mu-1}{2}\sum_{k=1}^{n}(r_k + r_{k+1})\left[\frac{1}{r_k^2}\frac{d\Phi(r_k)}{dr}\delta(r-r_k) - \frac{1}{r_{k+1}^2}\frac{d\Phi(r_{k+1})}{dr}\delta(r-r_{k+1})\right]; \quad (8)$$

$$\frac{d^4\omega}{dr^4} + \frac{4}{r}\frac{d^3\omega}{dr^3} = \frac{Q}{r} + Q_1 - \frac{1+\mu}{2}\sum_{k=1}^{n}(r_k + r_{k+1})\left[\frac{1}{r_k^2}v(r_k)\delta(r-r_k) - \frac{1}{r_{k+1}^2}v(r_{k+1})\delta(r-r_{k+1})\right]; \quad (9)$$

where

$$\Omega = -\frac{1}{6}E_0\alpha_0 t_0 h^3; \quad Q = \frac{12(1-\mu^2)}{E_0 h^3}\left(q_0 - \frac{E_1\alpha_0 t_0 h^5}{20(1-\mu)}\right); \quad Q_1 = \frac{12(1-\mu^2)}{ah^3}q_1; \quad v = \frac{d^2\omega}{dr^2}. \quad (10)$$

The solutions of Eqs. (8) and (9) can be written as:

$$\frac{d\Phi}{dr} = \frac{\Omega}{12}r^4 - \frac{\mu-1}{2}\sum_{k=1}^{n}(r_k + r_{k+1})\left[\frac{1}{r_k^2}(r-r_k)\frac{d\Phi(r_k)}{dr}H(r-r_k) - \frac{1}{r_{k+1}^2}(r-r_{k+1})\frac{d\Phi(r_{k+1})}{dr}H(r-r_{k+1})\right] + \boldsymbol{C_1}r + \boldsymbol{C_2}; \quad (11)$$

$$\omega = \frac{Q}{24}r^3 + \frac{Q_1}{120}r^4 - \frac{1+\mu}{2}\sum_{k=1}^{n}(r_k + r_{k+1})\left[r_k^2\left(-\frac{1}{6r} + \frac{1}{6r_k} - \frac{r-r_k}{6r_k^2} + \frac{(r-r_k)^2}{6r_k^3}\right)v(r_k)H(r-r_k) - r_{k+1}^2\left(-\frac{1}{6r} + \frac{1}{6r_{k+1}} - \frac{r-r_{k+1}}{6r_{k+1}^2} + \frac{(r-r_{k+1})^2}{6r_{k+1}^3}\right)v(r_{k+1})H(r-r_{k+1})\right] - \frac{\boldsymbol{B_1}}{6r} + \boldsymbol{B_2}\frac{r^2}{2} + \boldsymbol{B_3}r + \boldsymbol{B_4}. \quad (12)$$

Two differential equations of the third and fourth orders should have seven boundary conditions. But here we can be limited with six conditions as the function Φ does not interest us: it is enough to define its first derivative with respect to *r*.

The problem is solved for rigidly fix the points the supporting the contour (Fig. 2, a) and hinged support along the contour of the plate (Fig. 2, b).

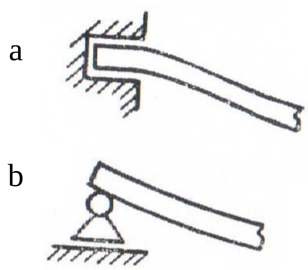

Fig. 2 Schemes of supports: a - rigidly fix the points the supporting the contour; b - hinged support along the contour of the plate

4. Solution of the problem for rigidly fixed points of the supporting contour plate

In rigid fixing adopt the following boundary conditions [11]:

$$\left.\frac{d\Phi}{dr}\right|_{r=0} = 0; \ \left.\left(\frac{d^2\Phi}{dr^2} - \frac{\mu}{r}\frac{d\Phi}{dr}\right)\right|_{r=R} = 0; \ \omega|_{r=R} = 0; \quad \left.\frac{d^2\omega}{dr^2}\right|_{r=0} = 0; \ \left.\frac{d\omega}{dr}\right|_{r=0} = 0; \ \left.\frac{d\omega}{dr}\right|_{r=R} = 0. \quad (13)$$

