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Natural oscillations of single span beam placed on cylindrical supports

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1. Introduction

Oscillations of a beam supported by a motionless hinges is a classical problem and have universally accepted solution. But in some cases beam can be simply placed on a support surface and line of contact will change its location. We assume the circular cylindrical shape of supports (Fig. 1). When radius r is small such support can be approximately replaced by hinge, but if r is much more than length of the beam, variation of the contact line can significantly influence oscillations of the beam. When alternation of the contact line position is significant and when it can be ignored is investigated in this paper.

It is proved that nonlinear mechanical systems with an analytical first integral allow periodic solutions which tend towards linear normal vibration modes as amplitudes tend to zero. Mechanical systems with soft nonlinearity are comprehensively covered by Kauderer [1]. Nonlinear statics and dynamics of a beam sliding on two knife edge supports is investigated by Somnay, Ibrahim [2] Somnay, Ibrahim, Banasic [3]. To simplify the dynamic modelling the mass is concentrated at the center of the beam and exact solution is given in terms of elliptic functions. Dynamic formulation for sliding beams that are deployed or retrieved through prismatic joint are presented by Vu-Quoc, Li [4]. The channel orifice is moving toward the beam, or beam can be sliding when joint is motionless. Geometrically similar problem is investigated by Turnbull, Perkin, Schulch [5], where a beam or string is contacting a circular surface of radius r. Behavior of frictional contact support of a vibrating beam is studied by Ahmadian, Jalali, Pourahmadian [6]. The frictional shear force at the support is identified using its nonlinear normal modes. The nonlinear normal vibration modes are discussed by Mikhlin [7]. Zajaczkowski, Lipinski, Yamada [8], [9] investigated stability of Euler-Bernoulli beams subjected to periodic sliding motions. The complex nature of instability is revealed.

In this paper damping is neglected. Dependence of velocity on deflections in a phase plane and dependence of deflection on time are investigated.

2. Equation of motion

The first mode of oscillating beam usually is of fundamental importance. If the line of a beam coincides with the sine curve deflections of the beam can be determined

$$u(x,t)+u_A=\left[u_C(t)+u_A\right]cos\frac{\pi x}{l}, \quad (1)$$

where $u_C(t)$ and u_A are deflections of the midpoint C and tangent point A (Fig. 1). Positive displacement of A is assumed to be down: $u_A=r(1-cos\,\varphi_A)$. If positive angle of the sine curve is clockwise then $tan\,\varphi=-\frac{du}{dx}=\frac{\pi}{x}(u_C+u_A)sin\frac{\pi x}{l}$ and $tan\,\varphi_A=\pi\frac{u_C+u_A}{l}$. As distance $l=l_o+2r\,sin\,\varphi_A$ the exact equation is $tan\,\varphi_A=\pi\frac{u_C+r(1-cos\,\varphi_A)}{l_o+2r\,sin\,\varphi_A}$. If deflections are small $|\varphi_A|<<1$, then:

$$1-cos\,\varphi_A\approx\frac{sin^2\varphi_A}{2} \quad (2)$$

and $sin\,\varphi_A\approx\pi q-\pi^2\frac{4-\pi}{2}\frac{r}{l_o}q^2$, where $q=\frac{u_C}{l_o}$. l_o is distance between tangent points in equilibrium position. Dynamic distance between the tangent points:

$$l=l_o\left(1+\alpha q-\frac{4-\pi}{4}\alpha^2q^2\right);\quad \alpha=2\pi\frac{r}{l_o} \quad (3)$$

and dependence of any beam point is expressed from Eq. (1):

$$\frac{u(x,t)}{l_o}=\left(q+\frac{\pi}{4}\alpha^2q^2\right)cos\frac{\pi x}{l}-\frac{\pi}{4}\alpha q^2. \quad (4)$$

Only of the second degree of the relative displacement q^2 will be taken into account in this investigation.

