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Modelling and analysis of systems with cylindrical piezoelectric transducers

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1. Introduction

A proposal of mathematical algorithm using in order to modelling and testing of torsional vibrating mechatronic system is presented in this work. A mechanical subsystem is a shaft. On the shaft an sectional, cylindrical piezoelectric transducer is glued [1]. A dynamic flexibility of the mechanical subsystem was calculated twice using an exact Fourier and approximate Galerkin method. It is impossible to use the exact method of separation of variables in order to analyse mechatronic systems, this is why only the approximate method was used in this case. The approximate method was first verified [2, 3]. Obtained results were juxtaposed and presented on charts.

Nowadays, a lot of applications of both direct and reverse piezoelectric effects are known. Benefits of using piezoelectric transducers for control or damping of vibrations are broad band of transmitted signals and high efficiency of conversion of mechanical energy into electrical energy and in the opposite direction. The possibility of designing and produce the piezoelectric transducer with any shape, suitable for the application is also important as well as simplicity of this type of systems, especially in case of passive vibration damping. In the presented case of torsional vibrating mechatronic system with piezoelectric transducer used as the vibration actuator, application of the sectional, cylindrical transducer is proposed [1]. Using such kind of innovate solutions it is possible to use piezoelectric materials in new, designed technical devices. It is also possible to obtain development of their effectiveness and less energy consumption, what is consistent with the direction of the current activities of constructors [4]. Knowledge of the dynamic characteristics of the designed systems is essential for the proper operation and should be taken into consideration during the design phase as well as verified during operation of the system [5].

Presented paper is a part of research works realized by scientists from Gliwice related with analysis and synthesis of mechatronic systems [2, 3, 6-11]. Works are realized using classical and non-classical methods and also computer aided [12]. Both discrete and discrete – continuous systems were being taken into consideration [13, 14]. Proposed methods can be successfully used to the analysis of various piezoelectric materials, including composite transducers, as it was presented in the previous papers [2, 3]. The presented work is an introduction to the synthesis of considered, complex, vibrating mechatronic systems with piezoelectric transducers.

2. The dynamic flexibility of the mechanical subsystem using the approximate method

Galerkin method is an approximate method of solving problems with continuous operators. The purpose is to bring to the weak variational form, function space discretization and receiving a system of linear equations. A solution of differential equation of motion was assumed as a product of eigenfunctions of displacement and time [2, 3]:

$$\varphi(x,t)=\sum_{n=1}^{\infty}\Phi(x)T(t),\quad n=1,2,3... \tag{1}$$

In agreement with the approximate method the eigenfunction of displacement was assumed as:

$$\Phi(x)=A\,sin(k\,x),\ \text{where}\ k=\frac{(2n-1)\pi}{2L}. \tag{2}$$

Boundary conditions were written as follows:

$$\varphi(x,t)=0\big|_{x=0};\qquad \frac{\partial\varphi(x,t)}{\partial x}=A\big|_{x=L}, \tag{3}$$

where A denotes an amplitude of vibration. Taking into account boundary and initial conditions the assumed equation of displacement can be written down as:

$$\varphi(x,t)=\sum_{n=1}^{\infty}sin(k\,x)cos(\omega t). \tag{4}$$

In order to simplify the notation in the rest of the paper the sum sign is omitted. Vibrations are result of a externally applied torque $M(t)=M_0cos(\omega t)$, and the shaft displacement was assumed in accordance with the phase of extortion. The differential equation of motion was written down as:

$$\frac{\partial^2\varphi(x,t)}{\partial t^2}=c^2\frac{\partial^2\varphi(x,t)}{\partial x^2}+\frac{M(t)\delta(x,t)}{\rho I_0}, \tag{5}$$

where

$$c^2=\frac{G}{\rho}. \tag{6}$$

Dirac delta function $\delta(x-L)$ was introduced to describe a distribution of externally applied torque. By substituting Eq. (4) in the Eq (5), after simplification the vibration amplitude can be calculated:

$$A = \frac{M_0 \delta(x-L)}{\rho I_0 \sin(kx)\left(k^2c^2 - \omega^2\right)}. \tag{7}$$

Taking into account obtained amplitude (7) Eq. (4) can be written down as:

$$\varphi(x,t) = \frac{M_0 \delta(x-L)\cos(\omega t)}{\rho I_0 \left(k^2c^2 - \omega^2\right)}. \tag{8}$$

