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Determination of mixed-mode fracture characteristics due dynamic opening and in-plane shear cases

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1. Introduction

In fracture mechanics the crack propagation problem is mainly discussed for individual cases of deformation, i.e. opening and affecting with shear [1, 2]. However, in practice often mixed cases are encountered when the crack is affected by opening and shear stresses. Most of the fracture mechanics of crack instability theories are based on the idea of Griffith [2]. For pure-mode cases, it has been commonly accepted that fracture will occur when the corresponding stress intensity factor reaches its critical value. Stress state ahead of a crack is often of the mixed type where both K_I^d and K_{II}^d are present. Practical engineering cracked structures are subjected to mixed mode loading, thus in general K_I and K_{II} are both nonzero, yet we usually measure only mode I fracture toughness K_{IC}. In this cases we have the so-called cumulative effect of modes I and II. The mode I describes the opening and normal stress effect and the mode II – the shear cases and shear stress effect [1-5]. The determination of a fracture initiation criterion for an existing crack in mode I and mode II would require a relationship between K_I, K_{II} and K_{IC} of the form:

$$F\left(K_I, K_{II}, K_{IC}\right) = 0 \tag{1}$$

and would be analogous to the between the two principal stress and yield stress (Fig. 1):

$$F_Y\left(\sigma_1, \sigma_2, \sigma_Y\right) = 0 . \tag{2}$$

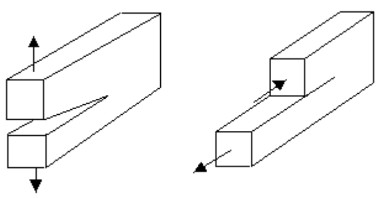

Fig. 1 Mixed mode crack propagation under opening (left side) and in-plane shear (right side) fracture modes

These tests become more important when dynamic effects arise because this far, dynamic fracture patterns are least examined in the fracture mechanics. The known works [6, 7] consider not only the characteristics of fracture, i.e. dynamic stress intensity factors K_I^d and K_{II}^d, but also focus on the crack path, i.e. when the crack propagation angle depends on both loads: opening and shear. Sih at al [8] proposed a mixed-mode criterion of fracture, which states, that the combination of mode I and mode II stress intensity factors present will cause crack initiation upon reaching some critical value K_{IC}. This critical intensity of the local stress field is a material constant and not depends on geometry of specimen. Also Sih at al [9] discussed the dynamic counterpart of Griffith crack configuration.

The dynamic crack propagation behaviour has attracted extensive attention during the past decades [10]. There are a number of experiments, theoretical models and simulations constructed and performed to understand the phenomena of dynamic fractures. Zhang at al investigated dynamic crack growth and branching of running crack under mixed-mode loading.

In order to predict the fracture loadings of cracked materials under the general mixed-mode state Chang at al [11] proposed general fracture criterion based on the concept of maximum potential energy release rate.

However, the obtained dependences show little representation of mechanical properties of materials, which determine the fracture process. Therefore the work is focused on the parameters of the material strength and fracture that allow easier assessment of the crack growth specimens depending on the complex stress condition in the tip of the crack. This would allow describing critical dangerous levels also in the general load case.

Unlike the static case, solution to the dynamic problem is more difficult to obtain. The effects of dynamic loading on the distribution of stresses around a crack – like imperfection have not received sufficient attention. The elasto-dynamic problems are fundamental interest in fracture mechanics.

2. Testing procedures

Loading the specimen with an inclined crack, i.e. pulling and affected by shear according to mode I and mode II (Fig. 2).

In the work [1] from maximum circumferential tensile stress theory for mixed modes is writing:

$$\frac{K_I^d}{K_{IC}^d} cos^3 \frac{\theta_0}{2} - \frac{3}{2} \frac{K_{II}^d}{K_{IC}^d} cos \frac{\theta_0}{2} sin\,\theta_0 = 1 , \tag{3}$$

where: θ_0 is crack angle from the crack position perpendicular to the load; K_I^d, K_{II}^d are dynamic stress intensity factors in the opening (mode I) and in-plane shear (mode II) cases.