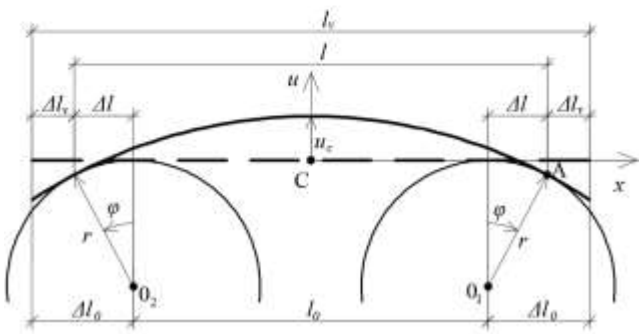

Fig. 1 Beam placed on cylidrical supports

Kinetic energy of the beam $T=\frac{m}{2l_v}\int_{-l_v/2}^{+l_v/2}\dot{u}^2dx$, where m is mass of the beam, l_v – length of the beam and $l_v \ge l$ at any moment. Therefore $l_v - l = 2\Delta l_v \ge 0$, where Δl_v is dependent on time cantilever length, also assumed to be small. When velocity $\dot{u}=du/dt$ from Eq. (4) is calculated dependence of l on time Eq. (3) also should be considered. Kinetic energy of the cantilevers can be calculated assuming that it is a portion of the sine curve or as an absolutely rigid straight line: outcome within the set accuracy is the same. Kinetic energy of the beam:

$$T=\frac{m_o l_o^3}{2}\frac{q^2}{2}\left(1+c_1\alpha q-c_2\alpha^2q^2\right), \quad (5)$$

where $m_o = m/l_v$ is mass per unit length, $c_1=\pi-2=1.142$, $c_2=11-\frac{\pi}{2}-\frac{53\pi^2}{60}=0.7111$.

Potential energy of the beam:

$$\Pi=\frac{EI}{2}\int_{-l/2}^{+l/2}\left(\frac{\partial^2u}{\partial x^2}\right)^2dx=\frac{EI}{2}\frac{\pi^4l_o^2}{2l^3}\left(q+\frac{\pi}{4}\alpha q^2\right)^2, \quad (6)$$

where EI is the stiffness and distance l is presented in Eq. (3). The first natural circular frequency of the simply hinged beam $\lambda=\frac{\pi^2}{l_o^2}\sqrt{\frac{EI}{m_o}}$, therefore the potential energy Eq. (6) $\Pi=\frac{m_o l_o^3}{2}\lambda^2\frac{q^2}{2}\left(1-2\frac{c_3}{3}\alpha q+\frac{c_4}{2}\alpha^2q^2\right)$, where $c_3=3\frac{6-\pi}{4}=2.144$, $c_4=\frac{144-36\pi+\pi^2}{8}=2.548$. Lagrange's equation for the beam is:

$$\ddot{q}\left(1+c_1\alpha q-c_2\alpha^2q^2\right)+\dot{q}^2\left(0.5c_1\alpha-c_2\alpha^2q\right)+$$
$$+\lambda^2q\left(1-c_3\alpha q+c_4\alpha^2q^2\right)=0. \quad (7)$$

Mechanical system is conservative in spite of the term, dependent on velocity.

3. Velocities and displacements

If radius of the cylinder $r\to 0$ and supports can be assumed as a hinges Eq. (7) is $\ddot{q}+\lambda^2q=0$ and after substitution $\ddot{q}=\dot{q}\left(d\dot{q}/dq\right)$ the first integral is $\dot{q}^2+\lambda^2q^2=q_o^2$ or

$$\frac{\dot{q}^2}{\dot{q}_o^2}+\frac{q^2}{q_s^2}=1, \quad q_s=\frac{\dot{q}_o}{\lambda}=const. \quad (8)$$

If relative displacement of the beam center point $x=\frac{q}{q_s}=\frac{\lambda}{\dot{q}_o}q$ is applied the velocity $\frac{dx}{dt}=\pm\lambda\sqrt{1-x^2}$ and therefore $\lambda t=-\int_{\pi/2}^{x}\frac{dx}{\sqrt{1-x^2}}=arc\cos x$, $x=\cos\lambda t$. The positive sign presents solution $x=\sin x$.