Using the boundary conditions (13) one can find the integration coefficients after that the solution can be written as:

$$\frac{d\Phi}{dr}=\frac{\Omega}{12}r^4+\left(\frac{\mu}{1-\mu}\frac{\Omega}{12}-\frac{1}{1-\mu}\frac{\Omega}{3}\right)R^3r-\frac{\mu-1}{2}\sum\left(r_k+r_{k+1}\right)\left[\beta_k\left(r-r_k\right)\theta\left(r_k\right)H\left(r-r_k\right)-\beta_{k+1}\left(r-r_{k+1}\right)\times\right.$$
$$\left.\times\theta\left(r_{k+1}\right)H\left(r-r_{k+1}\right)\right]-\frac{\mu(\mu-1)}{2(1-\mu)}\frac{r}{R}\sum\left(r_k+r_{k+1}\right)\left[\beta_k\left(R-r_k\right)\theta\left(r_k\right)H\left(R-r_k\right)-\beta_{k+1}\left(R-r_{k+1}\right)\theta\left(r_{k+1}\right)H\left(R-r_{k+1}\right)\right]+$$
$$+\frac{\mu-1}{2(1-\mu)}r\sum\left(r_k+r_{k+1}\right)\left[\beta_k\theta\left(r_k\right)H\left(R-r_k\right)-\beta_{k+1}\theta\left(r_{k+1}\right)H\left(R-r_{k+1}\right)\right]; \qquad (14)$$

$$\omega=\frac{Q}{24}r^3+\frac{Q_1}{120}r^4-\frac{Q}{24}R^3-\frac{Q_1}{120}R^4+\left(\frac{R}{2}-\frac{r^2}{2R}\right)\left(\frac{Q}{8}R^2+\frac{Q_1}{30}R^3\right)-\frac{1+\mu}{2}\sum_{k=1}^{n}\left(r_k+r_{k+1}\right)\left[r_k^2\left(-\frac{1}{6r}+\frac{1}{6r_k}-\right.\right.$$
$$\left.-\frac{r-r_k}{6r_k^2}+\frac{\left(r-r_k\right)^2}{6r_k^3}\right)v\left(r_k\right)H\left(r-r_k\right)-r_{k+1}^2\left(-\frac{1}{6r}+\frac{1}{6r_{k+1}}-\frac{r-r_{k+1}}{6r_{k+1}^2}+\frac{\left(r-r_{k+1}\right)^2}{6r_{k+1}^3}\right)v\left(r_{k+1}\right)H\left(r-r_{k+1}\right)\Bigg]+$$
$$+\frac{1+\mu}{2}\sum_{k=1}^{n}\left(r_k+r_{k+1}\right)\left[r_k^2\left(-\frac{1}{6R}+\frac{1}{6r_k}-\frac{R-r_k}{6r_k^2}+\frac{\left(R-r_k\right)^2}{6r_k^3}\right)v\left(r_k\right)H\left(R-r_k\right)-r_{k+1}^2\left(-\frac{1}{6R}+\frac{1}{6r_{k+1}}-\frac{R-r_{k+1}}{6r_{k+1}^2}+\right.\right.$$
$$\left.\left.+\frac{\left(R-r_{k+1}\right)^2}{6r_{k+1}^3}\right)v\left(r_{k+1}\right)H\left(R-r_{k+1}\right)\right]-\frac{1+\mu}{2}\left(\frac{R}{2}-\frac{r^2}{2R}\right)\sum_{k=1}^{n}\left(r_k+r_{k+1}\right)\left[r_k^2\left(\frac{1}{6R^2}-\frac{1}{6r_k^2}+\frac{R-r_k}{3r_k^3}\right)v\left(r_k\right)H\left(R-r_k\right)-\right.$$
$$\left.-r_{k+1}^2\left(\frac{1}{6R^2}-\frac{1}{6r_{k+1}^2}+\frac{R-r_{k+1}}{3r_{k+1}^3}\right)v\left(r_{k+1}\right)H\left(R-r_{k+1}\right)\right], \qquad (15)$$

