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An improved method for optimal shakedown design of circular plates

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1. Introduction

The paper focuses on the methodology for determining the distribution of the optimal limit bending moment M_0 of an elastic-plastic circular plate subjected to variable repeated loading at shakedown. The behaviour of materials is described by an ideal elastic plastic model; a possible unloading phenomenon of a cross section is ignored (variable repeated loading is defined only by its upper and lower bounds). In the previous paper [1], the optimization of the plates was performed applying Rozen's gradient projection method [2] which is not very convenient for a direct use of the influence matrices $\boldsymbol{H}$ and $\boldsymbol{G}$ of residual displacements $\boldsymbol{u}_r$ and forces $\boldsymbol{S}_r$ when nonlinear von Mises yield criterion is applied. Newly derived compatibility conditions for residual deformations $\boldsymbol{\Theta}_r$ improve the quality of the mathematical model of the optimization problem and avoid a direct use of the abovementioned influence matrices. A new algorithm, along with *MATLAB* nonlinear optimization tools [3], substantially improved the numerical implementation of a shakedown plate optimization problem and allowed for a more distinct interpretation of the results in terms of ultimate and serviceability limit states. These qualities of optimization are important in civil and transport engineering where round plates under cyclic loading are commonly designed.

The paper formulates the analysis problem of the internal residual forces of the structure at shakedown (when loading and limit moments of the plate are known) as a convex nonlinear mathematical programming problem. The interpolation functions of the internal forces of equilibrium finite elements strongly agree with the discretization of differential equilibrium equations [4, 5].

A complete system of equations for the shakedown state of the elastic-plastic structure is obtained when Kuhn-Tucker optimality conditions [2] are applied for the analysis problem. If an appropriate optimality criterion is chosen, the system becomes the basis for the optimization problem of plate optimization at shakedown [6-11]. Then, the prime unknowns are limit bending moments of plate finite elements $\boldsymbol{M}_0$, self-balanced residual moments $\boldsymbol{M}_r$ and plastic multipliers $\boldsymbol{\lambda}$ (terms and notations are the same as those in our earlier works [1, 12]).

The methodology is illustrated with a numerical example of circular plate optimization. The obtained results are based on the assumption of small deformation.

2. Discrete model of a circular plate at shakedown

The geometry of the plate under consideration is known, and the actual load process $\boldsymbol{F}(t)$ is described via time-independent upper $\boldsymbol{F}_{sup}$ and lower $\boldsymbol{F}_{inf}$ variation bounds ($\boldsymbol{F}_{inf} \le \boldsymbol{F}(t) \le \boldsymbol{F}_{sup}$). The discrete model is derived dividing the plate into s finite elements, every of which contains ν nodal points. Thus, the total number of sections in the discrete model of the plate is $\zeta = s \times \nu$. The stress-strain field of the discrete model is described by ζ-size vectors of bending moments $\boldsymbol{M} = \left[\boldsymbol{M}_1 \ \boldsymbol{M}_2 \ ... \ \boldsymbol{M}_\zeta \right]^T$ and deformations $\boldsymbol{\Theta} = \left[\boldsymbol{\Theta}_1 \ \boldsymbol{\Theta}_2 \ ... \ \boldsymbol{\Theta}_\zeta \right]^T$. The equilibrium and geometric equations for the plate are [4, 12]:

$$\sum_k \boldsymbol{A}_k \boldsymbol{M}_k = \boldsymbol{F} \ \text{ or } \ \boldsymbol{A}\boldsymbol{M} = \boldsymbol{F}\,; \tag{1}$$

$$\left.\begin{array}{l} \boldsymbol{A}_k^T \boldsymbol{u} - \boldsymbol{D}_k \boldsymbol{M}_k = 0 \ \text{ or } \ \boldsymbol{A}^T \boldsymbol{u} - \boldsymbol{D}\boldsymbol{M} = 0; \\ k = 1, 2, ..., s; \ k \in K, \end{array}\right\} \tag{2}$$