Nonlinear differential Eq. (7) when q is considered as a function of time can be transformed to the linear equation of $\dot{q}^2$ as function of variable q. Let independent variable is $z=\alpha q$, the function $w=\dot{z}^2$ and Eq. (7) is multiplied by $\left(1+2\frac{c_2}{c_1}z\right)$. Then the equation of beam motion is:

$$\frac{dw}{dz}\left(1+b_1z+b_3z^2\right)+c_1w+2\lambda^2z\left(1-b_2z-b_4z^2\right)=0, \quad (9)$$

where $b_1=c_1+2c_2/c_1$, $b_2=c_3-2c_2/c_1$, $b_3=c_2$, $b_4=\left(2c_2c_3/c_1\right)-c_4$. A particular solution of the inhomogeneous equation $w_a=B_o+B_1z+B_2z^2+B_3z^3$.

$$\frac{\dot{x}^2}{\lambda^2}=1-x^2-c_1a_sx+\eta_oa_sx^3+\frac{c_1\theta_1}{2}a_s^2x^2-\eta_1a_s^2x^4, \quad (10)$$

satisfies Eq. (9) if $\begin{cases}c_1B_o+B_1=0;\\ \theta_1B_1+2B_2=-2\lambda^2;\\ b_3B_1+\theta_2B_2+3B_3=2\lambda^2b_2;\\ 2b_3B_2+\theta_3B_3=2\lambda^2b_4,\end{cases}$ but the term $3b_3B_3z^4$ is neglected and $\theta_1=b_1+c_1$, $\theta_2=2b_1+c_1$, $\theta_3=3b_1+c_1$. Solution of this linear equations is $B_3=2\lambda^2\kappa_3$, $B_2=2\lambda^2\kappa_2$, $B_1=-2\lambda^2\left(1+2\kappa_2\right)/\theta_1$, $B_o=-B_1/c_1$, where $\eta_4\kappa_2=\left(b_2\theta_1+b_3\right)\theta_3-b_4\theta_1$, $\theta_3\kappa_3=b_4-2b_3\kappa_2$, $\eta_4=\theta_1\theta_2\theta_3-2b_3\left(3\theta_1+\theta_3\right)$. Particular solution (10) is expressed $w_a=B_o\left(1-c_1z\right)+$ $+2\lambda^2z^2\left(\kappa_2+\kappa_3z\right)$. Homogeneous differential Eq. (9) can be expressed as:

$$\frac{dw}{dz}\left(1+s_1z\right)\left(1+s_2z\right)+c_1w=0, \quad (11)$$

where $1+b_1z+b_3z^2=\left(1+s_1z\right)\left(1+s_2z\right)$, therefore s_1 and s_2 are roots of the equation $s^2-b_1s+b_3=0$. The exact solution of (11) is $w_b=C_oF\left(z\right)$, where $F\left(z\right)=\left(\frac{1+s_1z}{1+s_2z}\right)^{-c_1/(s_1-s_2)}$. Derivative and the function are related by $c_1F\left(z\right)+\left(1+b_1z+b_3z^2\right)F'\left(z\right)=0$. By differentiation of this identity derivatives of the higher order can be deduced and Taylor's formula gives $F\left(z\right)=1-c_1z+c_1\theta_1z^2/2-c_1\eta_3z^3/6+c_1\eta_4z^4/24-\ldots$, where $\eta_3=\theta_1\theta_2-2b_3$. The general solution $w=C_oF\left(z\right)+w_a\left(z\right)$ of the inhomogeneous Eq. (9) satisfies the initial value $w_o=\dot{z}_o^2=\alpha^2\dot{q}^2$ when $q=0$ if $C_o=w_o-B_o$. After substitution the general solution and the first integral is:

