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Research on the dynamics of single-acting crank pump with damless hydro turbine drive

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1. Introduction

Development of the renewable energy and the ever-increasing application of power plants which uses renewable energy sources are recognized by scientists as one of the most important areas of sustainable development. Thus, in accordance with the state of technique and technology progress the most effective power plants based on renewable energy sources keep to be various types of hydroturbines. To their indisputable advantages can be included a high value of efficiency and low cost, as well as several others. In many hard-to-get communities settled along banks of rivers sundry types of hydro pumps must be used to supply water to the irrigated fields. For such hydro pumps motors or internal combustion engines are used primarily as a drive, which seems to be disadvantageous in economic terms. The solution to this problem could be the use of damless hydro turbine as a drive. Therefore, we formulate the objectives of the dynamics study of the piston crank pump with hydro turbine drive and with radial blades as follows: it is required to find regularities that characterize the work of a piston pump with damless hydro turbine drive.

In the books [1, 2] fundamentals of computation theory and theory of operation of crank piston pumps are illuminated. They show typical pump designs, considered design features and strength calculation of details of hydraulic block.

In article [3] torque and power of damless hydro turbine blades in the beginning of the dive and the possibility of their maximum values depending on the position of the blades oriented with respect to the axis of rotation of the hydro turbine were investigated.

In the book [4] describes the design, outlines the issues of calculation, design, manufacture and testing of positive displacement pumps and hydraulic motors used in power hydraulic systems of machines and mechanical equipment.

To ensure the translational motion of the single-acting crank-driven pump as the pump drive can be used crank-rod, rocker mechanisms. Output links of these mechanisms can make reciprocating motion [5].

In the book [6] outlined the basics of mathematical modelling of dynamics of mechanical systems, mechanisms of machines, devices and instruments.

The authors [7] studied the dynamics of the vibratory movement of the body on the swinging plane for automatic assembly of parts. They investigated the motion of the body when the stiffness coefficient and rolling angle had been changed, also identified laws for different trajectories of body on the rocking plane.

The paper [8] is dedicated to modeling of the thermal-hydraulic piston pump which is used in the airplane. Lumped parameter mathematical models are developed which are based on conservation of energy. Heat transfer analysis for the piston pump is also given in the paper.

In the article [9] investigation of a single-piston model of the aviation axial-piston pump is presented. This single-piston model comprehensively considers fluid compressibility, orifice restriction effect, fluid resistance in the capillary tube, and the leakage flow. Besides, the instantaneous discharge areas used in the single-piston model have been calculated in detail.

In the article [10] a mathematical model for swash plate axial piston pumps with conical cylinder blocks is presented. Simulation runs are carried out using this model to evaluate the performance of a pump with certain design parameters under different operating conditions. The results show that both the moment acting on the swash plate in the direction perpendicular to its inclination and the torque acting on the driving shaft are nearly constants under certain operating conditions, and increase linearly with the increase of the delivery pressure and/or increase of the swash plate inclination angle.

The paper [11] discusses the basic principles of hydraulic turbines, with special emphasis on the use of computational fluid dynamics (CFD). The basic fluid mechanics is briefly treated for the three main types of hydraulic turbine: Pelton, Francis and axial turbines.

The TurboPiston Pump [12] was invented to make use of merits such as, high flow rates often seen in centrifugal pumps and high pressures associated with positive displacement pumps. The objective of this study is to manufacture a plastic model 12” TurboPiston Pump to demonstrate the working principle and a metal prototype for performance testing. In addition, this research includes the study of the discharge valve to estimate the valve closing time and fluid mass being recycled back into the cylinder through hand calculations.

2. Equations of part movement

Make the following assumption, simplification and allowance: there is no dead volume, the fluid sucked into is considered to be incompressible, and there is no friction in the mechanical parts of the pump.

The law of motion of the piston (plunger) of the crank pumps is defined by kinematics of the crank - connecting rod mechanism. If we neglect the influence of the finite length of the connecting rod, the path traveled by the piston can be related to the angle φ of rotation of the crank by the following relationship (Fig. 1):

$$x = r(1 - cos\,\varphi), \qquad (1)$$

where r is the radius of the crank.