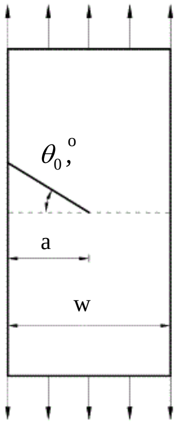

Fig. 2 Specimen with initial inclined crack under tension

Evaluating the characteristics of material strength, from minimum strain energy density criteria [1] can be expressed as:

$$\frac{8\mu_d}{\kappa-1}\left[a_{11}\left(\frac{K_I^d}{K_{IC}^d}\right)^2+2a_{12}\left(\frac{K_I^d K_{II}^d}{\left(K_{IC}^d\right)^2}\right)+a_{22}\left(\frac{K_{II}^d}{K_{IC}^d}\right)^2\right]=1, \quad (4)$$

where μ_d is dynamic sear modulus; coefficients a_{11}, a_{12} and a_{22} are calculated from:

$$a_{11}=\frac{1}{16\mu_d}\left(1+cos\,\theta_0\right)\left(\kappa-cos\,\theta_0\right); \quad (5)$$

$$a_{12}=\frac{sin\,\theta_0}{16\mu_d}\left[2cos\,\theta_0-\left(\kappa-1\right)\right]; \quad (6)$$

$$a_{22}=\frac{1}{16\mu_d}\left[\begin{array}{l}\left(\kappa+1\right)\left(1-cos\,\theta_0\right)+\\+\left(1+cos\,\theta_0\right)\left(3cos\,\theta_0-1\right)\end{array}\right]; \quad (7)$$

$$\begin{cases}\kappa=\dfrac{3-\nu_d}{1+\nu_d} & \text{plane stress}\\ \kappa=3-4\nu_d & \text{plane strain}\end{cases}.$$

Solving the system of Eqs. (3) and (4), from the Eq. (3) we get:

$$K_I^d=\frac{1+\frac{3}{2}\frac{K_{II}^d}{K_{IC}^d}cos\frac{\theta_0}{2}sin\,\theta_0}{cos^3\frac{\theta_0}{2}}K_{IC}^d. \quad (8)$$

Upon inserting the Eq. (8) to Eq. (4), it follows:

$$\frac{8\mu_d}{\kappa-1}\left(a_{11}\frac{1+\frac{3}{2}\frac{K_{II}^d}{K_{IC}^d}cos\frac{\theta_0}{2}sin\,\theta_0}{cos^3\frac{\theta_0}{2}}+\right.$$
$$\left.+2a_{12}\frac{1+\frac{3}{2}\frac{K_{II}^d}{K_{IC}^d}cos\frac{\theta_0}{2}sin\,\theta_0}{cos^3\frac{\theta_0}{2}K_{IC}^d}+a_{22}\left(\frac{K_{II}^d}{K_{IC}^d}\right)^2\right)=1 \quad (9)$$

From the obtained Eq. (9), having K_{IC}^d, θ_0 and μ_d, κ and ν_d and coefficients a_{11}, a_{12} and a_{21}, we can calculate K_{II}^d, and from Eq. (8) also K_I^d.

Control of the solution can be carried out upon entering the angle $\theta_0=0$ and the Eq. (9) then it will be written as:

$$1+\frac{2}{\kappa-1}\left(\frac{K_{II}^d}{K_{IC}^d}\right)^2=1, \quad (10)$$

from here we get:

$$\frac{2}{\kappa-1}\left(\frac{K_{II}^d}{K_{IC}^d}\right)^2=0, \quad (11)$$

i.e. $K_{II}^d=0$ and it shows that in this case there is no shear. We only have the case of opening $K_I^d=K_{IC}^d$, as evidenced by Eq. (8).

In order to evaluate different opening and shear influence on fracture, the experiments with steel plates under mixed-mode loading have been carried out. Specimens are made of S355 grade construction steel, with statitac modulus of elasticity $E=200$ GPa, yield stress $\sigma_y=350$ MPa, ultimate stress $\sigma_u=500$ MPa, Poison‘s ratio $\nu_d=0{,}30$. Size length × width × thickness of the pre-cracked specimens are respectively $40\times30\times3$ mm. In center of the specimen 0.25 mm width and 8 mm length initial crack are made. This initial crack imitates fatigue crack.