$$\frac{\dot{x}^2}{\lambda^2}=1-x^2-c_1a_sx+\eta_oa_sx^3+\frac{c_1\theta_1}{2}a_s^2x^2-\eta_1a_s^2x^4, \quad (12)$$

where $a_s=\alpha q_s$ can be considered as the principal parameter to assess the magnitude of a term. Factors c_1, $\eta_o=\left(\theta_2-\frac{2b_3}{\theta_1}\right)\frac{1+2\kappa_2}{3}+2\kappa_3$, $\frac{c_1\theta_1}{2}$, $\eta_1=\frac{\eta_4}{\theta_1}\frac{1+2\kappa_2}{12}$ and the variable $x=q/q_s$ takes the values approximately from the interval [-1, +1].

If $a_s=0$ equation coincides with Eq. (8), because $\frac{\dot{x}}{\lambda}=\frac{\dot{q}}{\dot{q}_o}$. Second in importance are the two terms with factor a_s, next are two last terms with factor a_s^2. Roots of the polynomial:

$$P(x)=1-c_1a_sx-\left(1-\frac{c_1\theta_1}{2}a_s^2\right)x^2+\eta_oa_sx^3-\eta_1a_s^2x^4, \quad (13)$$

depend on parameter a_s. When $a_s=0$ we have quadratic equation and roots $x_1=-1$, $x_2=+1$. If $a_s=0.001$ the roots of Eq. (13) are $x_1=-0.993$, $x_2=1.007$, $x_3=26.15+37.18i$, $x_4=26.15-37.18i$. If $a_s=0.2$ then $x_1=-0.889$, $x_2=1.118$, $x_3=1.193+1.942i$, $x_4=1.193-1.942i$. It can be shown that $P(x)\geq 0$ if and only if $x_1\leq x\leq x_2$, so. $x_1=x_{min}$, $x_2=x_{max}$ and do not exist real velocity solutions in other intervals. In Fig. 2 are depicted harmonic vibration dependence Eq. (8) and two approximations when a_s and a_s^2 are taken into account. The cubic equation $P(x)=0$ is to be solved when the first approximation is examined and all three roots are real if $a_s\leq a_{s3}\approx 0.1831$. Displacement x_v is when velocity of the beam $\dot{x}=\dot{x}_{max}$ and depends on the parameter a_s. All values $x_v<0$ (Table 1). If upper amplitude $A_+=x_v-x_{min}$ and lower amplitude $A_-=x_v-x_{min}$, the ratio $r_A=A_+/A_-$ approaches 1.5 when $a_s\approx 0.15$ even though the whole displacement $A_++A_-=2A_o\approx 2$ for every a_s. The displacement average $x_0=(x_{max}+x_{min})/2$ is positive for all a_s. When $a_s\leq 0.1$, difference between the first and the second approximations is insignificant. If $a_s\geq 1.5$, difference between the oscillations being studied and harmonic dependence of velocity on displacement is substantial.

Table 1

Dependence of average displacement x_v, x_0 and ratio r_A on a_s

a_s	$n = 3$			$n = 4$			$\beta_0 a_s$
	x_0	x_v	r_A	x_0	x_v	r_A	
0.0100	0.0070	-0.0057	1.0257	0.0070	-0.0057	1.0256	0.0070
0.0200	0.0140	-0.0114	1.0520	0.0139	-0.0114	1.0519	0.0139
0.0500	0.0357	-0.0284	1.1361	0.0345	-0.0285	1.1343	0.0348
0.1000	0.0778	-0.0559	1.3005	0.0672	-0.0570	1.2820	0.0700
0.1500	0.1430	-0.0818	1.5326	0.0951	-0.0852	1.4364	0.104
0.2000				0.1148	-0.1130	1.5873	