Piston's velocity is derivative of the path with respect to time, i.e.:

$$u = dx/dt = r\,sin\,\varphi\,d\varphi/dt = r\omega\,sin\,\varphi, \qquad (2)$$

where $\omega = d\varphi/dt$ is the angular velocity of rotational motion of the crank. Piston acceleration is

$$j = du/dt = r\omega\cos\varphi\,d\varphi/dt = r\omega^2\cos\varphi. \qquad (3)$$

Using the equation of unsteady flow of a real fluid, which has the form:

$$\frac{\partial}{\partial l}\left(z + \frac{p}{pg} + \frac{\vartheta^2}{2g}\right) + \frac{1}{g}\frac{\partial\vartheta}{\partial t} + i_\omega = 0, \qquad (4)$$

we will determine the pressure under the piston expressed in meters from the column of fluid in the form of [1, 5]:

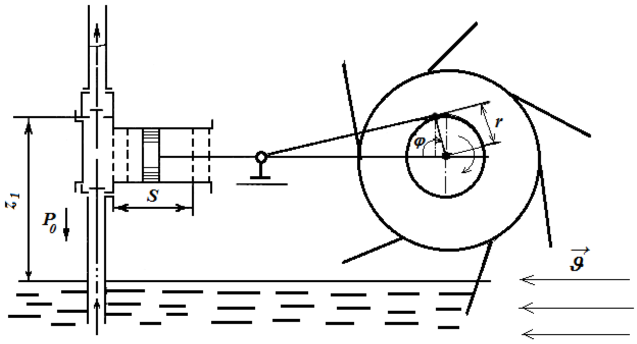

Fig. 1 A dynamic model of single-acting crank pump

for the suction period:

$$\frac{p_B}{\rho g} = \frac{p_0}{\rho g} - \left[z_1 + x + (W_B + 1)\frac{u^2}{2g} + h_B + \frac{1}{g}(L_x + x)\frac{\partial u}{\partial t}\right], \qquad (5)$$

for the period of discharge:

$$\frac{p_i}{\rho g} = \frac{p_B}{\rho g} + \left[z_2 - (S - x) + \frac{\vartheta_B^2}{2g} + (W_i - 1)\frac{u^2}{2g} + h_i + \frac{1}{g}(L_i + S - x)\frac{\partial u}{\partial t}\right], \qquad (6)$$

where p_B is pressure obtained by the piston of the crank pump during the suction period, p_0 is pressure on the free surface of a liquid in the receiving reservoir, z_1 is piston elevation above a liquid level in the receiving reservoir, ϑ is fluid flow velocity at any cross-section of the pipe, x is instantaneous state of the piston with respect to its rightmost position, i_ω is harmful resistance, ξ is the length of the fluid path when lifting, h_B is pressure expended to overcome resistance in the suction valve, W_B is coefficient of the suction pipe, L_B is reduced length of the suction pipe, ρ is density of the fluid, g is acceleration of free fall, z_2 is elevation of discharge hole of the pressure pipe above the lower position of the piston, p_H is pressure under the piston during discharge, p_e is the absolute pressure in the reservoir where the liquid is supplied, L_H is reduced length of the discharge pipe, W_H is reduced coefficient of resistance of the pressure pipe, ϑ_e is velocity of the fluid flow from discharge hole, S is the length of the path of piston motion. We derive the equation of motion of the piston crank-driven pump with damless hydro turbine drive as a mechanical system in the form of Lagrange equations of the second kind.

The kinetic energy of this system is:

$$T = \frac{J\dot{\varphi}^2}{2} + \frac{mu^2}{2} + T_s, \qquad (7)$$

where m is sum of the masses of the piston and rod, T_s is kinetic energy of the connecting rod, J is moment of inertia of the hydro turbine impeller is given by [2]:

$$J = \frac{1}{2}M_0R^2 + 6\rho hl\left(R^2 + \frac{1}{3}l^2 + Rl\,cos\,\alpha_2\right),$$

where R and l is radius of the impeller and hydro turbine blade length respectively, α_2 is angle of the blades, h is blade width, M_0 is mass of water turbine impeller.

The kinetic energy of a connecting rod is determined by the formula:

$$T_s = \frac{1}{2}M_s\vartheta_c^2 + \frac{1}{2}J_c\omega_I^2, \qquad (8)$$

where ω_I is instantaneous angular velocity of the connecting rod, J_c is moment of inertia of the connecting rod relative to the axis passing through the centre of gravity of the connecting rod perpendicularly to the connecting rod.