The dynamic flexibility Y_G can be calculated in agreement with definition:

$$\varphi(x,t) = Y_G M(t) \tag{9}$$

and written down as:

$$Y_G = \frac{\delta(x-L)}{\rho I_0 \left(k^2c^2 - \omega^2\right)}, \tag{10}$$

where:

$$\delta(x-L) = Lim_{\varepsilon \to 0}\left(\frac{1}{\pi}\frac{\varepsilon}{(x-L)^2 + \varepsilon^2}\right). \tag{11}$$

In order to analyse mechanical subsystem it is possible to use the exact method. In case of mechatronic systems it is impossible, this is why the approximate method is used. It is important to verify accuracy of the approximate method by juxtaposed results of the mechanical subsystem analysis using both methods. The process of the dynamical flexibility calculation using the exact method of separation of variables is well known and not presented in this work.

A value of the coefficient ε was selected to draw the dynamic flexibility. The characteristic was calculated using different values and obtained results were juxtaposed on charts with results obtained using the exact method. The value of coefficient ε was selected to obtain the most similar results. Results obtained using obtained using exact and approximate methods - the dynamic flexibility of the mechanical subsystem is presented in Fig. 1. The optimal value of the coefficient ε was assumed in this case. One can see that obtained results are very similar for both methods. In Fig. 2 and Fig. 3 results obtained for different values of coefficient ε and geometric coefficient x are presented. They are also juxtaposed with results obtained using the exact method. In order to more precise results presentation an absolute value of the system's dynamic flexibility is presented. In presented figures results obtained for the mechanical subsystem using the exact method are presented by using a continuous line, while for the approximate method a dotted line is used.

It was proved that the approximate Galerkin method is very precise and can be successfully used to analyse mechatronic systems. It should be mentioned that inexactness of the Galerkin method depends of the analysed system's form of vibration and the method of fixing the system [2, 3].

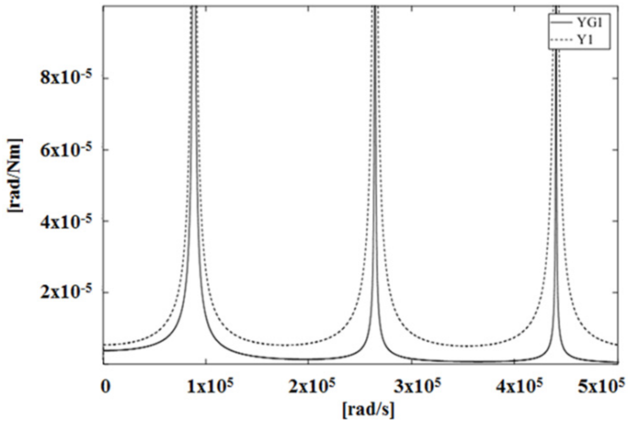

Fig. 1 Juxtaposed results obtained using exact and approximate methods, when $x = 0.1L$, $\varepsilon = 0.05$

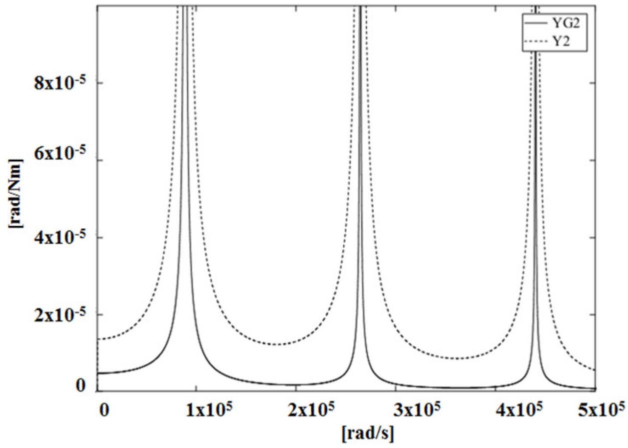

Fig. 2 Juxtaposed results obtained using exact and approximate methods, when $x = 0.25L$, $\varepsilon = 0.04$