Fig. 3 Specimen with oblique angles slits every 5°, starting from 0° to 45°

Testing was done in Kazimieras Vasiliauskas Strength of Materials lab at the Kaunas University of Technology. During impact testing a swinging pendulum of 30 kg m potential energy impact tester was used.

Specimen was embedded in swinging pendulum with special fixture and lifted to starting position (Fig. 4). Released pendulum swings through and strikes the specimen causing fracture. In the scale of impact tester the breaking energy C_V are shown. Using specimens with different angles of initial cracks, mixed-mode loading are obtained. The experimental results under various mixed-mode loading conditions are given in Table 1.

Impact test results (Table 1) are shown in Fig. 5. It should be mentioned that increasing initial crack angle increases the fracture energy.

Fig. 4 Impact tester

Table 1

Results of the impact testing

No.	θ_0, °	C_V, kg·m		Mode
1	0	13.6	avr.: 12.9	I mode
2	0	12.8		I mode
3	0	12.2		I mode
4	5	12.8	avr.: 13.4	I, II mode
5	5	13.9		I, II mode
6	5	13.4		I, II mode
7	10	13.9	avr.: 14.3	I, II mode
8	10	14.8		I, II mode
9	10	14.1		I, II mode
10	15	14.3	avr.: 14.3	I, II mode
11	15	14.7		I, II mode
12	15	14.0		I, II mode
13	20	15.7	avr.: 16.1	I, II mode
14	20	16.5		I, II mode
15	20	16.1		I, II mode
16	25	16.5	avr.: 17.1	I, II mode
17	25	17.9		I, II mode
18	25	17.5		I, II mode
19	30	16.4	avr.: 17.1	I, II mode
20	30	17.8		I, II mode
21	30	17.1		I, II mode
22	35	18.2	avr.: 18.9	I, II mode
23	35	19.7		I, II mode
24	35	18.7		I, II mode
25	40	19.5	avr.: 19.8	I, II mode
26	40	20.0		I, II mode
27	40	19.8		I, II mode
28	45	22.1	avr.: 21.5	I, II mode
29	45	21.2		I, II mode
30	45	21.3		I, II mode

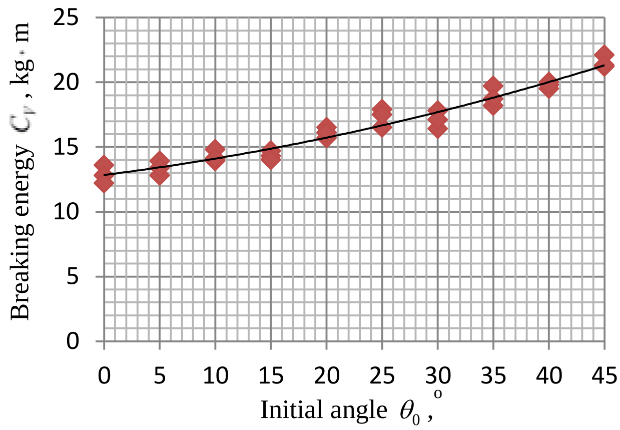

Fig. 5 Breaking energy C_V due to initial angle θ_0, °

3. Evaluation of mixed-mode characteristics

Dynamic characteristics of material were obtained from a specimen without a crack. During the tests, the dynamic shear modulus $\mu_d = 1\cdot 10^5$ MPa, and dynamic Poisson's ratio $\nu_d = 0.25$ were identified. In the case of plane stress $\kappa = 3 - 4\nu_d = 2$, and in case of plane strain $\kappa = (3-\nu_d)/(1+\nu_d) = 2.2$.

Empirical equation which relates fracture energy C_V to fracture toughness K_{IC} [12]:

$$K_{IC} = 12.36\, C_V^2, \tag{12}$$

where stress intensity factor K_{IC} is for static loading. Empiric formulas having impact strength characteristics can be expressed in terms of critical stress intensity factor K_{IC}^d [12]:

$$K_{IC}^d = 1.06\, K_{IC}. \tag{13}$$

In our case the critical stress intensity factor K_{IC}^d from Eq. (13) is equal to $K_{IC} = 47.21\,\text{MPa}\cdot\text{m}^{1/2}$.