The following relations have taken place:

$$\omega_I = \frac{r}{l_s}\frac{cos\,\varphi}{\sqrt{1 - \frac{r^2}{l_s^2}sin^2\,\varphi}}\dot{\varphi},$$

$$\vartheta_c^2 = r^2\dot{\varphi}^2\left[\left(1 - \frac{r}{2l_s}\frac{cos\varphi}{\sqrt{1 - \frac{r^2}{l_s^2}sin^2\varphi}}\right)^2 + \frac{1}{4}cos^2\varphi\right],$$

where l_s is the connecting rod length. Therefore, the total kinetic energy has the form:

$$T=\frac{1}{2}\left[\frac{1}{2}M_0R^2+6\rho hl\left(R^2+\frac{1}{3}l^2+Rl\cos\alpha_2\right)\dot{\varphi}^2+\frac{1}{2}mr^2\dot{\varphi}^2\cos^2\varphi\right]+$$

$$+\frac{1}{2}M_sr^2\dot{\varphi}^2\left[\left(1-\frac{r}{2l_s}\frac{\cos\varphi}{\sqrt{1-\frac{r^2}{l_s^2}\sin^2\varphi}}\right)^2+\frac{1}{4}\cos^2\varphi\right]+\frac{M_s}{24}r^2\dot{\varphi}^2\frac{\cos^2\varphi}{1-\frac{r^2\sin^2\varphi}{l_s^2}}. \tag{9}$$

Now we define a generalized hydrodynamic forces acting on the piston during suction and discharge. Work of hydrodynamic forces acting while the piston is moving during the suction and discharge, respectively, will be:

$$\left.\begin{aligned}\delta A_B&=\left(P_0-P_B\right)Fr\sin\varphi\,\delta\varphi,\\ \delta A_H&=\left(P_e-P_H\right)Fr\sin\varphi\,\delta\varphi,\end{aligned}\right\} \tag{10}$$

Consequently, the generalized hydrodynamic forces are determined in the form:

$$\left.\begin{aligned}Q_B&=\left[z_1+d+\left(W_B+1\right)\frac{r^2}{2g}\sin^2\varphi\dot{\varphi}^2+h_B+\frac{1}{g}\left(L_B+d\right)\left(r\ddot{\varphi}\sin\varphi+r\dot{\varphi}^2\cos\varphi\right)\right]F\rho gr\sin\varphi,\\ Q_H&=\left[z_2+\left(W_H-1+F^2/f_e^2\right)\frac{r^2}{2g}\dot{\varphi}^2\sin^2\varphi+h_H+\frac{1}{g}\left(L_H+d\right)\left(r\ddot{\varphi}\sin\varphi+r\dot{\varphi}^2\cos\varphi\right)\right]F\rho gr\sin\varphi,\end{aligned}\right\} \tag{11}$$

where F and d are area and diameter of the piston, f_e is cross-sectional area of discharge hole.

The equations of motion of the piston crank pump with damless hydro turbine drive become:

for suction period:

$$\left\{\frac{1}{2}M_0R^2+6\rho hl\left(R^2+\frac{1}{3}l^2+Rl\cos\alpha_2\right)+mr^2\cos^2\varphi+M_sr^2\left[\left(1-\frac{r}{2l_s}\frac{\cos\varphi}{\sqrt{1-\frac{r^2}{l_s^2}\sin^2\varphi}}\right)^2+\frac{1}{4}\cos^2\varphi\right]+\right.$$

$$\left.+\frac{1}{12}M_sr^2\frac{\cos^2\varphi}{1-\frac{r^2}{l_s^2}\sin^2\varphi}\right\}\ddot{\varphi}+\left\{M_sr^2\left[\left(1-\frac{r}{2l_s}\frac{\cos\varphi}{\sqrt{1-\frac{r^2}{l_s^2}\sin^2\varphi}}\right)\left(\frac{r}{2l_s}\frac{\sin\varphi}{\sqrt{1-\frac{r^2}{l_s^2}\sin^2\varphi}}-\frac{r^3}{4l_s^3}\frac{\cos\varphi\sin2\varphi}{\left(1-\frac{r^2}{l_s^2}\sin^2\varphi\right)^{\frac{3}{2}}}\right)-\frac{1}{8}\sin2\varphi\right]+\right.$$

$$\left.+F\left(W_B+1\right)\frac{\rho r^3}{2}\sin^3\varphi+F\rho\left(L_B+d\right)r^2\cos\varphi\sin\varphi\right\}\dot{\varphi}^2+F\rho r^2\left(L_B+d\right)\sin^2\varphi\ddot{\varphi}+\left(z_1+d+h_B\right)Fr\rho g\sin\varphi=M, \tag{12}$$

