Optimization of fixation elements parameters for ring type piezoelectric actuator generating elliptical movement

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1. Introduction

Development of mechanisms based on non-traditional principles finds wider applications in modern mechatronics systems. Piezoelectric drive – the mechanism functioning of which possible due to features of smart materials is an example of modern tendencies in mechanisms development. Being of simple and convenient for electronic control structure they are favorable to be integrated into mechatronics structures [1-6].

A wave piezoelectric drive functions due to conversion of the energy of high frequency travelling wave generated in a resonator into mechanical energy of output's link directional motion (rectilinear or rotational). The most important part of these piezoelectric drives is its stator. In our case – it is ring type piezoelectric actuator. A lot of design and operating principles are investigated to transform mechanical vibrations of piezoceramic elements into elliptical movement of the contact zone of actuator [7, 8].

A piezoelectric drive with a ring-shaped exciter is a complicated electromechanical system, when considered analytically. Mechanical and electrical parameters and high frequency vibration transportation methods should be included into its mathematical model. The type of fixture of the piezoceramic ring has a great impact on dynamic characteristics and quality of vibrations (the travelling wave parameters) excited in it [9, 10].

A novel design of ring type piezoelectric actuator with waves reflecting active elements (the fixation elements) and dynamic processes in it is presented and analyzed.

Manufacture of this type piezoelectric actuator is complicated however there is a possibility to reduce the energy losses of the travelling wave at fixing points of the actuator (piezoceramic ring). Dynamic processes in this piezoelectric actuator of elliptical movement of the contact zone are analyzed by means of the method of finite elements [11-13].

Numerical analysis with the aim to determine natural frequencies and modal shapes of the ring-shaped resonator and to find the path of the interaction point were carried out case different excitation patterns were applied.

Dynamic processes in the ring type piezoelectric actuator without fixation elements are investigated [14].

The present paper addresses the special novel structural solution of the ring shaped piezoelectric actuator generating elliptical movement.

2. Numerical modeling of the actuator

Modal frequency and harmonic response analysis of ring-shaped piezoelectric actuator was performed and trajectories of the contact points were obtained by FEM (Finite Element Method) [15, 16].

Mathematic model evaluating the coupled mechanical and electrical behavior of the actuator was constructed applying the principal of minimum potential energy by means of variation functional:

$$[M]\{\ddot{u}\}+[C]\{\dot{u}\}+[K]\{u\}+[T]\{\varphi\}=\{F\}$$
$$[T]^T\{u\}+[S]\{\varphi\}=\{Q\} \quad , \qquad (1)$$

where $[M]$, $[C]$, $[K]$, $[T]$, $[S]$ are matrixes of mass, damping, stiffness, electro elasticity, capacity accordingly $\{u\}$, $\{F\}$, $\{Q_1\}$ are vectors of nodes displacements, external forces and charges on electrodes $\{\varphi\}$ is vector of nodal potentials of the nodes associated with electrodes.

The applied to piezoelectric actuator mechanical and electrical boundary conditions are displacement of fixed points of the actuator and the charge of piezoelectric elements not coupled with electrodes equals zero.

Natural frequencies and modal shapes of the ring-shaped resonator are obtained solving the equation of piezoelectric system:

$$det\left([K^*]-\omega^2[M]\right)=\{0\}, \qquad (2)$$

where K^* is modified stiffness matrix.

A trajectory of the contact point movement is determined from the displacements obtained as the result of harmonic response analysis.

3. Structure and operating principle of ring type piezoelectric actuator with fixation elements

The model of a ring with fixation elements is made of piezoceramics material PZT-8 (Fig. 1). The polarization vector is directed along the height of the ring. The detailed properties of this material are provided in Table 1.