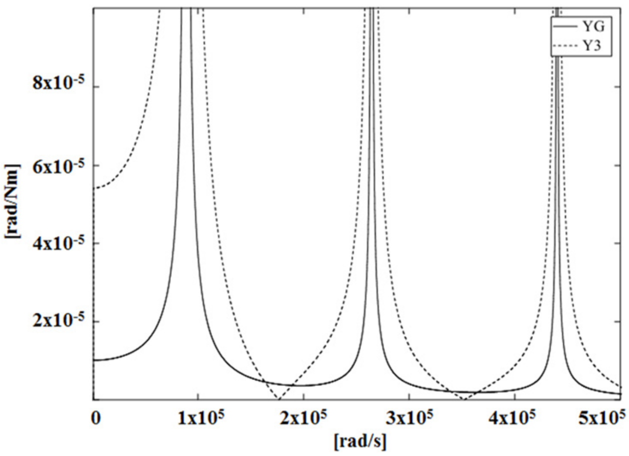

Fig. 3 Juxtaposed results obtained using exact and approximate methods, when $x = L$, $\varepsilon = 0.03$

4. Characteristic of the mechatronic system

The analysed system was created by development of the mechanical subsystem. The sectional, cylindrical piezoelectric transducer is glued on the shaft surface. Obtained system is a one-dimensional, discrete – continuous torsional vibrating mechatronic system. It was assumed that the transducer is perfectly bonded to the shaft surface – influence of a glue layer was neglected. Deformation of the transducer is equal to the shaft surface's deformation. The considered system with bonded cylindrical piezoelectric transducer is presented in Fig. 4.

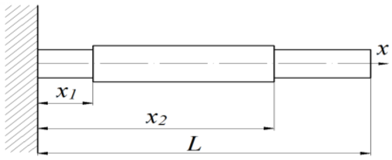

Fig. 4 Scheme of the considered mechatronic system – place of the transducer application

Equation of motion was written down taking into consideration arrangement of torques acting in the system:

$$\frac{\partial^2\varphi(x,t)}{\partial t^2}=c^2\frac{\partial^2\varphi(x,t)}{\partial x^2}-\frac{\partial M_p(x,t)}{\partial x}+\alpha M(t),\quad(12)$$

where:

$$\alpha=\frac{\delta(x-L)}{\rho I_0}.\quad(13)$$

$M_p(x, t)$ denotes torque generated by the transducer and $M(x, t)$ externally applied torque. The Heaviside's function was introduced into Eq. (14) to carb the working space of the piezoelectric transducer to partition from x_1 to x_2:

$$\frac{\partial^2\varphi(x,t)}{\partial t^2}=c^2\frac{\partial^2\varphi(x,t)}{\partial x^2}-\\-\frac{\partial}{\partial x}\left[M_p(x,t)H(x)\right]+\alpha M(t),\quad(14)$$

where:

$$H(x)=H(x-x_1)-H(x-x_2).\quad(15)$$

In this work it was assumed that the transducer is used as actuator this is why the externally applied $M(x, t)$ torque was eliminated. The system is excited by the torque generated by the transducer so Eq. (14) can be simplified and written down as:

$$\frac{\partial^2\varphi(x,t)}{\partial t^2}=c^2\frac{\partial^2\varphi(x,t)}{\partial x^2}-\frac{\partial\left[M_p(x,t)H(x)\right]}{\partial x}.\quad(16)$$

In this case characteristic Y_p that describe relations between externally applied electric voltage and angle of rotation of the mechanical subsystem. This relation can be described by the equation:

$$\varphi(x,t)=Y_pV(t).\quad(17)$$

The angular displacement of the piezoelectric transducer was written as [1]:

$$\beta=\frac{L_p}{R_Z}d_{15}E,\quad(18)$$

where L_p denotes length of the transducer, E the electric field intensity, d_{15} is the piezoelectric constant and R_Z denotes the outer radius of the cylindrical transducer. The process of power supply of the transducer and its deformation is presented in Fig. 5 [1].

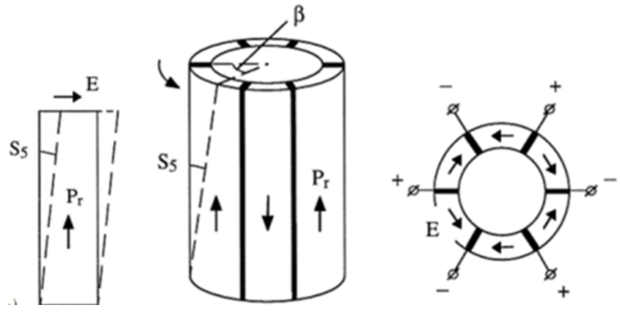

Fig. 5 Construction and interactions in the cylindrical piezoelectric transducer [1]

In Fig. 5 are presented respectively:

a) an electric field vector E is perpendicular to the vector of the residual polarization P_r what causes stress S_5 described as:

$$S_5=d_{15}E;\quad(19)$$

b) The piezoelectric element in the form of a segmented ring in which segments are supplied alternately. The polarization inside the ring is represented by arrows. Depending on the set electric voltage the upper side will turn clockwise or counter-clockwise relative to the bottom by an angle β [1].

