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Thermally induced stability and vibration of initially stressed laminated composite plates

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1. Introduction

For the past decades, the composite materials are widely used in spacecraft and engineering industries because of their higher tensile strength and lower weight. Composite plate structures are often applied at elevated temperature environments. In such thermal circumstances, the thermal induced compressive stresses will be developed in the composite plates and consequently lead to the change in mechanical behaviors. The thermally induced behavior of composite plate plays an important role in the design of structural components in thermal environments. Thus, the studies on thermal vibration and buckling of composite plates are increasing considerably in recent years.

Many investigations on thermally induced behaviors of composite plates are concerned with the thermal stability and vibration. The critical buckling temperatures of laminated plates based on a finite strip method were studied by Dawe and Ge [1]. In the pre-buckling stage, an in-plane thermal stress analysis was conducted first, and a buckling analysis was followed using the determined in-plane stress distribution. Wang et al. [2] presented the local thermal buckling of laminated plate using the delaminated buckling model. The analytical predictions for the critical temperature yielding the local delamination buckling are shown to correlate well with experimental results. Shian and Kuo [3] developed a thermal buckling analysis method for composite sandwich plates. The results show that the buckling mode of sandwich plate depends on the fiber orientation in the faces and the aspect ratio of the plate. Thermal buckling analysis of cross-ply laminated hybrid composite plates with a hole subjected to a uniform temperature rise was investigated by Avci et al. [4]. The effects of hole size, lay-up sequences and boundary conditions on the thermal buckling temperatures were investigated. The equivalent mechanical loading concept was used to study various thermal buckling problems of simple laminated plate configurations by Jones [5]. The results were given in the form of buckling temperature change from the stress-free temperature against plate aspect ratio curves. Matsunaga [6] presented the thermal buckling of laminated plates using the principle of virtual work. Several sets of truncated mth order approximate theories were applied to solve the eigenvalue problems. Modal transverse shear and normal stresses could be calculated by integrating the equilibrium equations.

The governing equations for determining thermal buckling of imperfect sandwich plates were developed by Zakeri and Alinia [7]. The buckling thermal stress remains unchanged for aspect ratios greater than five. The structural optimization of a laminated plate subjected to thermal and shear loading was considered by Teters [8]. The optimization criteria depend on two variable design parameters of composite properties and temperature. Thermal buckling analysis of composite laminated plates under uniform temperature rise was investigated by Shariyat [9]. A numerical scheme and a modified instability criterion are used to determine the buckling temperature in a computerized solution. A thermal buckling response of symmetric laminated plates subjected to a uniformly distributed temperature load was presented by Kabir et al. [10]. The numerical results were presented for various significant effects such as length-to-thickness ratio, plate aspect ratio and modulus ratio. Thermal buckling behavior of imperfect laminated plates based on first order plate theory was studied by Pradeep and Ganesan [11]. A decoupled thermo-mechanical analysis is used to deal with the thermal buckling and vibration behavior of sandwich plates. The variation of natural frequency and loss factor with temperature was studied by Owhadi and Shariat [12]. The plate was assumed to be under the longitudinal temperature rise. The effects of initial imperfections on buckling loads were discussed. A perturbation technique was used by Verma and Singh [13] to find the buckling temperature of laminated composite plates subjected to a uniform temperature rise. It was found that small variations in material and geometric properties of the composite plate significantly affect the buckling temperature of the laminated composite plate. Wu [14] investigated the stresses and deflections of a laminated plate under thermal vibration using the moving least squares differential quadrature method. The method provides rapidly convergent and accurate solutions for calculating the stresses and deflections.

The thermal buckling behavior of the laminated plates subjected to uniform and/or non-uniform temperature fields was studied by Ghomshei [15]. The influence parameters of plate aspect ratio, cross-ply ratio and stiffness ratio on the critical temperature were presented. Rath [16] the free behavior of laminated plates subjected to varying temperature and moisture. A simple laminated plate model is developed for the vibration of composite plates subjected to hygrothermal loading. The results showed the effects of geometry, material and lamination parameters of woven fiber laminate on the vibration of composite plates for different temperature. Ghugal [17] presented the flexural response of cross-ply laminated plates subjected to thermo-mechanical loads. The shear deformation theory satisfies the shear stress free boundary conditions on the top and bottom surfaces of the plate. Thermal stresses for three-layer symmetric cross-ply laminated plates subjected to uniform linear and nonlinear and thermo-mechanical loads are obtained. The governing equations for laminated beams subjected to uniform temperature rise are derived by Fu [18]. The effects of the transverse shear effects and boundary conditions on the thermal buckling and post-buckling of the beams are discussed. A differential quadrature method is applied to obtain the maximum buckling temperature of laminated composite by Malekzadeh [19]. The direct iterative method in conjunction with genetic algorithms is used to determine the optimum fiber orientation for the maximum buckling temperature.