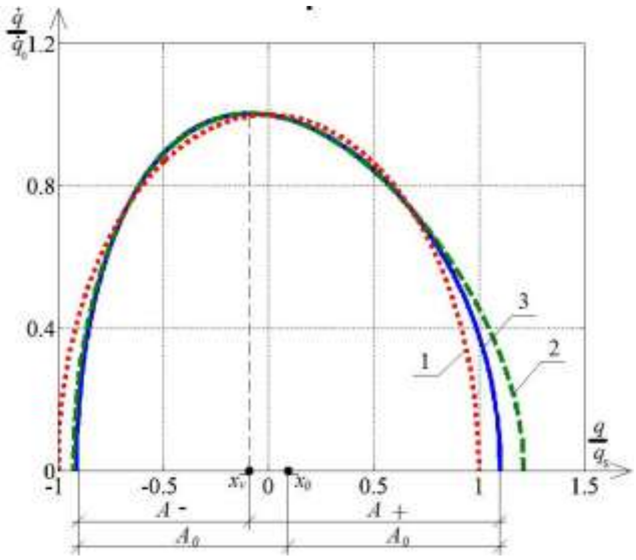

Fig. 2 Dependence of velocity on displacement in phase plane: *1* – harmonic oscillations, *2* – the first approximation, *3* – the second approximation. Parameter $a_s=0.15$

4. Displacement as function of time

If terms with a_s^2 are neglected and $a_s<a_{s3}$ the polynomial Eq. (13) $P(x)=\eta_oa_s(x_3-x)(x_2-x)(x-x_1)$ where all roots x_1,x_2,x_3 are real. Eq. (12) then can be presented $\frac{dx}{\sqrt{(x_3-x)(x_2-x)(x-x_1)}}=\sqrt{\eta_oa_s}\lambda dt$. Integration yields [10, 11] $\sqrt{\eta_oa_s}\lambda t=\frac{2}{\sqrt{x_3-x_1}}F(\varphi_i,k_i)$, where $F(\varphi_i,k_i)=\int_0^{\varphi_i}\frac{d\varphi_i}{\sqrt{1-k_i^2\sin^2\varphi_i}}$ is elliptic integral of the first kind, $\sin^2\varphi_i=\frac{\delta_2}{\delta_1}\cdot\frac{x_2-x}{x_3-x}$, $k_i^2=\delta_1/\delta_2$, $\delta_1=x_2-x_1$, $\delta_2=x_3-x_1$. If $F(\varphi_i,k_i)=\sqrt{\eta_oa_s\delta_2}\,\lambda t/2\equiv u$ then the invers function is the Jacobian elliptic sine: $sn(u,k_i)=\sin\varphi_i=\sqrt{\frac{\delta_2}{\delta_1}\cdot\frac{x_2-x}{x_3-x}}$. The displacement as function of the time is deduced from this equation:

$x = \frac{\delta_1 x_3 sn^2 u - \delta_2 x_2}{\delta_1 sn^2 u - \delta_2}$. Initial values of the first two roots can be $-1, +1$, the first iteration step yields $x_1 = -1 + \frac{\eta_o - c_1}{2} a_s$, $x_2 = 1 + \frac{\eta_o - c_1}{2} a_s$. The third root is approximated $x_3 = 1/(\eta_o a_s)$. Complete elliptic integral $K = \left(1 + \frac{k_i^2}{4}\right)\frac{\pi}{2} = \left(1 + \frac{\eta_o a_s}{2}\right)\frac{\pi}{2}$, $u = \left(1 + \frac{\eta_o a_s}{2}\right)\frac{\lambda t}{2}$, $y = \frac{\pi u}{2K} = \frac{\lambda t}{2}$, therefore the elliptic sine $snu = sin\, y \cdot (1 + 4 q_i cos^2 y)$, where elliptic Jakobi's parameter $q_i = exp(-\pi K'/K) \approx \frac{1}{2}\frac{1 - \sqrt{k_i'}}{1 + \sqrt{k_i'}} \approx \frac{\eta_o a_s}{2}$, k_i' and K' are complementary module and complete elliptic integral. Substitution to (14) yields the first approximation:

$$x = \frac{\eta_o a_s}{2} a_s + cos\, \lambda t\,, \qquad (14)$$

the shifted harmonic oscillations. Applying given above values we have $\eta_o = 2.532$ and $\beta_0 = (\eta_o - c_1)/2 = 0.695$, so $x_0 = \beta_0$ perfectly coincides with data given in Table 1 when $a_s \le 0.02$ and coincides satisfactory when $a_s \le 0.10$. But asymmetry of the real oscillations, displayed by x_v, r_A in Table 1, is not presented in Eq. (14).

The real values of $x_0 < \beta_0$ and solution of the second approximation ($n = 4$) better corresponds to the theoretical values, presented in Eq. (14).

The second approximation of the polynomial (13) has four roots, two of which are complex numbers: $P(x) = \eta_1 a_s^2 G(x)$, $G(x) = \left[(x_r - x)^2 + x_i^2\right](x_2 - x)(x - x_1)$, where $x_3 = x_r + i x_i$, $x_4 = x_r - i x_i$. Eq. (12) can be expressed [12] $\int_{x_2}^{x} \frac{dx}{\sqrt{G(x)}} = \mu_i F(\varphi_i, k_i) = a_s \sqrt{\eta_1} \lambda t$, where $\mu_i = -\sqrt{cos\, \vartheta_1\ cos\, \vartheta_2}/x_i$, $k_i = sin \frac{\vartheta_1 - \vartheta_2}{2}$, $tan\, \vartheta_1 = \frac{x_2 - x_x}{x_i}$, $tan\, \vartheta_2 = \frac{x_1 - x_r}{x_i}$. Solution is:

$$x = \frac{x_2 + x_1}{2} - \frac{x_2 - x_1}{2} \frac{v_i - cnu}{1 - v_i cnu}\,, \qquad (15)$$

where $v_i = tan \frac{\vartheta_2 - \vartheta_1}{2} tan \frac{\vartheta_2 + \vartheta_1}{2}$, $u = \frac{a_s \sqrt{\mu_1}}{\mu_i} \lambda t$, $cnu \equiv cn(u, k_i)$ is Jacobian elliptic cosine. Dependence of relative displacement $x = q/q_s$ on ratio time t and period τ, calculated from Eq. (15), is depicted in Fig. 3. The dashed line discloses difference between harmonic oscillation and solution (15).

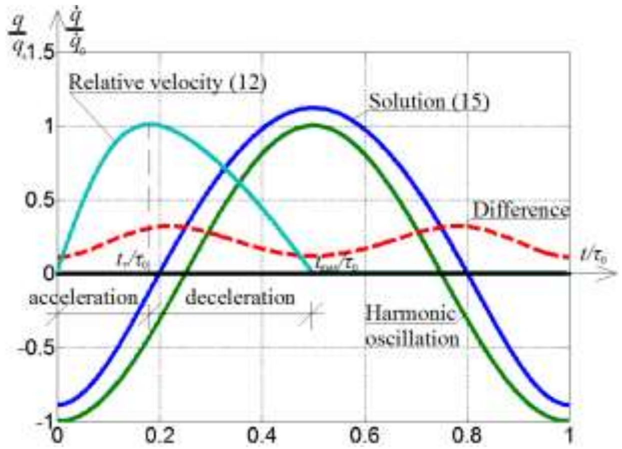

Fig. 3 Dependence of relative displacement q/q_s on time t and period τ ratio when $a_s = 0.2$

Relative velocities $\dot q/\dot q_o$ calculated from Eq. (12) indicates the maximal velocity time t_v. Ratio of the deceleration duration to the acceleration duration $r_\tau = (t_{min} - t_v)/(t_{max} - t_v)$ is presented in Table 2 and strongly depends on a_s, while ratio of the whole period τ with the period of harmonic oscillations τ_0 is approximately constant. This corresponds with the first approximation (14). Dependence of the ratio r_τ on a_s is even greater than r_A in Table 1.