The calculated value is K_I^d and K_{II}^d according to the Eqs. (8) and (9) are presented in Table 2.

Table 2

Results of the impact testing

θ_0, °	K_I^d, MPa·m$^{1/2}$	K_{II}^d, MPa·m$^{1/2}$	K_I^d/K_{IC}^d	K_{II}^d/K_{IC}^d
0	47.21	0	1	0
5	46.21	8.67	0.99	0.18
10	44.17	13.65	0.95	0.29
15	40.61	19.83	0.86	0.42
20	36.06	25.27	0.76	0.54
25	30.79	29.99	0.65	0.64
30	25.08	33.98	0.53	0.72
35	19.13	37.31	0.41	0.9
40	13.09	40.12	0.28	0.85
45	7.01	42.53	0.15	0.90

The obtained results were used to make a graph of changes in the dynamic stress intensity factors K_I^d / K_{IC}^d and K_{II}^d / K_{IC}^d depending on θ_0 which is presented in Fig. 6.

As can be seen from the graphs, the effect of stress intensity factor K_{II}^d increases with increasing initial angle of the crack. The ratio K_I^d / K_{IC}^d increases from 0 to 0.48 increasing the crack angle from 0° to 45° and K_{II}^d / K_{IC} increases from 0 to 0.9. It may be seen that K_{II}^d increases more than K_I^d.

The intersection of K_I^d / K_{IC}^d and K_{II}^d / K_{IC}^d is approximately at 25.8°.

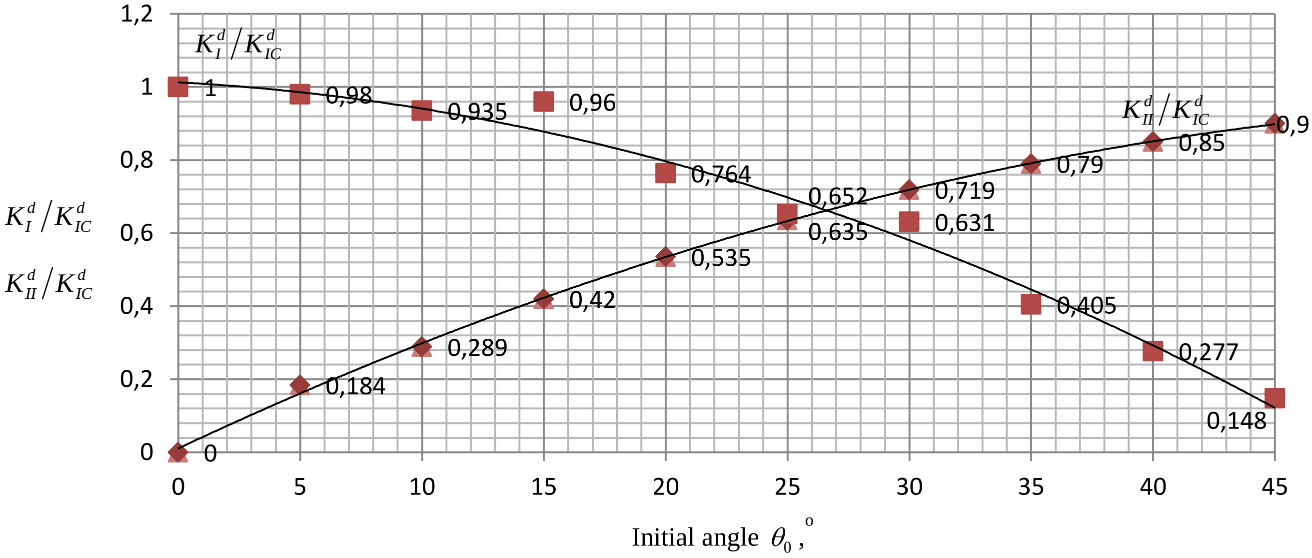

Fig. 6 K_I^d / K_{IC}^d and K_{II}^d / K_{IC}^d dependences at different angles of the crack