Geometric parameters of the piezoceramic ring are chosen (proportionally) in such a way that the Eigen frequency of the 2nd flexional form is as high as possible, since this way its rapidity is guaranteed.
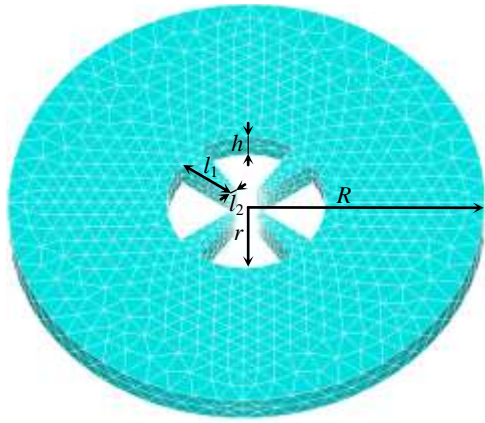
Fig. 1 Principle scheme of a ring type piezoelectric actuator with fixation elements, where R – outer radius, r –inner radius, h – height, l_1 – width of fixation element, l_2 – length of fixation element

Table 1

The properties of the piezoceramics used for modeling

Material property	Piezoceramics PZT-8
Elasticity modulus, N/m^2	8.2764×10^{10}
Poisson′s Ratio	0.33
Density, kg/m^3	7600
Dielectric permittivity, x10^3 F/m	$\varepsilon_{11}=1.2$; $\varepsilon_{22}=1.2$; $\varepsilon_{33}=1.1$
Piezoelectric matrix, x10^{-3} C/m^2	$e_{13}=-13.6$; $e_{23}=-13.6$
	$e_{33}=27.1$; $e_{42}=37.0$
	$e_{51}=37.0$

Geometric parameter's proportions used in the finite element model modal analysis (Fig. 1) are provided in Table 2.

Table 2

Geometric parameters of piezoceramic ring

	Measurement of ring type actuator			
	Model 1	Model 2	Model 3	Model 4
Outer radius R, m	0.015	0.015	0.015	0.015
Inner radius r, m	0.005	0.005	0.005	0.005
Height h, m	0.002	0.002	0.002	0.002
Length of fixation element l_2, m	0.004	0.004	0.004	0.004
Width of fixation element l_1, m	0.002	0.003	0.005	0.007

4. Dominations coefficient analysis

When performing the numerical analysis a changed geometric parameters of the actuator changes the sequence of eigenforms what leads to the possibility of unsuitable eigenvalue choice according the scheme (Fig. 2) for rotational geometric parameters determining. It is a typical situation that a vibrational device operates in one of its eigenfrequencies and with the change of eigenforms sequence the numerical analysis becomes meaningless as the solution usually does not converge.

By the algorithm of eigenvalue problem eigenfrequencies for systems are sorted in the ascending order; thereby the sequences of eigenforms change. This rule for sorting frequencies is disadvantageous when numerical analysis of multidimensional piezoelectric actuators needs

to be automated. This problem is also important for optimization, since calculations are tied both to eigenfrequencies and eigenforms. If the Eigen frequency is chosen incorrectly, the piezoelectric actuator will not function, so it is very important to numerically determine eigenforms and place them inside the eigenform matrix of the construction model [15].
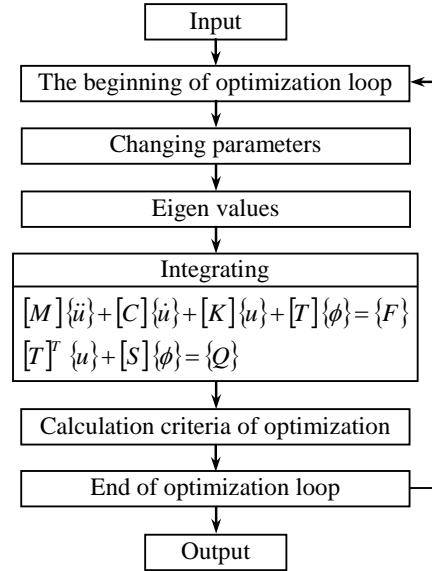

Fig. 2 The scheme for determining rational geometric parameters

An algorithm for the solution of the problem is proposed as follows: the sum of squares of amplitudes of piezoelectric actuator oscillations in the directions of all freedoms of a point is calculated and used for the determination of the total energy of the system in all directions [17]:

$$S_k^n = \sum_{i=1}^{r} \left(A_{ik}^n \right)^2 , \qquad (3)$$

where n is the eigenfrequency for a system, k is the number of degrees of freedom in a node, A_{ik}^n is the value of the eigenform vector for the ith element.