for discharge period:

$$\left\{\frac{1}{2}M_0R^2+6\rho hl\left(R^2+\frac{1}{3}l^2+Rl\cos\alpha_2\right)+mr^2\cos^2\varphi+M_sr^2\left[\left(1-\frac{r}{2l_s}\frac{\cos\varphi}{\sqrt{1-\frac{r^2}{l_s^2}\sin^2\varphi}}\right)^2+\frac{1}{4}\cos^2\varphi\right]+\right.$$

$$\left.+\frac{1}{12}M_sr^2\frac{\cos^2\varphi}{1-\frac{r^2}{l_s^2}\sin^2\varphi}\right\}\ddot{\varphi}+\left\{M_sr^2\left[\left(1-\frac{r}{2l_s}\frac{\cos\varphi}{\sqrt{1-\frac{r^2}{l_s^2}\sin^2\varphi}}\right)\left(\frac{r}{2l_s}\frac{\sin\varphi}{\sqrt{1-\frac{r^2}{l_s^2}\sin^2\varphi}}-\frac{r^3}{4l_s^3}\frac{\cos\varphi\sin2\varphi}{\left(1-\frac{r^2}{l_s^2}\sin^2\varphi\right)^{\frac{3}{2}}}\right)-\frac{1}{8}\sin2\varphi\right]+\right.$$

$$\left.+F\left(W_H-1+\frac{F}{f_e}\right)\frac{\rho r^3}{2}\sin^3\varphi+F\rho\left(L_H+d\right)r^2\cos\varphi\sin\varphi\right\}\dot{\varphi}^2+F\rho r^2\left(L_H+d\right)\sin^2\varphi\,\ddot{\varphi}+\left(z_2+h_H\right)Fr\rho g\sin\varphi=M. \quad (13)$$

For simplicity, assume that the following conditions occur:

$$\frac{r}{l_s}<<1 \text{ and } \frac{m}{M_0}<<1.$$

Under these conditions, the equations of motion of the piston of the crank pump Eqs. (12) and (13) are transformed to:

for suction period:

$$\ddot{\varphi}+\left[\left(\varepsilon_2-\varepsilon_1\right)\sin2\varphi+\mu\sin^3\varphi\right]\varphi^2+\omega_{OB}^2\sin\varphi=\frac{M}{J}, \quad (14)$$

for discharge period:

$$\ddot{\varphi}+\left[\left(\varepsilon_3-\varepsilon_1\right)\sin2\varphi+\mu\sin^3\varphi\right]\varphi^2+\omega_{OH}^2\sin\varphi=\frac{M}{J}, \quad (15)$$

where $\varepsilon_1=\frac{M_sr^2}{8J}, \varepsilon_2=\frac{m_Br\left(L_B+d\right)}{4J}, \mu=\frac{m_Br^2}{4J}$,

$\varepsilon_3=\frac{m_Br\left(L_H+d\right)}{4J},\ \omega_{OB}^2=\frac{m_Bg\left(z_1+d+h_B\right)}{2J}$,

$\omega_{OH}^2=\frac{m_Bg\left(z_2+h_H\right)}{2J},\ m_B=2\rho Fr.$

In [2-7] the characteristics of the driving torque of the hydro turbine with six flat and radial blades are determined. Forms of the averaged moments of the driving forces for such damless hydro turbines are:

$$M=a-b\dot{\varphi}, \quad (16)$$

where:

$$a=\frac{\rho Q_0^2}{h}\left(3w-T_1\mu_{11}+D_1\mu_{12}\right),\ b=\frac{\rho Q_0^2}{h}\left(E_1\lambda_{11}+K_1\lambda_{21}\right),$$

$$T_1=\frac{1}{2}\frac{R}{l}\left[\sin\alpha_2+\sin\left(2\alpha_1+\alpha_2\right)+\sin\left(4\alpha_1+\alpha_2\right)\right]+$$
$$+\frac{1}{4}\left[\sin2\alpha_2+\sin2\left(\alpha_1+\alpha_2\right)+\sin2\left(2\alpha_1+\alpha_2\right)\right],$$