From the literature reviewed, researches on the vibration and buckling of initially stressed laminate plates under thermal environmental condition seem to be lacking. The vibration and stability behaviors of initially-stressed laminate plates have been investigated by Chen et al. [20-21] in recent years. The studies revealed that the initial stress in structures may significantly influence the behaviors of laminated plates. Therefore, while studying the thermal buckling and vibration behavior of laminate plates, the effect of initial stress should be taken into account. In this paper, the equilibrium equations for a laminated plate subjected to the arbitrary initial stress and thermal condition are established by using variation method. The temperature field is assumed to be uniform plus linearly distributed through the plate thickness. The effects of various parameters on the critical temperature, natural frequencies and buckling loads in thermal environments are presented.

2. Equilibrium equations

Following a similar technique described by Brunelle and Robertson [22] and Chen et al. [20-21], Hamilton's principle is applied to derive nonlinear equations of the composite plate including the effects of rotary inertia and transverse shear. For an initially stressed body which is in equilibrium and subjected to a time-varying incremental deformation, the Hamilton's principle can be expressed as

$$\delta\int_{t_0}^{t_1}\left(U-K-W_e-W_i\right)dt=0\,, \tag{1}$$

where

$$\left.\begin{aligned}&U=\int_{V_0}\sigma_{ij}\varepsilon_{ij}\,dV;\ K=\frac{1}{2}\int_{V_0}\rho\dot{v}_i\dot{v}_i\,dV;\\&W_e=\int_{S_0}p_iv_i\,dS;W_i=\int_{v_o}X_iv_i\,dV,\end{aligned}\right\} \tag{2}$$

where U, K, W_e and W_i are the strain energy, kinetic energy, work of external forces and internal forces, respectively; σ_{ij} and ε_{ij} are the stresses and strains; v_i are the displacements referred to the spatial frame; ρ is the density; X_i is the body force per unit initial volume and p_i is the external force per unit initial surface area. The application of the minimum total energy principle leads to the general equations and boundary conditions. Assume that the stresses and applied forces are constant, and substitute Eq. (2) into Eq. (1). Then taking the variation and integrating the kinetic energy term by parts with respected to time, Eq. (1) becomes:

$$\int_{t_0}^{t_1}\left[\int_{V_0}\left(\sigma_{ij}\delta\varepsilon_{ij}-X_i\delta v_i-\rho\ddot{v}\delta v_i\right)dV-\int_{S_0}p_i\delta v_i\,dS\right]dt=0\,. \tag{3}$$

If a rectangular plate is considered, the equations can be rephrased in xy coordinates. The incremental displacements are assumed to be of the following forms:

$$\left.\begin{aligned}v_x(x,y,z,t)&=u_x(x,y,t)+z\varphi_x(x,y,t);\\v_y(x,y,z,t)&=u_y(x,y,t)+z\varphi_y(x,y,t);\\v_z(x,y,z,t)&=w(x,y,t),\end{aligned}\right\} \tag{4}$$

where u_x, u_y and w are the displacements of the middle surface in the x, y and z direction, respectively; φ_x and φ_y denotes the rotation angle about y and x axis, respectively. The two edges of a rectangular plate are set along x and y axes, respectively. The stress-strain relations are taken to be those of uncoupled linear thermal elasticity. Hence, the constitutive relations for the kth lamina including the thermal effect can be written as:

$$\begin{bmatrix}\sigma_{xx}\\\sigma_{yy}\\\sigma_{xy}\\\sigma_{yx}\\\sigma_{zx}\end{bmatrix}^{(k)}=\begin{bmatrix}C_{11}&C_{12}&C_{16}&0&0\\C_{12}&C_{22}&C_{26}&0&0\\C_{16}&C_{26}&C_{66}&0&0\\0&0&0&C_{44}&C_{45}\\0&0&0&C_{45}&C_{55}\end{bmatrix}^{(k)}\begin{bmatrix}\varepsilon_{xx}-\alpha_{xx}\Delta T\\\varepsilon_{yy}-\alpha_{yy}\Delta T\\\varepsilon_{xy}-\alpha_{xy}\Delta T\\\varepsilon_{yx}\\\varepsilon_{zx}\end{bmatrix}, \tag{5}$$

where C_{ij} are the elastic constants of lamina; α_{ij} are thermal expansion coefficients and ΔT is the temperature rise. The stress-displacement relations are found to be:

$$\left.\begin{aligned}\sigma_{xx}&=C_{11}\left(u_{x,x}+z\varphi_{x,x}-\alpha_{xx}\Delta T\right)+C_{12}\left(u_{y,y}+z\varphi_{y,y}-\alpha_{yy}\Delta T\right)+C_{16}\left(u_{x,y}+z\varphi_{x,y}+u_{y,x}+z\varphi_{y,x}+w_{,x}w_{,y}-\alpha_{xy}\Delta T\right);\\\sigma_{yy}&=C_{12}\left(u_{x,x}+z\varphi_{x,x}-\alpha_{xx}\Delta T\right)+C_{22}\left(u_{y,y}+z\varphi_{y,y}-\alpha_{yy}\Delta T\right)+C_{26}\left(u_{x,y}+z\varphi_{x,y}+u_{y,x}+z\varphi_{y,x}+w_{,x}w_{,y}-\alpha_{xy}\Delta T\right);\\\sigma_{xy}&=C_{16}\left(u_{x,x}+z\varphi_{x,x}-\alpha_{xx}\Delta T\right)+C_{26}\left(u_{y,y}+z\varphi_{y,y}-\alpha_{yy}\Delta T\right)+C_{66}\left(u_{x,y}+z\varphi_{x,y}+u_{y,x}+z\varphi_{y,x}+w_{,x}w_{,y}-\alpha_{xy}\Delta T\right);\\\sigma_{yz}&=C_{44}\left(\varphi_y+w_{,y}\right);\\\sigma_{zx}&=C_{55}\left(\varphi_x+w_{,x}\right),\end{aligned}\right\} \tag{6}$$

Substitute Eqs. (4)-(6) into Eq. (3), perform all necessary partial integrations and group the terms by the five independent displacement variations, δu_x, δu_y, δw, $\delta\varphi_x$ and $\delta\varphi_y$, to yield the following five governing equations:

$$\begin{bmatrix} A_{11}u_{x,x} + A_{16}(u_{x,y} + u_{y,x}) + A_{12}u_{y,y} + B_{11}\varphi_{x,x} + B_{16}(\varphi_{x,y} + \varphi_{y,x}) + B_{12}\varphi_{y,y} + N_{xx}u_{x,x} + \\ + M_{xx}\varphi_{x,x} + N_{xy}u_{x,y} + M_{xy}\varphi_{x,y} + N_{xz}u_{z,x} + N^{\mathrm{T}}_{xx}u_{x,x} + M^{T}_{xx}\varphi_{x,x} + N^{T}_{xy}u_{x,y} + M^{T}_{xy}\varphi_{x,y} \end{bmatrix}_{,x} +$$
$$+ \begin{bmatrix} A_{16}u_{x,x} + A_{26}u_{y,y} + A_{66}(u_{x,y} + u_{y,x}) + B_{16}\varphi_{x,x} + B_{66}(\varphi_{x,y} + \varphi_{y,x}) + B_{26}\varphi_{y,y} + N_{yy}u_{x,y} + \\ + M_{yy}\varphi_{x,y} + N_{xy}u_{x,x} + M_{xy}\varphi_{x,x} + N_{yz}u_{z,x} + N^{T}_{yy}u_{x,y} + M^{T}_{yy}\varphi_{x,y} + N^{T}_{xy}u_{x,x} + M^{T}_{xy}\varphi_{x,x} \end{bmatrix}_{,y} + f_x = I_1\ddot{u}_x; \tag{7}$$

$$\begin{bmatrix} A_{16}u_{x,x} + A_{66}(u_{x,y} + u_{y,x}) + A_{26}u_{y,y} + B_{16}\varphi_{x,x} + B_{66}(\varphi_{x,y} + \varphi_{y,x}) + B_{26}\varphi_{y,y} + N_{xx}u_{y,x} + \\ + M_{xx}\varphi_{y,x} + N_{xy}u_{y,y} + M_{xy}\varphi_{y,y} + N_{xz}u_{z,y} + N^{\mathrm{T}}_{xx}u_{y,x} + M^{T}_{xx}\varphi_{y,x} + N^{T}_{xy}u_{y,y} + M^{T}_{xy}\varphi_{y,y} \end{bmatrix}_{,x} +$$
$$+ \begin{bmatrix} A_{12}u_{x,x} + A_{22}u_{y,y} + A_{26}(u_{x,y} + u_{y,x}) + B_{12}\varphi_{x,x} + B_{26}(\varphi_{x,y} + \varphi_{y,x}) + B_{22}\varphi_{y,y} + N_{yy}u_{y,y} + \\ + M_{yy}\varphi_{y,y} + N_{xy}u_{y,x} + M_{xy}\varphi_{y,x} + N_{xz}u_{z,y} + N^{T}_{yy}u_{y,y} + M^{T}_{yy}\varphi_{y,y} + N^{T}_{xy}u_{y,x} + M^{T}_{xy}\varphi_{y,x} \end{bmatrix}_{,y} + f_y = \rho h\ddot{u}_y; \tag{8}$$

$$\left[A_{55}(w_{,x} + \varphi_x) + A_{45}(w_{,y} + \varphi_y) + N_{xx}w_{,x} + N_{xy}w_{,y} + N^{T}_{xx}w_{,x} + N^{T}_{xy}w_{,y} \right]_{,x} +$$
$$+ \left[A_{45}(w_{,x} + \varphi_x) + A_{44}(w_{,y} + \varphi_y) + N_{xx}w_{,x} + N_{yy}w_{,y} + N^{T}_{xy}w_{,x} + N^{T}_{yy}w_{,y} \right]_{,y} + f_z = I_1\ddot{w}; \tag{9}$$