where $\boldsymbol{A}(m \times n)$ is the matrix of the coefficients of equilibrium equations, m is a degree of freedom of the discrete model of the plate, n is the number of internal forces; $\boldsymbol{D}(n \times n)$ is a block-diagonal matrix of elemental flexibilities $\boldsymbol{D}_k = \int_{A_k} \boldsymbol{N}_k^T(\rho)\, \boldsymbol{d}_k\, \boldsymbol{N}_k(\rho) dA$ and $\boldsymbol{u} = \left[u_1 \ u_2 \ ... \ u_m \right]^T$ is the global displacement vector. The interpolation function of bending moments applying the finite element k shape function $\boldsymbol{N}_k(\rho)$ is $\boldsymbol{M}_k(\rho) = \boldsymbol{N}_k(\rho) \cdot \boldsymbol{M}_k$; $\boldsymbol{d}_k$ is the matrix of the physical regularities (elasticity coefficients) of an element. Von Mises nonlinear yield condition will be verified in all nodal points $i = 1, 2, ..., \zeta$ ($i \in I$):

$$\varphi_i = (M_{0i})^2 - \boldsymbol{M}_i^T \boldsymbol{\Pi}_i \boldsymbol{M}_i \ge 0 \tag{2}$$

or

$$\varphi_i = (M_{0i})^2 - \begin{bmatrix} M_{\rho i} & M_{\theta i} \end{bmatrix} \begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix} \begin{bmatrix} M_{\rho i} \\ M_{\theta i} \end{bmatrix} \ge 0 \,. \tag{3}$$

Limit bending moment M_{0i} is assumed to be constant per finite element area. Radial M_ρ and circular M_θ bending moments describe the stress state of the circular plate.

It is convenient to pick out residual bending moments $\boldsymbol{M}_r$, displacements $\boldsymbol{u}_r$ and strains $\boldsymbol{\Theta}_r = \boldsymbol{D}\boldsymbol{M}_r + \boldsymbol{\Theta}_p$ when analysing the plate at shakedown. If the $j=1, 2, ..., p$ ($j \in J$) vertices of the elastic force $\boldsymbol{F}(t)$ locus exist, then, the combinations of elastic bending moments $\boldsymbol{M}_e$ and displacements $\boldsymbol{u}_e$ are determined by equations $\boldsymbol{M}_{ej} = \boldsymbol{\alpha}\boldsymbol{F}_j$, $\boldsymbol{u}_{ej} = \boldsymbol{\beta}\boldsymbol{F}_j$, where $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ are the influence matrices of elastic response. When omitting detailed investigation into loading history, yield conditions (2) take a form:

$$\varphi_{i,j} = \left(M_{0i}\right)^2 - \boldsymbol{M}_{i,j}^T \boldsymbol{\Pi}_i \boldsymbol{M}_{i,j} \geq 0 ; \tag{4}$$

$$\boldsymbol{M}_{i,j} = \boldsymbol{M}_{ei,j} + \boldsymbol{M}_{ri} ; \; i \in I ; \; j \in J . \tag{5}$$

In this case, equilibrium (1) and geometrical **Error! Reference source not found.** equations are:

$$\boldsymbol{A}\boldsymbol{M}_r = \boldsymbol{0} \tag{6}$$

and

$$\boldsymbol{A}^T \boldsymbol{u}_r = \boldsymbol{D}\boldsymbol{M}_r + \boldsymbol{\Theta}_p , \tag{7}$$

where $\boldsymbol{\Theta}_p = \left(\boldsymbol{\Theta}_{pi}\right)^T$ is a vector of plastic strains. For each cross-section, $\boldsymbol{\Theta}_{pi}$ is equal to:

$$\boldsymbol{\Theta}_{pi} = 2\sum_j \lambda_{i,j} \boldsymbol{\Pi}_i \boldsymbol{M}_{i,j} , \tag{8}$$

where $\lambda_{i,j} \geq 0$, $i \in I$, $j \in J$ are plastic multipliers.