By rearranging Eq. (16) taking into account Eq. (4), after similar transformations as in case of mechanical subsystem analysis it can be written down:

$$-A\omega^2\sin(kx)\cos(\omega t)=-c^2Ak^2\sin(kx)\cos(\omega t)-\\-\frac{1}{\rho I_0}\left[\frac{\partial M_p(x,t)}{\partial x}H(x)+M_p(x,t)\frac{\partial H(x)}{\partial x}\right].\quad(20)$$

Using the properties of the Heaviside's function:

$$\frac{\partial H(x)}{\partial x}=\delta(x),\quad(21)$$

where $\delta(x)$ is the Dirac delta function and assuming that characteristic will be calculated when $x = L$ it can be written down:

$$-A\omega^2\sin(kx)\cos(\omega t)=-c^2Ak^2\sin(kx)\cos(\omega t)-\\-\frac{1}{\rho I_0}M_p(x,t)\delta(x).\quad(22)$$

The torque generated by the piezoelectric transducer can be described by formula [1]:

$$M_p(x,t)=\frac{n_p(R_Z-R_W)}{s_{44}^E}d_{15}V(t)\frac{(R_Z-R_W)}{2},\quad(23)$$

where R_Z and R_W denote outer and inner radius of the cylindrical transducer, n_p is the number of transducer's segments, s_{44}^E is the elastic constant determined at zero/constant electric field. It was assumed that externally applied voltage is:

$$V(t)=V_0\cos(\omega t).\quad(24)$$

Vibration amplitude can be calculated similarly as in the previous case:

$$A = \frac{n_p \left(R_Z - R_W\right) d_{15} V_0 R_a \delta\left(x\right)}{sin\left(k\,x\right)\left(k^2 c^2 - \omega^2\right)\rho I_0 s_{44}^E}\,, \tag{25}$$

where R_a denotes the average value of the radius of the transducer. In accordance with the Eq. (4) angle of rotation can be written down as:

$$\varphi\left(x,t\right) = \frac{n_p \left(R_Z - R_W\right) d_{15} R_a \delta\left(x\right)}{\left(k^2 c^2 - \omega^2\right)\rho I_0 s_{44}^E} cos\left(\omega t\right), \tag{26}$$

and characteristic of the considered mechatronic system can be described by the equation:

$$Y_P = \frac{n_p \left(R_Z - R_W\right) d_{15} R_a \delta\left(x\right)}{\left(k^2 c^2 - \omega^2\right)\rho I_0 s_{44}^E}\,. \tag{27}$$

Obtained results – characteristic of the considered mechatronic system excited by cylindrical piezoelectric transducer is presented in Fig. 6. In presented case the characteristic was calculated when length of the transducer $L_p = L/3$. This characteristic describes relation between value and frequency of electric voltage that supplies the piezoelectric actuator and an angle of rotation of the shaft, measured in rad/V. Results are juxtaposed with the dynamic flexibility of mechanical subsystem, measured in rad/Nm, obtained using the exact method. One can observe that the biggest values of the angle of rotation can be obtained in resonance zones.

It should be mentioned that in this work very simply mathematical model was used. There was no influence of the glue layer between the transducer and surface of the mechanical subsystem on the obtained characteristic. In the future work more precise model will be used. Obtained results should be also verify and juxtaposed with results of the experimental tests. It will be presented in future works.