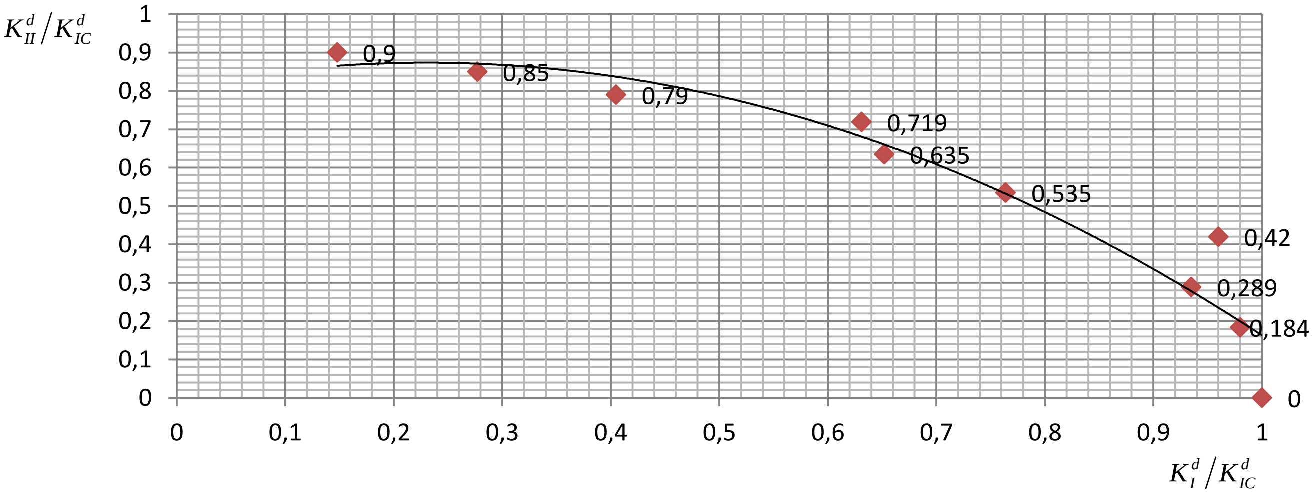

Fig. 7 Two dimensional fracture curve

4. Conclusions

1. In mixed load: for opening and shear strength, the dynamic fracture with the crack can be calculated. The obtained Eqs. (8) and (9) are able to calculate dynamic fracture parameters K_I^d and K_{II}^d, using fracture characteristic K_{IC}^d, shear modulus and Puason's ratio.

2. The obtained test and calculation results show different effect of the opening and shear to fracture, and the presented to dependences allow to assess the effect in qualitative indicators.

3. Test and calculation results can be applied not only to cracks with a clear path of spreading, but also for bifurcations on the tip of the main crack, where a mixed opening and shear fracture develops.

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MIŠRAUS IRIMO CHARAKTERISTIKŲ NUSTATYMAS DINAMINIO ATPLĖŠIMO IR ŠLYTIES ATVEJAIS

R e z i u m ė

Straipsnyje nagrinėjama medžiagų irimo problema mišraus deformavimo atvejais: atplėšiant ir veikiant šlyčiai dinaminio apkrovimo atveju. Nagrinėjami irimo kriterijai ir gaunami nauji dėsningumai, jog mišrų irimą galima įvertinti panaudojant ribines medžiagų irimo ir stiprumo charakteristikas: šlyties modulį bei Puasono koeficientą. Pateikiamos įtempių intensyvumo koeficientų esant atplėšimui ir šlyčiai priklausomybės, kai kinta plyšio plitimo trajektorija. Parodomas atplėšimo ir šlyties irimo įtakos santykis

P. Šadreika, A. Žiliukas

DETERMINATION OF MIXED-MODE FRACTURE CHARACTERISTICS DUE DYNAMIC OPENING AND IN-PLANE SHEAR CASES

S u m m a r y

This work analyses the material fracture problem due to mixed-mode: dynamic opening and in-plane shear cases. Fracture criterions are analysed and new regularities are established, that mixed-mode fracture can be determined using critical characteristics of fracture and toughness: shear modulus and Poison’s ratio.

Stress intensity factor dependence curves are determined due opening and in-plane shear cases with different angle of initial crack trajectory. The opening and in-plane shear fracture influence ratio is set.

Keywords: fracture toughness, welded joints materials.

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