3. Complete set of equations for the analysis problem of the plate

The analysis problem defines the determination of the stress-strain state of the plate when physical parameters and variable repeated loading is known in advance. The problem of static formulation represents the principle of minimum complementary energy: of all statically admissible vectors $\boldsymbol{M}_r$ of residual bending moments, the actual one corresponds to the minimum of complementary deformation energy of the structure at shakedown. The mathematical model of the problem stated on the basis of above-mentioned principle, reads:

find

$$\min \; \frac{1}{2}\boldsymbol{M}_r^T \boldsymbol{D}\boldsymbol{M}_r ; \tag{9}$$

subject to

$$\boldsymbol{A}\boldsymbol{M}_r = \boldsymbol{0} ; \tag{10}$$

$$\boldsymbol{\varphi}_j = \left(\boldsymbol{M}_0\right)^2 - \boldsymbol{\Gamma}\left(\boldsymbol{M}_j^T\right)\boldsymbol{\Pi}\boldsymbol{M}_j \geq \boldsymbol{0} ; \tag{11}$$

$$\boldsymbol{M}_j = \boldsymbol{M}_r + \boldsymbol{M}_{ej} ; \; \boldsymbol{M}_{ej} = \boldsymbol{\alpha}\boldsymbol{F}_j ; \; j \in J . \tag{12}$$

The optimal solution to the problem (9)-(12) is residual bending moments $\boldsymbol{M}_r^*$ that ensure the state of shakedown; a possibility of determining sections where plastic deformations $\boldsymbol{\Theta}_p$ appear opens up. Block-diagonal matrix $\boldsymbol{\Pi}(n \times n)$ consists of blocks $\boldsymbol{\Pi}_i$. Operator $\boldsymbol{\Gamma}\left(\boldsymbol{M}_j^T\right)$ arranges the components of vector $\boldsymbol{M}_j^T$ in such a way, that yield conditions (3) would be verified in every section i of the discrete model.

The constraints (10)-(12) of the problem (9)-(12) along with Kuhn-Tucker conditions constitute the complete system of equations defining the stress-strain state of the plate at shakedown:

$$\boldsymbol{A}\boldsymbol{M}_r^* = \boldsymbol{0} ; \tag{13}$$

$$\left(\boldsymbol{M}_0\right)^2 - \boldsymbol{\Gamma}\left(\boldsymbol{M}_r^* + \boldsymbol{M}_{ej}\right)^T \boldsymbol{\Pi}\left(\boldsymbol{M}_r^* + \boldsymbol{M}_{ej}\right) \geq \boldsymbol{0} ; \tag{14}$$

$$\boldsymbol{D}\boldsymbol{M}_r^* + 2\sum_j \boldsymbol{\Pi}\boldsymbol{\Gamma}^T\left(\boldsymbol{M}_r^* + \boldsymbol{M}_{ej}\right)^T \boldsymbol{\lambda}_j - \boldsymbol{A}^T \boldsymbol{u}_r = \boldsymbol{0} ; \tag{15}$$

$$\boldsymbol{\lambda}_j^T \left[\left(\boldsymbol{M}_0\right)^2 - \boldsymbol{\Gamma}\left(\boldsymbol{M}_r^* + \boldsymbol{M}_{ej}\right)^T \boldsymbol{\Pi}\left(\boldsymbol{M}_r^* + \boldsymbol{M}_{ej}\right)\right] = 0 ; \tag{16}$$

$$\boldsymbol{\lambda}_j \geq \boldsymbol{0} ; \; \boldsymbol{\lambda}_j = \left[\lambda_{1j} \; \lambda_{2j} \; ... \; \lambda_{\zeta j}\right]^T ; \tag{17}$$

$$\boldsymbol{M}_{ej} = \boldsymbol{\alpha}\boldsymbol{F}_j ; \; \boldsymbol{F}_{inf} \leq \boldsymbol{F}_j \leq \boldsymbol{F}_{sup} ; \; j \in J . \tag{18}$$

The components of vector $\boldsymbol{\lambda}_j^T$ under conditions (15) are arrayed so that plastic deformations in every section i would be obtained according to Eq. (8): $\boldsymbol{\Theta}_{pi} = 2\sum_j \lambda_{i,j} \boldsymbol{\Pi}_i \boldsymbol{M}_{i,j}$. Recall that Kuhn-Tucker conditions state that solution $\boldsymbol{M}_r^*$ is global if multipliers $\boldsymbol{\lambda}_j \geq \boldsymbol{0}$ ($j \in J$) and displacements $\boldsymbol{u}_r$, satisfying conditions (15)-(17), exist [2].