where

$$\left.\begin{aligned}&\beta_k=\frac{1}{r_k^2};\qquad \theta_k=\frac{d\Phi(r_k)}{dr};\qquad v(r_k)=Q\left(\frac{r}{4}-\frac{R}{8}\right)+Q_1\left(\frac{r^2}{10}-\frac{R^2}{30}\right)-\\&-\frac{1+\mu}{2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k^2\left(-\frac{1}{3r^3}+\frac{1}{3r_k^3}\right)v(r_k)H(r-r_k)-r_{k+1}^2\left(-\frac{1}{3r^3}+\frac{1}{3r_{k+1}^3}\right)v(r_{k+1})H(r-r_{k+1})\right]+\\&+\frac{1+\mu}{2R}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k^2\left(\frac{1}{6R^2}-\frac{1}{6r_k^2}+\frac{R-r_k}{3r_k^3}\right)v(r_k)H(R-r_k)-r_{k+1}^2\left(\frac{1}{6R^2}-\frac{1}{6r_{k+1}^2}+\frac{R-r_{k+1}}{3r_{k+1}^3}\right)v(r_{k+1})H(R-r_{k+1})\right].\end{aligned}\right\} \qquad (16)$$

5. Solution of the problem for hinged support along the contour the plate

For pin joint support we adopt the following boundary conditions [11]:

$$\left.\begin{aligned}&\left.\frac{d\Phi}{dr}\right|_{r=0}=0;\ \left.\frac{1}{r}\frac{d\Phi}{dr}\right|_{r=R}=0;\ \left.\frac{d^2\omega}{dr^2}\right|_{r=0}=0;\\&\left.\frac{d\omega}{dr}\right|_{r=0}=0;\ \left.\omega\right|_{r=R}=0;\ \left.D_2\left(\frac{d^2\omega}{dr^2}+\frac{\mu}{r}\frac{d\omega}{dr}\right)\right|_{r=R}=0.\end{aligned}\right\} \qquad (17)$$

Using the boundary condition (17) one can find the integration coefficients after that the solution can be written as:

$$\begin{aligned}&\frac{d\Phi}{dr}=\frac{\Omega}{12}r^4-\frac{\Omega}{12}R^3r-\\&-\frac{\mu-1}{2}\sum\left(r_k+r_{k+1}\right)\left[\beta_k\left(r-r_k\right)\theta\left(r_k\right)H\left(r-r_k\right)-\right.\\&\left.-\beta_{k+1}\left(r-r_{k+1}\right)\theta\left(r_{k+1}\right)H\left(r-r_{k+1}\right)\right]+\\&+\frac{\mu-1}{2}\frac{r}{R}\sum\left(r_k+r_{k+1}\right)\left[\beta_k\left(R-r_k\right)\theta\left(r_k\right)H\left(R-r_k\right)-\right.\\&\left.-\beta_{k+1}\left(R-r_{k+1}\right)\theta\left(r_{k+1}\right)H\left(R-r_{k+1}\right)\right];\end{aligned} \qquad (18)$$

$$\omega=\frac{Q}{24}\left(r^3-R^3\right)+\frac{Q_1}{120}\left(r^4-R^4\right)+\frac{R^2-r^2}{2(1-\mu)}\frac{\mu}{R}\left(\frac{Q}{8}R^2+\frac{Q_1}{30}R^3\right)+\frac{R^2-r^2}{2(1-\mu)}\left(\frac{Q}{4}R+\frac{Q_1}{10}R^2\right)-\frac{1+\mu}{2}\sum_{k=1}^{n}\left(r_k+r_{k+1}\right)\times$$
$$\times\left[r_k^2\left(-\frac{1}{6r}+\frac{1}{6r_k}-\frac{r-r_k}{6r_k^2}+\frac{\left(r-r_k\right)^2}{6r_k^3}\right)v\left(r_k\right)H\left(r-r_k\right)-r_{k+1}^2\left(-\frac{1}{6r}+\frac{1}{6r_{k+1}}-\frac{r-r_{k+1}}{6r_{k+1}^2}+\frac{\left(r-r_{k+1}\right)^2}{6r_{k+1}^3}\right)v\left(r_{k+1}\right)H\left(r-r_{k+1}\right)\right]+$$
$$+\frac{1+\mu}{2}\sum_{k=1}^{n}\left(r_k+r_{k+1}\right)\left[r_k^2\left(-\frac{1}{6R}+\frac{1}{6r_k}-\frac{R-r_k}{6r_k^2}+\frac{\left(R-r_k\right)^2}{6r_k^3}\right)v\left(r_k\right)H\left(R-r_k\right)-r_{k+1}^2\left(-\frac{1}{6R}+\frac{1}{6r_{k+1}}-\frac{R-r_{k+1}}{6r_{k+1}^2}+\right.\right.$$