$$D_1=\frac{1}{2}\frac{R}{L}\left[\cos\alpha_2+\cos\left(2\alpha_1+\alpha_2\right)+\cos\left(4\alpha_1+\alpha_2\right)\right]+$$
$$+\frac{1}{4}\left[\cos2\alpha_2+\cos2\left(\alpha_1+\alpha_2\right)+\cos2\left(2\alpha_1+\alpha_2\right)\right],$$

$$E_1=\frac{JC}{\vartheta}\left[\cos\alpha_2+\cos\left(\alpha_1+\alpha_2\right)+\cos\left(2\alpha_1+\alpha_2\right)\right],$$

$$W=\frac{1}{2}\frac{R}{l}\cos\alpha_2+\frac{1}{4}, C=\frac{R^2}{l^2}+\frac{R}{l}\cos\alpha_2+\frac{1}{3}, Q=\vartheta hl,$$

$$K_1=\frac{JC}{\vartheta}\left[\sin\alpha_2+\sin\left(\alpha_1+\alpha_2\right)+\sin\left(2\alpha_1+\alpha_2\right)\right],$$

$$\mu_{11}=\frac{1}{\pi-2\alpha_1-\alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2}\sin2\varphi2\varphi,$$

$$\mu_{21}=\frac{1}{\pi-2\alpha_1-\alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2}\cos2\varphi2\varphi,$$

$$\lambda_{11}=\frac{1}{\pi-2\alpha_1-\alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2}\sin\varphi d\varphi,$$

$$\lambda_{21}=\frac{1}{\pi-2\alpha_1-\alpha_2}\int_0^{\pi-2\alpha_1-\alpha_2}\cos\varphi d\varphi.$$

Q_0 is fluid flow rate at the working part of the blade, α_1 is angle between the blades, ϑ is flow velocity of the liquid. The remaining parameters are specified in a scheme of hydro turbine.

3. Solution of the equation of motion

Eqs. (14) and (15) are essentially nonlinear, since the nonlinear term is included in the equation without a small parameter. For simplicity, assume that $\varepsilon_1\approx\varepsilon_2\approx\varepsilon_3$.

The differential Eqs. (14) and (15) differ from each other by coefficients, but the shape is identical, so it suffices to solve one of them. By substituting (16) into (14) and substituting $d\tau=\omega_{OB}dt$, transform the equation of motion (14) to the dimensionless form

$$\frac{d^2\varphi}{d\tau^2}+\frac{b}{J\omega_{OB}}-\frac{d\varphi}{d\tau}+\mu\sin^3\varphi\left(\frac{d\varphi}{d\tau}\right)^2+\sin\varphi=\frac{a}{J\omega_{OB}^2}. \quad (17)$$

Eq. (17) may be subjected to further simplification, if we take into account that during one period of change in the angle φ from 0 to 2π value $\omega=\frac{d\varphi}{d\tau}$ changes very little, then its derivative with respect to the angle of rotation φ can be considered equal to its average value [13]. Assuming for a variable angle φ and using the relation $d\varphi = \bar{\omega} d\tau,$ transform Eq. (17) to the form:

$$\frac{d\bar{\omega}}{d\varphi} = -\frac{\varepsilon}{2\pi}\int_0^{2\pi} d\varphi - \frac{\mu\bar{\omega}}{2\pi}\int_0^{2\pi} sin^3\varphi d\varphi - \frac{1}{2\pi\bar{\omega}}\int_0^{2\pi} sin\varphi d\varphi + \\ + \frac{B}{2\pi\bar{\omega}}\int_0^{2\pi} d\varphi,$$

where $\varepsilon = \frac{b}{J\omega_{OB}}$, $B = \frac{a}{J\omega_{OB}^2}$.

When performing the averaging quantities a value $\bar{\omega}$ is constant. Assuming that:

$$\int_0^{2\pi} sin^3\varphi d\varphi = 0, \int_0^{2\pi} sin\varphi d\varphi = 0, \frac{1}{2\pi}\int_0^{2\pi} d\varphi = 1.$$

After integration:

$$\frac{d\bar{\omega}}{d\varphi} = -\varepsilon + \frac{B}{\bar{\omega}}. \qquad (18)$$

Eq. (18) can be examined to determine the variable $\bar{\omega}$ under transient regimes. After averaging the Eq. (17) also takes the form (18).