4. Transformations of the mathematical optimization model

The problem of determining the distribution of optimal limit bending moment $\boldsymbol{M}_0 = \left[M_{01} \; M_{02} \; ... \; M_{0s}\right]^T$ is relevant to practical design. The problem of an optimal shakedown design of a circular plate is formulated as follows: for given load variation bounds $\boldsymbol{F}_{sup}$, $\boldsymbol{F}_{inf}$, the vector of limit forces $\boldsymbol{M}_0$, satisfying optimality criterion $\min f\left(\boldsymbol{M}_0\right)$ and the constraints of shakedown and stiffness, should be found [13]. A general mathematical model of plate optimization reads:

find

$$\min f\left(\boldsymbol{M}_0\right) ; \tag{19}$$

subject to

$$\boldsymbol{A}\boldsymbol{M}_r = \boldsymbol{0} ; \tag{20}$$

$$\boldsymbol{\varphi}_j = \left(\boldsymbol{M}_0\right)^2 - \boldsymbol{\Gamma}\left(\boldsymbol{M}_r + \boldsymbol{M}_{ej}\right)^T \boldsymbol{\Pi}\left(\boldsymbol{M}_r + \boldsymbol{M}_{ej}\right) \geq \boldsymbol{0} ; \tag{21}$$

$$\boldsymbol{D}\boldsymbol{M}_r + 2\sum_j \boldsymbol{\Pi}\boldsymbol{\Gamma}^T\left(\boldsymbol{M}_r + \boldsymbol{M}_{ej}\right)^T \boldsymbol{\lambda}_j - \boldsymbol{A}^T \boldsymbol{u}_r = \boldsymbol{0} ; \tag{22}$$

$$\lambda_j^T \varphi_j = 0\,;\ \lambda_j \geq 0\,; \tag{23}$$

$$M_{0,min} \leq M_0 \leq M_{0,max}\,; \tag{24}$$

$$u_{\min} \leq u_{ej} + u_r \leq u_{\max}\,; \tag{25}$$

$$M_{ej} = \alpha F_j\,;\ u_{ej} = \beta F_j\,;\ j \in J\,. \tag{26}$$

The objective function (19) can express the optimal distribution of limit forces (for example, $\min L^T M_0$, where L is a vector of element areas) or an optimal volume of the structure. It is a problem of continuous optimization where unknowns include M_0, M_r, u_r, λ_j. The multi-extremity of the problem is determined by complementary slackness conditions for mathematical programming (23). According to Eurocode [14] requirements, the ultimate limit state is secured byEq. (21) while serviceability limit state – by Eq. (25). A shortcoming of the model (19)-(26) is incapability to determine an unloading phenomenon, i.e. a vector of residual displacements u_r determined by the non-monotonic process of plastic deformations in the shakedown state may be non-unique.

The model (19)-(26) can be transformed by eliminating residual displacements u_r from geometric Eq. (22):

$$A^T u_r = D M_r + \Theta_p\,;\ \left[\frac{A^{(1)T}}{A^{(2)T}}\right] u_r = \left[\frac{D^{(1)}}{D^{(2)}}\right] M_r + \left[\frac{\Theta_p^{(1)}}{\Theta_p^{(2)}}\right]; \tag{27}$$

$$u_r = \left(A^{(1)T}\right)^{-1} D^{(1)} M_r + \left(A^{(1)T}\right)^{-1} \Theta_p^{(1)}, \tag{28}$$

where $A^{(1)T}$ is a sub-matrix of the matrix $A^T = \left[A^{(1)} \quad A^{(2)}\right]^T$ that has an inverse matrix (corresponds to the sub-matrix $D^{(1)}$ of the flexibility matrix D and to sub-vector $\Theta_p^{(1)}$; selection method of the lines for sub-matrix $A^{(1)T}$ is based only on the existence of its inverse matrix). Then, geometric Eq. (22) are interchanged with compatibility equations for residual deformations (number of these equations equals to the static indeterminacy of the structure $k_0 = n - m$):