$$\begin{bmatrix} B_{11}u_{x,x} + B_{16}(u_{x,y} + u_{y,x}) + B_{12}\varphi_{y,y} + D_{11}\varphi_{x,x} + D_{16}(\varphi_{x,y} + \varphi_{y,x}) + D_{12}\varphi_{y,y} + M_{xx}u_{x,x} + \\ + M^{*}_{xx}\varphi_{x,x} + M_{xy}u_{x,y} + M^{*}_{xy}\varphi_{x,y} + M_{xz}u_{z,x} + M^{\mathrm{T}}_{xx}u_{x,x} + M^{T*}_{xx}\varphi_{x,x} + M^{T}_{xy}u_{x,y} + M^{T*}_{xy}\varphi_{x,y} \end{bmatrix}_{,x} +$$
$$+ \begin{bmatrix} B_{16}u_{x,x} + B_{26}u_{y,y} + B_{66}\left(u_{x,y} + u_{y,x}\right) + D_{16}\varphi_{x,x} + D_{66}\left(\varphi_{x,y} + \varphi_{y,x}\right) + D_{22}\varphi_{y,y} + M_{yy}u_{x,y} + \\ + M^{*}_{yy}\varphi_{x,y} + M_{xy}u_{x,x} + M^{*}_{xy}\varphi_{x,x} + M_{yz}u_{z,x} + M^{T}_{yy}u_{x,y} + M^{T*}_{yy}\varphi_{x,y} + M^{T}_{xy}u_{x,x} + M^{T*}_{xy}\varphi_{x,x} \end{bmatrix}_{,y} -$$
$$- A_{55}\left(w_{,x} + \varphi_x\right) - \left(N_{xz}u_{x,x} + M_{xz}\varphi_{x,x} + N_{zz}\varphi_x + N_{zy}u_{x,y} + M_{zy}\varphi_{x,y}\right) + m_x = I_3\ddot{\varphi}_x; \tag{10}$$

$$\begin{bmatrix} B_{16}u_{x,x} + B_{66}\left(u_{x,y} + u_{y,x}\right) + B_{26}u_{y,y} + D_{16}\varphi_{x,x} + D_{66}\left(\varphi_{x,y} + \varphi_{y,x}\right) + D_{26}\varphi_{y,y} + M_{xx}u_{y,x} + \\ + M^{*}_{xx}\varphi_{y,x} + M_{xy}u_{y,y} + M^{*}_{xy}\varphi_{y,y} + M_{xz}u_{z,y} + M^{\mathrm{T}}_{xx}u_{y,x} + M^{T*}_{xx}\varphi_{y,x} + M^{T}_{xy}u_{y,y} + M^{T*}_{xy}\varphi_{y,y} \end{bmatrix}_{,x} +$$
$$+ \begin{bmatrix} B_{12}u_{x,x} + B_{22}u_{y,y} + B_{26}\left(u_{x,y} + u_{y,x}\right) + D_{12}\varphi_{x,x} + D_{26}\left(\varphi_{x,y} + \varphi_{y,x}\right) + D_{22}\varphi_{y,y} + M_{yy}u_{y,y} + \\ + M^{*}_{yy}\varphi_{y,y} + M^{*}_{xy}u_{y,x} + M_{xy}\varphi_{y,x} + M_{xz}u_{z,y} + M^{T}_{yy}u_{y,y} + M^{T*}_{yy}\varphi_{y,y} + M^{T}_{xy}\varphi_{y,x} + M^{T*}_{xy}u_{y,x} \end{bmatrix}_{,y} -$$
$$- A_{44}\left(w_{,x} + \varphi_x\right) - \left(N_{xz}u_{y,x} + M_{xz}\varphi_{y,x} + N_{zz}\varphi_y + N_{zy}u_{y,y} + M_{zy}\varphi_{y,y}\right) + m_y = I_3\ddot{\varphi}_y; \tag{11}$$

where f_x, f_y, f_z, m_x and m_y are the lateral loadings. The arbitrary initial stresses are included in the stress resultants N_{ij}, M_{ij} and M^{*}_{IJ}. N^{T}_{ij}, M^{T}_{ij} and M^{T*}_{jj} are thermal stress resultants. The coefficients associated with material parameters, initial stress, thermal stress resultants and rotary inertia are defined as:

$$\left.\begin{aligned} (A_{ij}, B_{ij}, D_{ij}) &= \int C_{ij}(1, z, z^2)dz, \qquad (i, j = 1,2,4,5,6); \\ (N_{ij}, M_{ij}, M^{*}_{ij}) &= \int \sigma_{ij}(1, z, z^2)dz, \qquad (i, j = x, y, z); \\ (N^{T}_{ij}, M^{T}_{ij}, M^{T*}_{ij}) &= \int \alpha_{ij}C_{ij}\Delta T(1, z, z^2)dz, \qquad (i = x, y); \\ (I_1, I_3) &= \int \rho(z)(1, z^2)dz, \end{aligned}\right\} \tag{12}$$

where all the integrals are integrated through the thickness h of the plate from $-h/2$ to $h/2$.