$$+\frac{(R-r_{k+1})^2}{6r_{k+1}^3}\Bigg)v(r_{k+1})H(R-r_{k+1})\Bigg]-\frac{R^2-r^2}{2(1-\mu)}\frac{\mu}{R}\frac{1+\mu}{2}\sum_{k=1}^{n}(r_k+r_{k+1})\Bigg[r_k^2\left(\frac{1}{6R^2}-\frac{1}{6r_k^2}+\frac{R-r_k}{3r_k^3}\right)v(r_k)H(R-r_k)-$$

$$-r_{k+1}^2\left(\frac{1}{6R^2}-\frac{1}{6r_{k+1}^2}+\frac{R-r_{k+1}}{3r_{k+1}^3}\right)v(r_{k+1})H(R-r_{k+1})\Bigg]-\frac{R^2-r^2}{2(1-\mu)}\frac{1+\mu}{2}\sum_{k=1}^{n}(r_k+r_{k+1})\Bigg[r_k^2\left(-\frac{1}{3R^3}+\frac{1}{3r_k^3}\right)v(r_k)H(R-r_k)-$$

$$-r_{k+1}^2\left(-\frac{1}{3R^3}-\frac{1}{3r_{k+1}^3}\right)v(r_{k+1})H(R-r_{k+1})\Bigg], \qquad (19)$$

where

$$\left.\begin{aligned}&\beta_k=\frac{1}{r_k^2};\qquad \theta_k=\frac{d\Phi(r_k)}{dr};\qquad v(r_k)=Q\frac{r}{4}+Q_1\frac{r^2}{10}-\frac{Q}{1+\mu}\left(\mu\frac{R}{8}+\frac{R}{4}\right)-\frac{Q_1}{1+\mu}\left(\mu\frac{R^2}{30}+\frac{R^2}{10}\right)+\\&+\frac{\mu}{2R}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k^2\left(\frac{1}{6R^2}-\frac{1}{6r_k^2}+\frac{R-r_k}{3r_k^3}\right)v(r_k)H(R-r_k)-r_{k+1}^2\left(\frac{1}{6R^2}-\frac{1}{6r_{k+1}^2}+\frac{R-r_{k+1}}{3r_{k+1}^3}\right)v(r_{k+1})H(R-r_{k+1})\right]-\\&-\frac{1+\mu}{2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k^2\left(-\frac{1}{3r^3}+\frac{1}{3r_k^3}\right)v(r_k)H(r-r_k)-r_{k+1}^2\left(-\frac{1}{3r^3}+\frac{1}{3r_{k+1}^3}\right)v(r_{k+1})H(r-r_{k+1})\right]+\\&-\frac{1}{2}\sum_{k=1}^{n}(r_k+r_{k+1})\left[r_k^2\left(-\frac{1}{3R^3}+\frac{1}{3r_k^3}\right)v(r_k)H(R-r_k)-r_{k+1}^2\left(-\frac{1}{3R^3}+\frac{1}{3r_{k+1}^3}\right)v(r_{k+1})H()\right](R-r_{k+1}).\end{aligned}\right\} \qquad (20)$$

From the solutions (14), (15) and (18), (19) using the mathematical induction method we find the formulas for $\theta_k=\frac{d\Phi(r_k)}{dr}$ and $\omega(r_k)$ at arbitrary point $r=r_k$.

6. The results of calculations

Using mathematical software for engineering calculations MathCAD the graphics profiles of bending of the circular plate along the radius with rigid and pin joint support along the contour are plotted (Figs. 3 and 4).