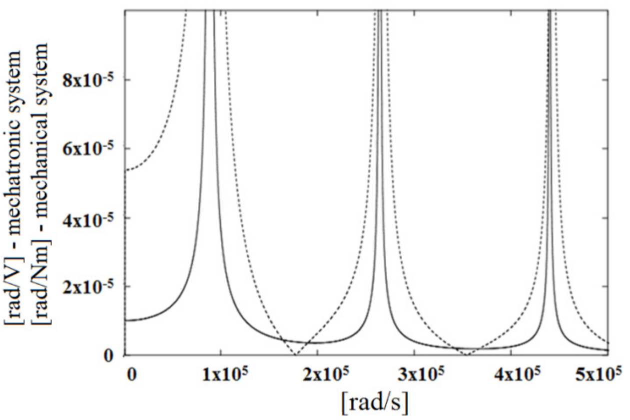

Fig. 6 Comparison of characteristic of mechatronic system excited by piezoelectric transducer (approximate method – the dotted line) with dynamic flexibility of the mechanical subsystem (exact method – the continuous line)

5. Conclusions

Due to the growing interest in the use of piezoelectric materials in modern technical devices, as well as works to develop new non-classical piezoelectric transducers the process of modelling and testing of such systems also becomes very essential issue. This paper is a proposal to use approximate Galerkin method for the determination of characteristics of vibrating systems containing non-classical, composite transducers. Using the proposed mathematical algorithm it is possible to determine the desired characteristics as well as analyse the effects of the parameters of the individual elements of the system on those characteristics including both the geometric and material parameters. It should be noted that in the assumed mathematical model of the considered torsional vibrating system significant simplifications were assumed. This is why in the future works more precise models will be proposed. Designing of technical systems containing piezoelectric transducers is a complex process, due to the phenomena occurring in them. A correct description of the given device in the form of a mathematical model, already in its design phase, is a fundamental condition for its proper functioning. It is very important to develop mathematical models of vibrating systems with piezoelectric transducers that can be used as actuators or vibration dampers, to allow an accurate description of the phenomena occurring in them with the maximum simplification of the calculations. It will significantly contribute to the development of this field of technology and facilitate the implementation of technical devices with piezoelectric transducers.

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CILINDRINIŲ PIEZOELEKTRINIŲ DAVIKLIŲ SISTEMŲ MODELIAVIMAS IR TYRIMAS

R e z i u m ė

Straipsnyje aprašomas virpančios mechatroninės sistemos tyrimo ir modeliavimo matematinis algoritmas. Velenas yra tiriamos sistemos mechaninė posistemė. Žiedo formos piezoelektrinis daviklis pritvirtintas prie veleno. Projektuojant sistemą, reikia žinoti jo dinamines charakteristikas.

Dėl to autoriai pristatė metodą, kuris naudingas analizuojant tokio tipo sistemas. Norint apskaičiuoti mechatroninės sistemos charakteristikas, pirmiausia buvo tiriamas mechaninis posistemis. Mechaninio posistemio dinaminis lankstumas apskaičiuotas naudojant tikslų ir apytikslį metodus. Kadangi mechatroninės sistemos analizei tikslų metodą taikyti neįmanoma, buvo naudotas apytikslis Galerkino metodas taikomas sistemos su piezoelektrine pavara analizei. Darbe pateikiama besisukanti vibruojanti mechatroninė sistema su piezoelektriniu davikliu. Tiriama sekcijinė piezoelektrinė pavara, kurios cilindrinis daviklis maitinamas išorine harmoniškai kintančia įtampa. Gautų mechaninių posistemių ir mechatroninių sistemų charakteristikų palyginamas atvaizduotas diagramoje.

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MODELLING AND ANALYSIS OF SYSTEMS WITH CYLINDRICAL PIEZOELECTRIC TRANSDUCERS

S u m m a r y

Paper presents a proposal of mathematical algorithm used in order to modelling and testing of vibrating mechatronic systems. A shaft is the mechanical subsystem of the considered system. A ring piezoelectric transducer is bonded on the shaft's surface. Knowledge of the dynamic characteristics of the designed systems is essential for the proper operation and should be taken into consideration during the design phase as well as verified during operation of the system. This is why authors decided to present a method that can be very useful for analysis of such kind of systems. In order to calculate the characteristic of mechatronic system a mechanical subsystem was analysed in the first step. The dynamic flexibility of mechanical subsystem was calculated using the exact and approximate methods. It is impossible to use the exact method in order to analyse mechatronic systems this is why the approximate Galerkin method was used to analyse the system with piezoelectric actuator. An exactness of the approximate method was verified. In the presented work a torsional vibrating mechatronic system with piezoelectric transducer used as the vibration actuator is presented. The considered piezoelectric actuator is the sectional, cylindrical transducer supplied by the external harmonic electric voltage. Obtained results – characteristics of mechanical subsystem and mechatronic system are juxtaposed on charts.

Keywords: modelling, analysis, vibrating mechatronic systems, piezoelectric transducer.

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