3. Solution of the governing equations

Because the stability and vibration behaviors of the investigated initially stressed laminate composite plate are affected by various parameters, it would be difficult to present results for all cases. Thus, only the initially-stressed simply supported cross-ply laminate plate under the combined uniform and linear thermal loading is investigated. The lateral loads and body forces f_x, f_y, f_z, m_x and m_y are taken to be zero. The only nonzero initial stress is assumed to be (Fig. 1)

$$\sigma_{xx} = \sigma_n + 2z\sigma_m/h, \tag{13}$$

which comprises of the constant uniaxial stress σ_n and bending stress σ_m. Hence, the nonzero axial stress resultants are $N_{xx} = h\sigma_n$, $M_{xx} = Sh^2\sigma_n/6$ and $M^{*}_{xx} = h^3\sigma_n/12$. The factor $S = \sigma_m/\sigma_n$ denotes the ratio of a bending stress to a normal stress. For the cross-ply plate, the stiffness coefficients C_{16}, C_{26} and C_{45} will be equal to zero in Eqs. (6) and (7).

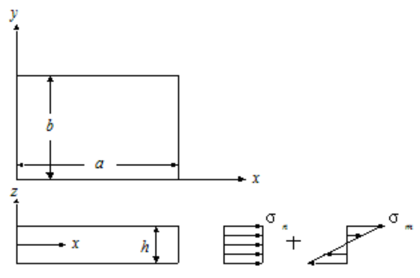

Fig. 1 A simply supported plate with initial stress

The combined uniform and linear temperature distribution is of the form as

$$\Delta T = T_o + 2zT_g , \tag{14}$$

where T_o is the uniform temperature rise and T_g is the temperature gradient. The nonzero thermal stress resultants are $N_{ij}^{T} = -\alpha_{ij} C_{ij} T_o h,$ $M_{ij}^{T} = -\alpha_{ij} C_{ij} T_g h^2 / 6$ and $M_{ij}^{T*} = -\alpha_{ij} C_{ij} T_o h^3 / 12$.

For a simply supported laminated plate, the boundary conditions along the x-constant edges are:

$$\left.\begin{aligned}
&u_y = 0;\ w = 0;\\
&A_{11}u_{x,x} + A_{16}\left(u_{x,y} + u_{y,x}\right) + A_{12}u_{y,y} + B_{11}\varphi_{x,x} +\\
&+B_{16}\left(\varphi_{x,y} + \varphi_{y,x}\right) + B_{12}\varphi_{y,y} + N_{xx}u_{x,x} + M_{xx}\varphi_{x,x} +\\
&+N_{xy}u_{x,y} + M_{xy}\varphi_{x,y} + N_{xz}u_{z,x} + N_{xx}^{T}u_{x,x} + M_{xx}^{T}\varphi_{x,x} +\\
&+N_{xy}^{T}u_{x,y} + M_{xy}^{T}\varphi_{x,y} = 0;
\end{aligned}\right\} \tag{15, a}$$

$$\begin{aligned}
&B_{11}u_{x,x} + B_{16}\left(u_{x,y} + u_{y,x}\right) + B_{12}\varphi_{y,y} + D_{11}\varphi_{x,x} +\\
&+D_{16}\left(\varphi_{x,y} + \varphi_{y,x}\right) + D_{12}\varphi_{y,y} + M_{xx}u_{x,x} + M_{xx}^{*}\varphi_{x,x} +\\
&+M_{xy}u_{x,y} + M_{xy}^{*}\varphi_{x,y} + M_{xz}u_{z,x} + M_{xx}^{T}u_{x,x} + M_{xx}^{T*}\varphi_{x,x} +\\
&+M_{xy}^{T}u_{x,y} + M_{xy}^{T*}\varphi_{x,y} = 0;
\end{aligned} \tag{15, b}$$

and along the y-constant edges are:

$$\left.\begin{aligned}
&u_x = 0;\ w = 0;\\
&A_{12}u_{x,x} + A_{26}\left(u_{x,y} + u_{y,x}\right) + A_{22}u_{y,y} + B_{12}\varphi_{x,x} +\\
&+B_{26}\left(\varphi_{x,y} + \varphi_{y,x}\right) + B_{22}\varphi_{y,y} + N_{yy}u_{y,y} + M_{yy}\varphi_{y,y} +\\
&+N_{xy}u_{y,x} + M_{xy}\varphi_{y,x} + N_{xz}u_{z,y} + N_{yy}^{T}u_{y,y} + M_{yy}^{T}\varphi_{y,y} +\\
&+N_{xy}^{T}u_{y,x} + M_{xy}^{T}\varphi_{y,x} = 0;
\end{aligned}\right\} \tag{16, a}$$

$$\begin{aligned}
&B_{12}u_{x,x} + B_{26}\left(u_{x,y} + u_{y,x}\right) + B_{22}u_{y,y} + D_{12}\varphi_{x,x} +\\
&+D_{26}\left(\varphi_{x,y} + \varphi_{y,x}\right) + D_{22}\varphi_{y,y} + M_{yy}u_{y,y} + M_{yy}^{*}\varphi_{y,y} +\\
&+M_{xy}^{*}u_{y,x} + M_{xy}\varphi_{y,x} + M_{xz}u_{z,y} + M_{yy}^{T}u_{y,y} + M_{yy}^{T*}\varphi_{y,y} +\\
&+M_{xy}^{T*}u_{y,x} + M_{xy}^{T}\varphi_{y,x} = 0.
\end{aligned} \tag{16, b}$$