Table 2

Dependence of ratio of periods τ/τ_0 on a_s

a_s	τ/τ_0	r_τ
0.01	1.000	1.031
0.02	1.001	1.073
0.05	1.003	1.187
0.10	1.009	1.398
0.20	1.044	1.764
0.30	0.950	2.000

5. Conclusions

When a single spanbeam is placed on cylindrical surface the contact line and length of the beam are periodically alternating. Nonlinear equation of the beam motion is reduced to linear parametric equation and solved in elliptic functions. The first approximation is harmonic oscillations about some center, elevated above the equilibrium position. Period of the oscillations is almost the same as when supports are hinges, but the half-periods and half-amplitudes in the upper and lower portions of the motion are significantly distorted. The level of distortion depends on the product $\alpha q = 2\pi r u_C/l_o^2$, where r is radius of the support cross section line, u_C – amplitude of the beam center oscillations, l_o – span in equilibrium position (Fig. 1). Therefore, if radius r is big and surface of the support cross section line is very close to straight line a small amplitudes can cause significant distortion of oscillations.

In a similar manner distortions of the oscillation regularity can appear when beam with fixed ends have some support surfaces tangent to the beam.

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ANT CILINDRINIŲ ATRAMŲ PADĖTOS DVIATRAMĖS SIJOS SAVIEJI VIRPESIAI

R e z i u m ė

Tiriami dviatramės sijos, laisvai padėtos ant dviejų apskritų cilindrinių atramų, savieji virpesiai. Sijos ir atramų lietimosi taškas keičia savo padėtį, todėl sijos tarpatramio atstumas taip pat kinta. Netiesinė virpesių diferencialinė lygtis transformuojama į tiesinę diferencialinę lygtį, kurioje nepriklausomas kintamasis yra nebe laikas, bet poslinkis, ir pirmasis judėjimo integralas nustatomas fazinėje plokštumoje. Poslinkio priklausomybė nuo laiko išvedama integruojant antrąjį kartą ir yra išreikšta Jakobi elipsinėmis funkcijomis. Gautas visas virpesių periodas yra beveik toks pat, kaip ir sijai remiantis į lankstus, bet viršutinio ir apatinio pusperiodžio bei viršutiniosios dalies ir apatiniosios judesio dalies amplitudės yra ryškiai pasikeitusios. Tokia virpėjimo proceso deformacija didesnė, kai yra didesnė virpesių amplitudė ir kai didesnis cilindrinių atramų kreivumo spindulys. Ant cilindrinių atramų uždėtos sijos virpesių centras perstumtas į viršų, palyginus su tos pat sijos, atremtos lankstais, virpesiais.

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NATURAL OSCILLATIONS OF SINGLE SPAN BEAM PLACED ON CYLINDRICAL SUPPORTS

S u m m a r y

Oscillations of a single spanbeam placed on two circular cylindrical supports are investigated. Contact line of the beam and the support surface changes its location therefore the span length also alternates. Nonlinear equation of motion is transformed to linear differential equation, where displacement instead of time is the independent variable and the first integral in a phase plane velocity-displacement is deduced. Dependence of displacement on time is expressed in Jacobian elliptic functions in the second integral. The complete period of the oscillations is almost the same as when supports are hinges but the half-periods and the half-amplitudes in upper and lower portions of the motion are significantly distorted. The level of the distortion is increasing when amplitude of oscillations and radius of the support circle are growing. The beam, placed on cylindrical supports, oscillates about a center shifted up compared with the same hinged beam.

Keywords: natural oscillations, simply supported beam, cylindrical support, nonlinear oscillations.

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