Condition for the existence of stationary regimes $\frac{d\bar{\omega}}{d\varphi} = 0.$

Under this condition, the equation of stationary regimes of motion is:

$$\bar{\omega}_c - \frac{B}{\varepsilon} = 0.$$

Hence, we find the average value of the angular velocity of the hydro turbine in a stationary mode:

$$\bar{\omega}_c = \frac{B}{\varepsilon} = \frac{a}{b\omega_{OB}}. \qquad (19)$$

Considering the ratio:

$$\frac{d\varphi}{d\tau} = \frac{d\varphi}{dt \cdot \omega_{OB}} = \frac{\overline{\Omega_c}}{\omega_{OB}}. \qquad (20)$$

From Eq. (20) we obtain the following expression:

$$\overline{\Omega_c} = \Omega_0,$$

where $\Omega_0 = \frac{a}{b}$ is the angular velocity of the hydro turbine at idle.

It follows that the mean value of the oscillation frequency of the piston of crank pump equals to the angular velocity of the hydro turbine power at idle.

Using the Eq. (19), we transform the Eq. (18) to the form:

$$\frac{d\bar{\omega}}{d\tau} = -\varepsilon\left(\bar{\omega} - \frac{\Omega_0}{\omega_{OH}}\right). \qquad (21)$$

Integrating (21) with the initial condition $\tau = 0, \bar{\omega} = 0$, we obtain for suction and discharge periods the following expressions:

$$\bar{\omega}_B = \frac{\Omega_0}{\omega_{OB}}\left(1 - e^{-\varepsilon\tau}\right), \ \bar{\omega}_H = \frac{\Omega_0}{\omega_{OH}}\left(1 - e^{-\varepsilon\tau}\right), \qquad (22)$$

whence

$$\overline{\Omega}_B = \Omega_0\left(1 - e^{-\frac{b}{J}t}\right), \ \overline{\Omega}_H = \Omega_0\left(1 - e^{-\frac{b}{J}t}\right). \qquad (23)$$

Average speed of the hydraulic turbine at $t \to \infty$ tends to Ω_0, i.e.

$$\Omega_0 = \lim_{t\to\infty} \overline{\Omega}.$$

Taking into account the relation (20), we write the integral Eq. (18) in the form

$$\varphi = \frac{J\Omega_0}{b} ln\frac{\Omega_0}{\Omega_0 - \overline{\Omega}} - \frac{J}{b}\overline{\Omega}. \qquad (24)$$

On Fig. 2 line 1 is plotted based on the results of numerical solutions of systems of Eq. (14), and line 2 is based on the results of analytical solutions The graphs show that the values of the numerical solutions of Eq. (14) oscillate around the mean value, and the analytical solutions of the averaged Eq. (14) pass through the midline of the graph 1.

Based on of this representation it can be judged about the proximity of the results of analytical and numerical solutions.

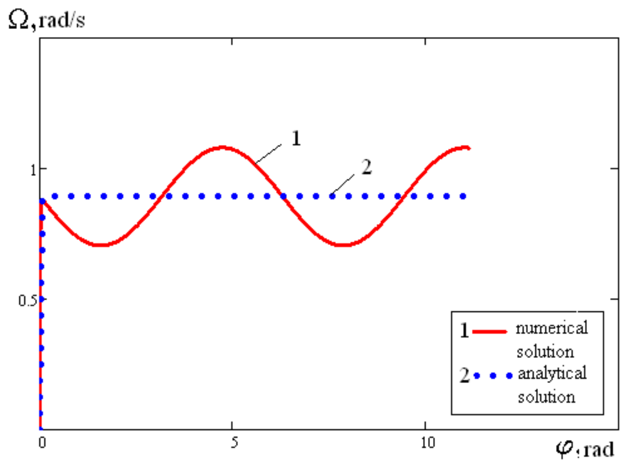

Fig. 2 Relation between the angular velocity and the rotation angle of hydraulic turbine

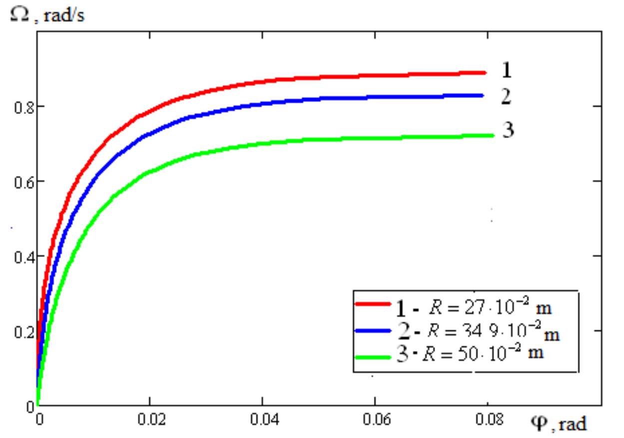

Fig. 3 Relation between the angular velocity and the rotation angle of the hydro turbine at different values of the impeller radius