For the simply supported plate, the displacement fields satisfying the geometric boundary conditions are given as follows:

$$\left.\begin{aligned}
u_x &= \sum\sum h\,U_{mn}\cos(m\pi x / a)\sin(n\pi y / b)e^{i\omega_{mn}t};\\
u_y &= \sum\sum h\,V_{mn}\sin(m\pi x / a)\cos(n\pi y / b)e^{i\omega_{mn}t};\\
w &= \sum\sum h\,W_{mn}\sin(m\pi x / a)\sin(n\pi y / b)e^{i\omega_{mn}t};\\
\varphi_x &= \sum\sum \Psi_{xmn}\cos(m\pi x / a)\sin(n\pi y / b)e^{i\omega_{mn}t};\\
\varphi_y &= \sum\sum \Psi_{ymn}\sin(m\pi x / a)\cos(n\pi y / b)e^{i\omega_{mn}t}.
\end{aligned}\right\} \tag{17}$$

All summations are summed up from m, $n = 1$ to ∞. For a buckling problem, $e^{i\omega_{mn}t}$ is neglected in Eq. (17). Substituting the initial stress (13), temperature distribution (14) and displacement fields (17) into the governing Eqs. (8)-(12), and collecting the coefficients for any fixed values of m and n leads to the following eigenvalue equation:

$$\left.\begin{aligned}
&\left([C] - \lambda[G]\right)\{\Delta\} = \{0\};\\
&\{\Delta\} = \left[U_{mn},\ V_{mn},\ W_{mn},\ \Psi_{xmn},\ \Psi_{ymn}\right]^T,
\end{aligned}\right\} \tag{18}$$

in which parameter λ refers to the corresponding frequency or buckling coefficient. For the vibration problems, the coefficients of the symmetric matrix $[C]$ and $[G]$ are expressed as follows:

$$C_{1,1} = -\left(A_{11} + N_{xx}\right)\alpha^2 - A_{66}\beta^2 + \alpha^2 N_{xx}^{T} + \beta^2 N_{yy}^{T};$$

$$C_{1,2} = -\left(A_{12} + A_{66}\right)\alpha\beta;$$

$$C_{1,4} = -\left(M_{xx}\alpha^2 + B_{66}\beta^2\right)/h - B_{11}\alpha^2/h + \alpha^2 M_{xx}^{T} + \beta^2 M_{yy}^{T};$$

$$C_{1,5} = -\left(B_{11} + B_{66}\right)\alpha\beta / h;$$

$$C_{2,2} = -\left(A_{66} + N_{xx}\right)\alpha^2 - A_{22}\beta^2 + \alpha^2 N_{xx}^{T} + \beta^2 N_{yy}^{T};$$

$$C_{2,4} = C_{1,5};$$

$$C_{2,5} = [-M_{xx}\alpha^2 + \left(B_{66}\alpha^2 + B_{22}\beta^2\right)]/h + \alpha^2 M_{xx}^{T} + \beta^2 M_{yy}^{T};$$

$$C_{3,4} = -A_{55}\alpha / h;$$

$$C_{3,3} = -\left(A_{55} + N_{xx}\right)\alpha^2 - A_{44}\beta^2 + \alpha^2 N_{xx}^{T} + \beta^2 N_{yy}^{T};$$

$$C_{3,5} = -A_{44}\beta / h;$$

$$C_{4,4} = -\left[\left(D_{11} + M_{xx}^{*}\right)\alpha_2 + D_{66}\beta^2 + A_{55}/h^2\right] + \alpha^2 M_{xx}^{T*} + \beta^2 M_{yy}^{T*};$$

$$C_{4,5} = -\left(D_{12} + D_{66}\right)\alpha\beta / h^2;$$

$$C_{5,5} = -\left[\left(D_{66} + M_{xx}^{*}\right)\alpha^2 + D_{22}\beta^2 + A_{44}\right]/h^2 + \alpha^2 M_{xx}^{T*} + \beta^2 M_{yy}^{T*};$$

$$G_{1,1} = G_{2,2} = G_{3,3} = -I_1;\quad G_{4,4} = G_{5,5} = -I_3 / h^2;$$

$$\alpha = m\pi / a;\quad \beta = n\pi / b.$$

For the thermal buckling problem, the coefficients of matrix $[C]$ are given by neglecting thermal induce stresses resultant terms in the stiffness matrix in Eq. (18) and the coefficients of matrix $[G]$ are:

$$G_{1,1} = G_{2,2} = G_{3,3} = \alpha^2 N_{xx}^{T} + \beta^2 N_{yy}^{T};$$

$$G_{1,4} = G_{2,5} = \alpha^2 M_{xx}^{T} + \beta^2 M_{yy}^{T};$$

$$G_{4,4} = G_{5,5} = \alpha^2 M_{xx}^{T*} + \beta^2 M_{yy}^{T*}.$$

As to the buckling load problems, the coefficients of the symmetric matrix $[C]$ are given by neglecting the initial stress resultant terms of the matrix $[C]$. The coefficients of the matrix $[G]$ are:

$$G_{1,1} = G_{2,2} = G_{3,3} = \alpha^2;$$

$$G_{1,4} = G_{2,5} = S\alpha^2 / 6h;$$

$$G_{4,4} = G_{5,5} = \alpha^2 / 12h^2.$$

4. Results and discussion

For verifying the present computer program, the close agreements between the present results and those in Matsunaga [23], Liu and Huang [24] for cross-ply plates as shown in Tables 1-2 demonstrate the accuracy and effectiveness of the present method. Parametric studies are carried out to examine the effects of various variables on the vibration and stability response of laminate plates under thermal environments. The following non- dimensional natural frequency ($\Omega = \omega b^2 \sqrt{\rho / h^2 E_y}$), buckling coefficient ($K_f = b^2 N_{xx} / E_y$) and thermal buckling coefficient ($T = \Delta T \alpha_{yy} 10^4$) are defined and used throughout the vibration and stability study. If the stress is tensile, then the buckling coefficient K_f is positive. There is no initial stress when $K_f = 0$ and $S = 0$. The critical buckling temperature is denoted by T_{cr}.

Table 1

Comparison of minimum critical temperatures of three-layer cross-ply laminated composite plates [0º/90º /0º]

a / *h*	Matsunaga [23]	Present
20/10	0.3334	0.3438
20/6	0.2465	0.2554
20/5	0.2184	0.2216
20/4	0.1763	0.1802
20/3	0.1294	0.1299
20/2	0.0746	0.0731
20	0.0230	0.0219

Table 2

Comparison of vibration frequencies of a $[0/90]_s$ square plate in thermal environment

T_o	Source	$\alpha_{xx} / \alpha_{yy}$			
		-0.05	0.1	0.2	0.3
-50	Liu [24]	15.149	15.247	15.320	15.394
	Present	15.165	15.277	15.351	15.425
0	Liu [24]	15.150	15.150	15.150	15.150
	Present	15.179	15.179	15.179	15.179
50	Liu [24]	15.164	15.052	14.978	14.902
	Present	15.193	15.081	15.006	14.930

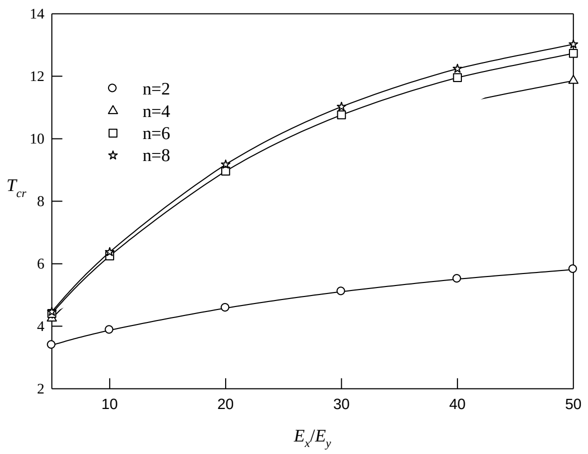

Fig. 2 Effect of modulus ratio on critical temperature parameter of plates ($a / b = 1$; $a / h = 10$; $K_f = 0$; $S = 0$; $T_g / T_o = 0$)

Fig. 2 presents the effect of modulus ratio on the thermal buckling temperature of plates with different stack layers. The critical temperature increases monotonically as the modulus ratio or/and layer number increase. The critical temperatures of eight-layer plates under different temperature gradient T_g are given in Table 3. The increasing temperature gradient reduces the thermal buckling temperature, and its influence on the critical temperature is less than the layer number. The effects of modulus ratio and span ratio on critical temperature parameters are shown in Fig. 3. The buckling temperature of plate with a smaller span ratio is always higher than that with a larger span ratio, especially for the plate with a higher modulus ratio. Thus, with a higher modulus, higher stacking number of layer, lower span ratio and lower gradient temperature, the laminated plate has a higher thermal buckling temperature.