$$A^{(2)T}\left[\left(A^{(1)T}\right)^{-1}\left(D^{(1)} S_r + \Theta_p^{(1)}\right)\right] = D^{(2)} S_r + \Theta_p^{(2)}\,;$$

$$\left[A^{(2)T}\left(A^{(1)T}\right)^{-1} D^{(1)} - D^{(2)}\right] S_r = \left[-A^{(2)T}\left(A^{(1)T}\right)^{-1};\ I\right] \Theta_p\,;$$

$$B_r S_r = B_p \Theta_p\,. \tag{29}$$

where $B_r = \left[A^{(2)T}\left(A^{(1)T}\right)^{-1} D^{(1)} - D^{(2)}\right]$ and $B_p = \left[-A^{(2)T}\left(A^{(1)T}\right)^{-1};\ I\right]$.

Then, the optimization problem (19)-(26) becomes:

find

$$\min f\left(M_0\right), \tag{30}$$

subject to

$$A M_r = 0\,; \tag{31}$$

$$\varphi_j = \left(M_0\right)^2 - \Gamma\left(M_r + M_{ej}\right)^T \Pi \left(M_r + M_{ej}\right) \geq 0\,; \tag{32}$$

$$B_r S_r = B_p \Theta_p\,; \tag{33}$$

$$\Theta_p = 2\sum_j \Pi \Gamma^T \left(M_r + M_{ej}\right)^T \lambda_j\,; \tag{34}$$

$$\lambda_j^T \varphi_j = 0\,;\ \lambda_j \geq 0\,; \tag{35}$$

$$M_{0,min} \leq M_0 \leq M_{0,max}\,; \tag{36}$$

$$u_{min} \leq u_{ej} + \left[\left(A^{(1)T}\right)^{-1}\left(D^{(1)} M_r + \Theta_p^{(1)}\right)\right] \leq u_{max}\,; \tag{37}$$

$$M_{ej} = \alpha F_j\,;\ u_{ej} = \beta F_j\,;\ j \in J\,. \tag{38}$$

The unknowns of the problem (30)-(38) are M_0, M_r, λ_j. The structure of plastic deformation vector θ_p is as follows: $\Theta_p = \left[\Theta_p^{(1)}\ \Theta_p^{(2)}\right]^T = \left[\Theta_{p1}\ \Theta_{p2}\ \ldots\ \Theta_{pn}\right]^T$.

The further rearrangement of the problem (19)-(26) is possible by the elimination of equilibrium equations $A M_r = 0$:

$$M_r = B M_r^{(2)} = B_p^T M_r^{(2)}\,. \tag{39}$$

Matrix $B = \begin{bmatrix} -\left(A^{(1)}\right)^{-1} A^{(2)} \\ I \end{bmatrix}$ is derived from equilibrium equations $A^{(1)} M_r^{(1)} + A^{(2)} M_r^{(2)} = 0$ ($M_r^{(2)}$ are the unknowns of the force method). Then, the problem (30) -(38) is simplified even more in terms of unknowns:

find

$$\min\ f\left(M_0\right); \tag{40}$$

subject to

$$\varphi_j = \left(M_0\right)^2 - \Gamma\left(B M_r^{(2)} + M_{ej}\right)^T \Pi \left(B M_r^{(2)} + M_{ej}\right) \geq 0\,; \tag{41}$$

$$B_r B M_r^{(2)} = B_p \Theta_p\,; \tag{42}$$

$$\Theta_p = 2\sum_j \Pi \Gamma^T \left(B M_r^{(2)} + M_{ej}\right)^T \lambda_j\,; \tag{43}$$

$$\lambda_j^T \varphi_j = 0\,;\ \lambda_j \geq 0\,; \tag{44}$$

$$M_{0,min} \leq M_0 \leq M_{0,max}\,; \tag{45}$$

$$u_{min} \leq u_{ej} + \left[\left(A_1^T\right)^{-1}\left(D^{(1)} B M_r^{(2)} + \Theta_p^{(1)}\right)\right] \leq u_{max}\,; \tag{46}$$