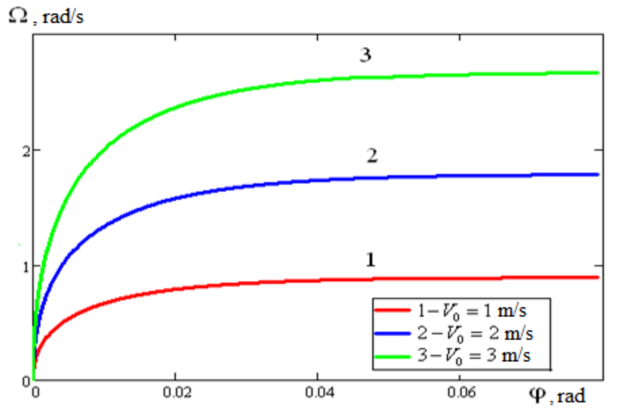

Fig. 4 Relation between the angular velocity and the rotation angle of the hydro turbine at different values of the watercourse speed

On Figs. 3 and 4 according to the Eq. (24) graphs of the relation between the average angular velocity and the rotation angle of the hydro turbine at different values of the radius of the rotor and at different values of the flow velocity of the watercourse are plotted. The average angular velocity of rotation of hydro turbine in a stationary mode equals to the angular velocity of the hydro turbine at idle and does not depend on the mode of work of piston pumps. Average speed of rotation depends only on the parameters of a hydro turbine and the flow speed of the watercourse.

4. Stability of motion of the system

Now consider the stationary regimes of the single-acting crank pump with damless hydro turbine drive, which are determined by the Eq. (18). To do so, we transform Eq. (18) to the form:

$$\left.\begin{aligned} \frac{d\bar{\omega}}{d\tau} &= -\varepsilon\left(\bar{\omega} - \frac{B}{\varepsilon}\right), \\ \frac{d\varphi}{d\tau} &= \bar{\omega}. \end{aligned}\right\} \tag{25}$$

Form a system of Eqs. (25) in variations. Assume that:

$$\left.\begin{aligned} \varphi &= \varphi^* + q, \\ \bar{\omega} &= \bar{\omega}^* + p, \end{aligned}\right\} \tag{26}$$

where $\bar{\omega}^*$ and φ^* are stationary solutions of Eqs. (25), which have form:

$$\bar{\omega}^* = \Omega, \ \varphi^* = \Omega\tau.$$

Substituting Eqs. (26) into (25) we obtain:

$$\left.\begin{aligned} \frac{dp}{d\tau} &= -\varepsilon p, \\ \frac{dq}{d\tau} &= p. \end{aligned}\right\} \tag{27}$$

Integrals of the system of Eqs. (27) take the form:

$$\begin{aligned} p &= Ce^{-\varepsilon\tau}, \\ q &= \frac{-C}{\varepsilon}e^{-\varepsilon\tau}, \end{aligned} \tag{28}$$

when $\tau \to \infty, \ p = 0, \ q = 0$. Consequently, the stationary motion of damless hydro turbine at discharge with crank piston pump is asymptotically stable.

5. Conclusions

The average angular velocity of rotation of hydro turbine in a stationary mode is equal to the angular velocity of the hydro turbine at idle and does not depend on the mode of work of piston pumps. Average speed depends only on the parameters of a hydro turbine and the flow speed of the watercourse. With the increase in the radius of hydro turbine impeller its average angular velocity decreases, while increasing the value of the watercourse speed, the average angular velocity increases.

References

1. **Chirnayev, I.A.** 1983. Crank piston pumps. – L: Mechanical Engineering, Leningrad dept, 176p. (in Russian).
2. **Chirnayev, I.A.** 1966. Piston pumps. Moscow-Leningrad.: Mechanical Engineering, 188p. (in Russian).
3. **Tuleshov, A.K.; Bissembayev, K.; Zhamenkeev, Y.K.** Torque and power of the hydro turbine in the beginning of the dive of blades // Vestnik of the Moscow City Pedagogical University. Series informatics and informatization of education, 2008, №4(14), p.154-161. (in Russian).
4. **Bashta, T.M.** 1971. Engineering Hydraulics. Handbook. 2 edition of Moscow.: Engineering, 672p. (in Russian).
5. **Krainev, A.F.** 1987. Dictionary-handbook by the mechanisms. Moscow: Mashinostroenie, 560p. (in Russian).
6. **Ualiev, G., Ualiev, Z.G.** 2006. Mathematical simulation of the dynamics of mechanical systems with the

variable parameters. Almaty: KazNPU, 275p. (in Russian).