Then the ratio is calculated [1]:

$$m_j^n = \frac{S_j^n}{\sum_{i=1}^{k} S_i^n} , \qquad (5)$$

where m_j^n is the oscillation domination coefficient corresponds to the nth eigenform. The index j of domination coefficients indicates, in which direction the energy under investigation is the largest: j can assume such values: 1 corresponds to the x coordinate, $2 - y$, and $3 - z$, etc. Having calculated domination coefficients all the freedom of motion and made their comparison the dominant oscillation type can be determined.

In order to make it convenient for the analysis of various parameters influence the calculated domination coefficients are normalized in order to vary in the range from 0 to 1.

Nevertheless determination of domination coefficients is not sufficient in order to clearly determine the eigenform and its place in the eigenform matrix of the model. They just help to sort eigenforms by oscillations that dominate, for example, radial, axial, tangential and other.

Therefore another additional criteria individual for each eigenform should be introduced, e.g. the number of nodal points or nodal lines for the form. This depends on the number of the eigenform.

The number of nodal points of beam-type or other type two dimensional piezoelectric actuators is determined as the number of sign changes of the oscillation amplitudes along coordinate axes of the piezoelectric actuator for its full length.

Such algorithm can be used for the analysis of the oscillation of any structure as it is not strictly related to multidimensional piezoelectric actuators.

Summarizing it can be stated that the algorithm consists of two integral phases: calculation of domination coefficients and determination of the number of nodal points or lines for an eigenform.

5. Calculations and results of numerical modeling

In order to validate design and operation principal of piezoelectric actuators its numerical modeling was performed.

For determining the proper resonance frequency of piezoelectric actuators actuator its modal analysis was performed. Material damping factor was also introduced in to FEM. Structural boundary conditions were not applied.

Simulation software package ANSYS was used for finite element model constructing and simulation.

The detailed geometric parameters of ring type piezoelectric actuator used in the FEM modal analysis are provided in Table 2.

Also, during analysis the oscillation amplitude has to remain unchanged or change insignificantly.

Domination coefficients (Table 3) and eigen frequencies (Table 4) have been also calculated. A more detailed analysis is provided below.

Table 3

The domination coefficients

Model	$S\tau$ (rotative)	$S\varphi$ (cross ply)	Sz (long)
1	0.718501	0.278450	0.003049
2	0.784067	0.214856	0.001077
3	0.515529	0.483265	0.001206
4	0.568482	0.396198	0.035320
5	0.807156	0.191465	0.001379

Table 4

The eigenfrequencies

Model	1	2	3	4	5
Frequency f, Hz	136067	90757	68080	135922	68061

Having compared the influence of geometric parameters on domination coefficients (Fig. 3) and eigenfrequencies (Fig. 4), it can be claimed that with the help of domination coefficients the eigenform of rotation can partially be determined.
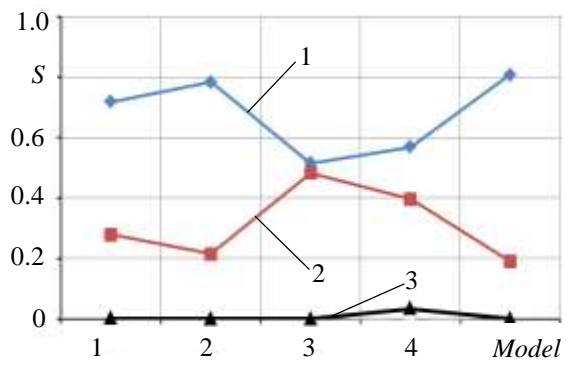

Fig. 3 Relationships between geometric parameters and domination coefficients: *1 − Sτ* (rotative), *2 − Sφ* (cross ply), *3 − Sz* (long)
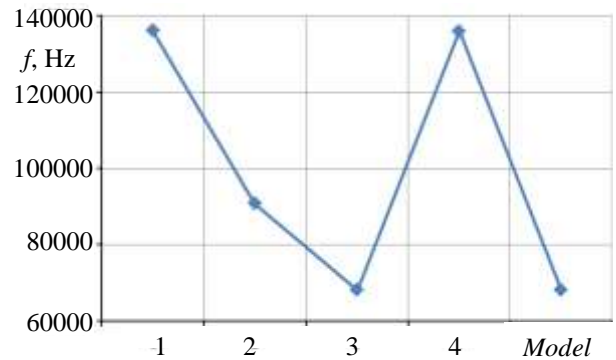

Fig. 4 Relationships between geometric parameters and frequencies *f*

After selecting the appropriate geometrical parameters of the piezoceramic ring, the optimization of geometrical parameters of the fixation elements was performed. The length l_1 and width of the stem were optimized at the fixing l_2 position (Table 2).