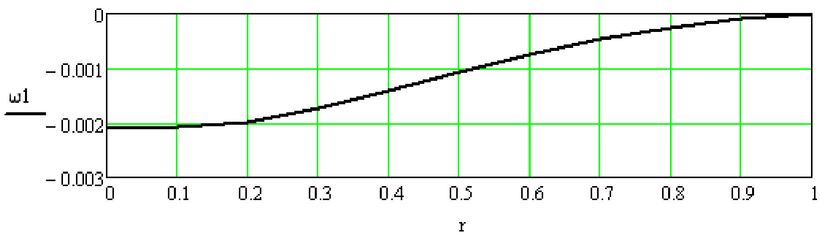

Fig. 3 The profile bending of the radius a circular plate for rigidly fixing

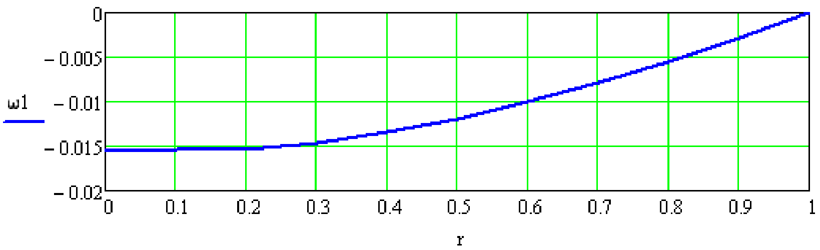

Fig. 4 The profile bending radius of the circular plate for case of simply supported

The same problem was solved for the case when a uniformly distributed normal load acts on a plate. Curve changes are given in Fig. 5 for comparison at normal and variable loading.

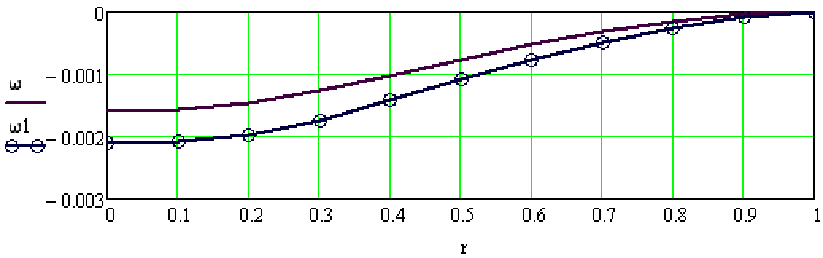

Fig. 5 Deflection curves of the change: $\omega-$deflection at the $q_z=const$, $\omega1-$deflection at the $q_z(r)=q_0+q_1r$

7. Conclusion

Expressions of the bending moments and effort in a median surface of a plate for two cases of fixing of a plate on a contour are received.

Deflections of a round plate (Figs. 3-5) shows deformation under the influence of constants and variable loadings for various boundary conditions. Thus, the method partial discretization of nonlinear differential the equation gives the chance to solve the differential equations for any law of change of mechanical characteristics and to receive the corresponding picture of deformations.

Proceeding from this can be formulated the novelty of research which is to obtain a solutions to the system of equations for the first time analytical method of partial discretization of nonlinear equations. Furthermore, these solutions are completely satisfied to studied system of the differential equations and boundary conditions are shown in high precision of the obtained data.

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V. B. Rystygulova

IŠORINĖS KINTAMOS APKROVOS VEIKIAMOS APSKRITOS TAMPRIOS PLOKŠTELĖS DEFORMACIJOS PROBLEMA

R e z i u m ė

Gautas lenkiamos plonos simetrinės plokštelės su kintamomis mechaninėmis charakteristikomis sprendinio bendras pavidalas. Uždavinys išspręstas netiesinių diferencialinių lygčių dalinės diskretizacijos metodu. Analizuojamos kelios pastovaus ir kintamo skerspjūvio plokštelės deformacijų, esant įvairioms kontūro įtvirtinimo sąlygoms, versijos.

V.B. Rystygulova

THE DEFORMATION PROBLEM OF A CIRCULAR ELASTIC FLEXIBLE PLATE UNDER VARIABLE LOAD

S u m m a r y

The general view of the solution of a bend of a thin axisymmetric plate with the variable mechanical characteristics are received. The task is solved by a method of partial discretization of the nonlinear differential equations. Some versions of the solution of problems of deformations of a plate of a constant and variable thickness for various fixing of a plate on a contour are considered.

Keywords: deformation, circular elastic flexible plate, variable load.

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