7. **Bakšys, B.; Kinzhebayeva, D.** 2008. Displacement of the body on the rocking plane, Mechanika 3(71): 31-37.
8. **Li Cheng-gong, Jiao Zong-xia**. 2006. Thermal-hydraulic Modeling and Simulation of Piston Pump, Chinese journal of aeronautics 19(4): 354-358. http://dx.doi.org/10.1016/S1000-9361(11)60340-3.
9. **Guan Changbin, Jiao Zongxia, He Shouzhan**. 2014. Theoretical study of flow ripple for an aviation axial-piston pump with damping holes in the valve plate, Journal of Aeronautics 27(1): 169-181. http://dx.doi.org/10.1016/j.cja.2013.07.044.
10. **Kassem, S.A**.; **Bahr, M.K.** On the dynamics of swash plate axial piston Pumps with conical cylinder blocks. Sixth Triennial International Symposium on Fluid Control Measurement and Visualization Flucome2000, August 13-17, 2000, Sherbrooke University, Sherbrooke, Canada.
11. **Drtina, P.; Sallaberger, M.** 1999. Hydraulic turbines—basic principles and state-of-theart computational fluid dynamics applications. Proc. Instn. Mech. Engrs, Vol. 213 Part C: 85-102.
12. **Jason A. Kent.** Numerical and Experimental Analysis of a TurboPiston Pump. A Thesis. University of New Orleans Theses and Dissertations. 2010. 85p.
13. **Levitsky, N.I.** 1988. Oscillations in mechanisms. Moscow: Nauka, 330p.

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VIENOS KRYPTIES SIURBLIO SU SKRIEJIKO NEUŽTVENKIAMA HIDROTURBINA DINAMINIO MODELIO TYRIMAI

R e z i u m ė

Šiame darbe pateikti vienos krypties skriejiko siurblio su neužtvenkiama hidroturbina dinaminio modelio tyrimo rezultatai. Sudaryti vienos krypties skriejiko siurblio su neužtvenkiama hidroturbina kompiliuoti dinaminis ir matematinis modeliai. Judėjimo lygtys išspręstos analitiškai ir skaitiniais metodais. Gauti rezultatai palyginti tarpusavyje. Neužtvenkiamos hidroturbinos charakteristikos su vienos krypties skriejiko siurblio apkrova įvertintos priklausomai nuo rotoriaus spindulio ir vandens tėkmės greičio. Buvo parodyta, kad hidroturbinos vidutinis kampinis sukimosi greitis stacionariam režime yra adekvatus neapkrautos hidroturbinos kampiniam greičiui, kuris nepriklauso nuo stūmoklinio siurblio poveikio. Vidutinis greitis priklauso tik nuo hidroturbinos parametrų ir vandens srauto greičio. Gauti teorinių tyrimų rezultatai gali būti sėkmingai panaudoti hidroturbinų, kaip vienos krypties skriejiko siurblių pavarų, projektavime.

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RESEARCH ON THE OF DYNAMIC MODEL OF SINGLE-ACTING CRANK PUMP WITH DAMLESS HYDRO TURBINE DRIVE

S u m m a r y

The research of the dynamic model of single-acting crank pump with damless hydro turbine drive has been made in the present work. Compiled dynamic and mathematical models of single-acting crank pump with damless hydro turbine drive have been constructed. The equations of motion have been solved using analytical and numerical methods and their results have been compared. The characteristics of the damless hydro turbine with the load of the crank pump have been evaluated depending on the radius of the rotor and on the speed of the watercourse. It has been revealed that the average angular velocity of rotation of hydro turbine in a stationary mode is the angular velocity of the hydro turbine at idle and does not depend on the mode of piston pumps. Average speed depends only on the parameters of a hydro turbine and the flow velocity of the watercourse.

The results obtained in the course of theoretical research can be successfully used for the design of hydro turbine as a drive for single-acting crank pump.

Keywords: single-acting crank pump, damless hydro turbine, watercourse speed.

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