The geometric parameters were selected in such a way, that the eigenform would not be lost (2nd flexional form).

Domination coefficients and eigenfrequencies have been also calculated, considering when cross ply and rotative movement is the most optimal, e.g. optimized geometrical parameters based on domination coefficients, it was examined at what frequency the rotation of the ring is the best and at what frequency it is most flexible.

A more detailed analysis of domination coefficients (according to which better flexibility was examined) is provided in Tables 5, 6 and Figs. 5, 6.
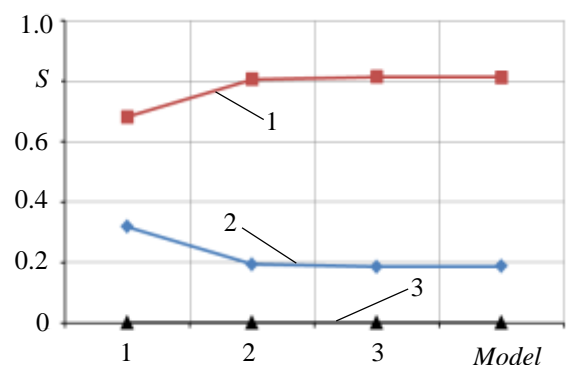

Fig. 5 Relationships between geometric parameters and domination coefficients in cross ply movement: *1 − Sτ* (rotative), *2 − Sφ* (cross ply), *3 − Sz* (long)

Table 5

The domination coefficients of cross ply movement

Model	$S\tau$ (rotative)	$S\varphi$ (cross ply)	Sz (long)
1	0.318357	0.681493	0.000149
2	0.194028	0.805757	0.000215
3	0.185279	0.814353	0.000368
4	0.187106	0.812637	0.000257

Table 6

The domination coefficients of rotative movement

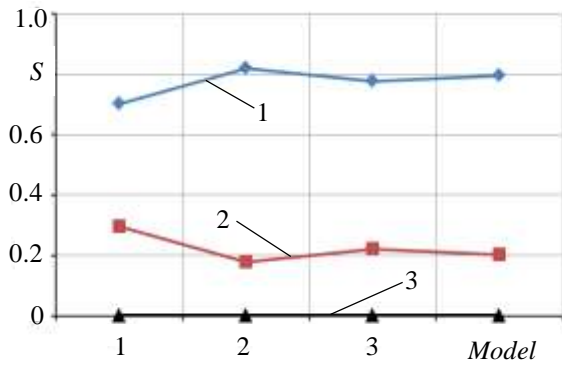| Model	$S\tau$ (rotative)	$S\varphi$ (cross ply)	Sz (long)
1	0.701586	0.296636	0.001779
2	0.819207	0.179199	0.001594
3	0.776393	0.222298	0.001310
4	0.795810	0.202962	0.001228

Fig. 6 Relationships between geometric parameters and domination coefficients of rotative movement: $1 – S\tau$ (rotative), $2 – S\varphi$ (cross ply), $3 – Sz$ (long)

A more detailed analysis of model eigenfrequencies (by cross ply and rotative movements) is provided in Table 7 and Fig. 7.

Table 7

The eigenfrequencies
(by cross ply and rotative movements)

Model	1	2	3	4
$S\varphi$ (cross ply) frequency f, Hz	30669	31483	33115	35319
$S\tau$ (rotative) frequency f, Hz	88992	89429	89810	90139