$$M_{ej} = \alpha F_j\,;\ u_{ej} = \beta F_j\,;\ j \in J\,. \tag{47}$$

The unknowns of the problem (40)-(47) are limit bending moments M_0, only a part of residual moments $M_r^{(2)}$ and plastic multipliers λ_j. The problem is solved in an iterative manner. Regarding the solution to the problem (40)-(47) of the initial data (initial matrix D), the vector of limit moments $\boldsymbol{M}_0^*$ is obtained with the help of which, a new flexibility matrix is formed, i.e. new influence matrixes $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$ and $\boldsymbol{B}_r$ are formed and new elastic forces $\boldsymbol{M}_{ej} = \boldsymbol{\alpha}\boldsymbol{F}_j$ and displacements $\boldsymbol{u}_{ej} = \boldsymbol{\beta}\boldsymbol{F}_j$ are calculated. The iterative process is continued until the change of the values of consecutive solutions is within desirable precision.

5. Numerical example of a circular plate optimization

The circular plate of radius $R = 0.9$ m hinge-supported at its outer contour is under consideration (Fig. 1). The plate is subjected to a symmetrically and uniformly distributed load varying in the range of $-95\ \text{kN/m}^2 \le q(t) \le 100\ \text{kN/m}^2$ and constant uniformly distributed bending moment $M = 36.25$ kN applied to the outer contour of the plate. The material modulus of elasticity is $E = 210$ GPa, yield stress – $\sigma_y = 210$ MPa, Poisson's ratio – $\nu = 1/3$ and the initial thickness - $t = 0.03$ m. Optimal limit bending moments of elements $M_{0,k}$, $k = 1,2,...,6$ are to be found. The moments directly determine the thickness of the plate: $M_{0,k} = \sigma_y t_k^2 / 4$.

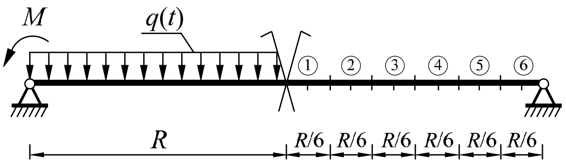

Fig. 1 Loading and discrete model of the circular plate

The discrete model of the structure is constituted of six uniformly distributed finite elements possessing three nodes each. Positive directions of internal forces are shown in Fig. 2.

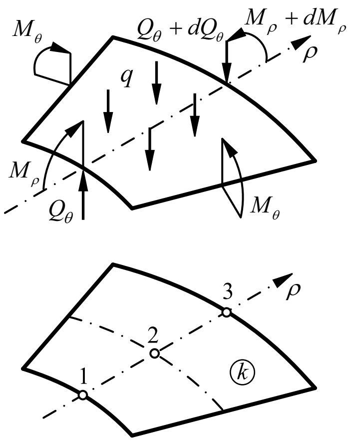

Fig. 2 The finite element of a plate

The polar coordinate system is located in the centre of the plate, and therefore it is enough to investigate only the radius of the plate, because internal forces and displacements do not depend on the angular coordinate.

Elastic bending moments $\boldsymbol{M}_{ej}$ ($j = 1,2$) are calculated applying influence matrix $\boldsymbol{\alpha}$. The optimal distribution of limit bending moments of the plate at shakedown is calculated using the mathematical model (30)-(38). An admissible plate deflection in the centre is bounded to $-0.03\ \text{m} \le u_{r,1} + u_{ej,1} \le 0.03\ \text{m}$. The results of optimization are presented in Table. The optimal thicknesses of the plate are indicated in the last row of the table. The optimal solution is achieved when the problem converges (Fig. 3).

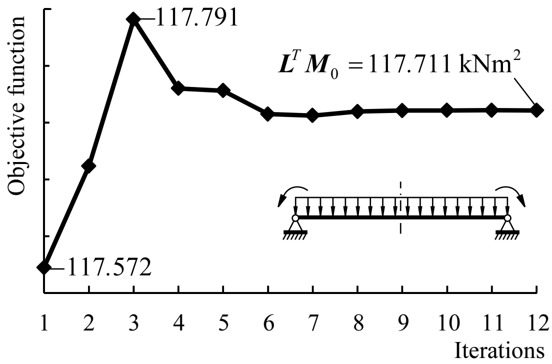

Fig. 3 The convergence of the objective function

Table

The convergence of limit bending moments (kN) and optimal plate thickness

Iter.	$M_{0,1}$	$M_{0,2}$	$M_{0,3}$	$M_{0,4}$	$M_{0,5}$	$M_{0,6}$
1	52.402	52.395	51.178	48.930	45.134	40.829
2	53.134	53.222	52.391	49.681	44.831	39.871
3	53.623	53.779	52.189	49.462	45.562	39.473
…	…	…	…	…	…	…
10	52.410	52.414	52.444	50.041	45.810	39.166
11	52.411	52.415	52.445	50.038	45.810	39.166
12	**52.411**	**52.415**	**52.445**	**50.038**	**45.810**	**39.166**
***t*, mm**	31.6	31.6	31.6	30.9	29.5	27.3

6. Conclusions

1. Compatibility conditions for residual deformations allow finding efficient solutions to optimization and analysis problems of circular plates at shakedown with reference to nonlinear von Mises yield criterion.
2. The new improved methodology reduces the number of unknowns and provides successful convergence of the iterative optimization problem of the circular plate with realistic geometric and physical parameters.
3. The presented methodology is suitable for practical applications of steel plate design problems (cover requirements for ultimate and serviceability limit states).

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PAGERINTA PRISITAIKANČIŲ APVALIŲ PLOKŠČIŲ OPTIMALAUS PROJEKTAVIMO METODIKA

R e z i u m ė

Straipsnyje nagrinėjamas prisitaikančių tampriai plastinių apvalių ir žiedinių lenkiamų plokščių ribinio momento optimalaus pasiskirstymo uždavinys. Plokštės geometrija žinoma, kintama-kartotinė apkrova apibūdinama tik viršutinėmis ir apatinėmis nuo laiko nepriklausančiomis kitimo ribomis (galimas skerspjūvių nusikrovimas ignoruojamas). Diskretizacijai pasitelkti pusiausviri baigtiniai elementai, plokštės saugos ribinį būvį apsprendžia netiesinė Mizeso takumo sąlyga, tinkamumo – įlinkių ribojimo reikalavimai. Konstrukcijos įrąžų ir deformacijų skaičiavimo (analizės) uždavinys formuluojamas kaip pilnutinė tampriai-plastinės plokštės prisitaikomumo būvio lygčių sistema. Liekamųjų deformacijų darnos lygčių ir MATLAB komplekso netiesinių uždavinių sprendimo galimybių panaudojimas įgalino sukurti pagerintą prisitaikančių plokščių optimizavimo uždavinio matematinį modelį, o kartu ir patobulinti sprendimo algoritmą. Tyrimai atlikti ir skaitinių eksperimentų rezultatai gauti, laikantis mažų poslinkių prielaidos.

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AN IMPROVED METHOD FOR AN OPTIMAL SHAKEDOWN DESIGN OF CIRCULAR PLATES

S u m m a r y

The paper analyzes the problem of distributing an optimal limit bending moment of elastic-plastic circular and annular plates subjected to variable repeated loading at shakedown. The geometry of the plate is known and variable repeated loading is defined by time-independent upper and lower bounds (unloading phenomenon of a cross section is ignored). Equilibrium finite elements are applied for a discrete model. The safety of the plate is described with reference to nonlinear von Mises yield criterion while serviceability – by displacement limitations. The analysis problem of internal forces and deformations is formulated as a complete system of equations for an elastic-plastic plate in the shakedown state. The implementation of compatibility conditions for residual displacements and MATLAB nonlinear optimization tools allowed creating an improved mathematical model for optimizing the plate and its effective numerical realization. Research methods and numerical results are based on the assumption of small deformations.

Keywords: optimal shakedown design, plates, finite equilibrium elements, energy principles, mathematical programming, MATLAB.

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