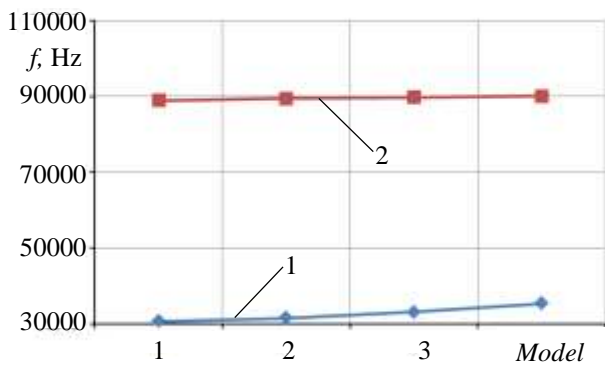Fig. 7 Relationships between geometric parameters and natural frequencies by cross ply and rotative movements: $1 – S\tau$ (rotative), $2 – S\varphi$ (cross ply)

The images of FEM simulation below provide the cases when cross ply and rotation movements are the best (Fig. 8).
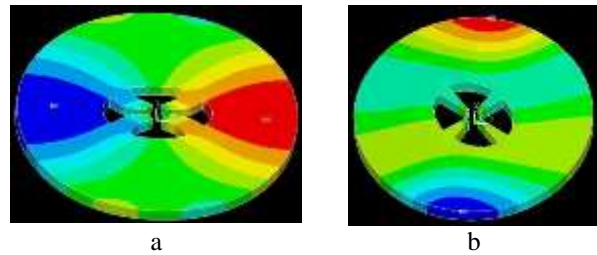
a   b

Fig. 8 2nd flexional form in ring shaped piezoelectic actuators: a – Model 3 (best cross ply movement; 33115 Hz; $S\varphi$ (cross ply) 0.814353); b – Model 2 (best rota – tive movement; 89429Hz; $S\tau$ (rotative) 0.819207)

After comparison of the relationships of geometric parameters with domination coefficients and natural frequencies in two iterations of calculations, we can claim that with the help of domination coefficients we can partially determine the eigenform of elliptical rotation.

6. Conclusions

The results of numerical analysis of ring-shaped piezoelectric actuator with fixation active elements are presented and analyzed in this paper.

The geometric parameters were selected in such a way, that the eigenform would not be lost and the 2nd flexional form would be as high as possible, since this way its rapidity is guaranteed.

In modal analysis, when the piezoelectric ring was optimized, geometric parameters of its fixation elements, domination coefficients and eigenfrequencies were calculated.

It was found out that the best result of cross ply movement was obtained in model 3 (table 5) with coefficient 0.814353, which was achieved with eigenfrequency of 33115 Hz.

In model 2 (Table 6) the best parameters of rotative movement were achieved with coefficient 0.819207, corresponding the natural frequency of 89429 Hz.

The condition for the oscillation amplitude to be stable or change insignificantly is set. The construction developed on such basis satisfies technical characteristics of the system and is technological from the viewpoint of manufacture.

Acknowledgement

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ELIPSINIAM JUDESIUI GAUTI ŽIEDO FORMOS PIEZOELEKTRINIO ŽADINTUVO TVIRTINIMO ELEMENTŲ PARAMETRŲ OPTIMIZAVIMAS

R e z i u m ė

Straipsnyje pasiūlyta žiedo formos pjezoelektrinio žadintuvo su bangas atspindinčiais aktyviais tvirtinimo elementais, skirto elipsiniam judesiui generuoti inovatyvi konstrukcija ir atlikta jos analizė. Pjezoelektrinio žiedinio žadintuvo su aktyviais tvirtinimo elementais rezonansinių savųjų dažnių ir žadinimo formų bei kontakto taškų trajek-torijų nustatymui atliktas skaitinis modeliavimas baigtinių elementų metodu. Iš gautų analizės rezultatų nustatytos tinkamiausios žiedinio žadintuvo skersinių ir sukamųjų virpesių formos. Aptarti skaitinio modeliavimo rezultatai.

G. Baurienė, G. Kulvietis, A. Grigoravičius

OPTIMIZATION OF FIXATION ELEMENT PARAMETERS FOR RING TYPE PIEZOELECTRIC ACTUATOR GENERATING ELLIPTICAL MOVEMENT

S u m m a r y

A novel design of ring type piezoelectric actuator with waves reflecting active elements (the fixation ele-ments) generating elliptical movement is proposed and analyzed in the paper. Numerical analysis by FEM with the aim to determine natural frequencies and modal shapes of the resonator was carried out and motion path of interac-tion contact point found. The best results of model analysis of cross ply and rota4tive movement were found. Results of numerical studies are discussed.