Table 3

Effect of gradient temperature on critical temperature parameter of plates with different modulus ratio ($a / b = 1$; $a / h = 10$; $n = 8$; $K_f = 0$; $S = 0$)

T_g / T_o	E_x / E_y					
	5	10	20	30	40	50
0	4.4621	6.3727	9.1758	11.0261	2.2410	13.0226
5	4.4545	6.3634	9.1653	11.0161	12.2320	13.0147
10	4.4321	6.3355	9.1340	10.9861	12.2050	12.9911
20	4.3467	6.2285	9.0125	10.8688	12.0990	12.8981
40	4.0580	5.8585	8.5775	10.4380	11.7016	12.5443

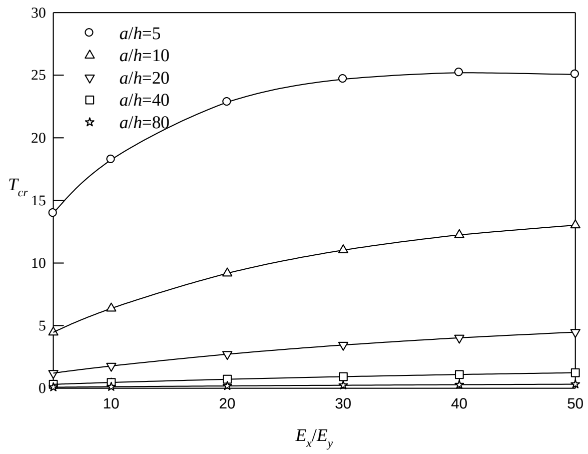

Fig. 3 Effect of span ratio and modulus ratio on critical temperature parameter under uniform temperature rise ($a / b = 1$; $n = 8$; $K_f = 0$; $S = 0$; $T_g / T_o = 0$)

The effect of buckling coefficient on the natural frequency of plates under various uniform temperature rises can be observed in Fig. 4. The natural frequency decreases with the increasing initial compressive stress and temperature rise. The buckling load can be obtained when the natural frequency approaches zero. Meanwhile, the plate under a lower temperature rise has a greater buckling coefficient. Fig. 5 shows the effect of modulus ratio on the natural frequency of laminated plates. The laminate plate with higher modulus ratio has a larger vibration frequency and higher buckling load.

The buckling load and natural frequency of laminate plate with different layer numbers and modulus ratios under uniform temperature rise are shown in Tables 4-5. The plate with larger stack layer number or/and higher modulus ratio has a higher critical buckling load and natural frequency. It can also be observed that the buckling load and natural frequency decreases steadily with the increasing uniform temperature rise. Thus, the two-layered plate with smallest modulus ratio and under higher temperature rise will possess the smallest buckling load and natural frequency.

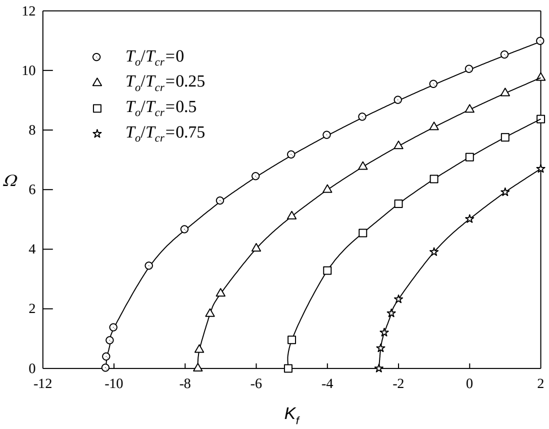

Fig. 4 The vibration frequency versus the buckling coefficient under uniform temperature rise ($a/b = 1$; $a/h = 10$; $n = 8$; $S = 0$; $E_x/E_y = 10$; $T_g/T_o = 0$)

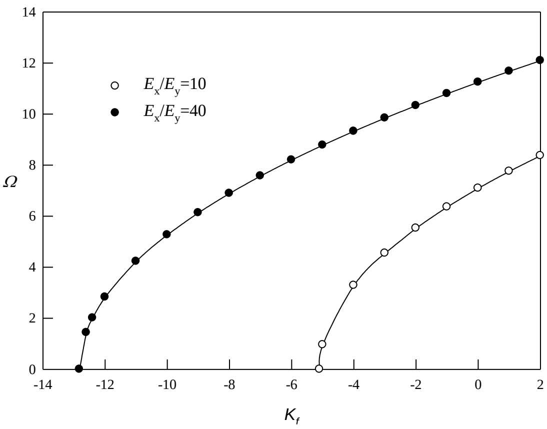

Fig. 5 The vibration frequency versus the buckling coefficient under various modulus ratio ($a/b = 1$; $a/h = 10$; $n = 8$; $S = 0$; $T_o/T_{cr} = 0.5$; $T_g/T_o = 0$)

Table 4

Effect of layer number and modulus ratio on critical buckling coefficient of plates under various uniform temperature rise ($a/b = 1$, $a/h = 10$, $S = 0$, $T_g/T_o = 0$)

n	T_o/T_{cr}	E_x/E_y					
		5	10	20	30	40	50
2	0	5.1392	6.1869	8.0733	9.8419	11.5230	13.1266
	0.25	3.8544	4.6402	6.0549	7.3815	8.6422	9.8450
	0.5	2.5696	3.0935	4.0366	4.9210	5.7615	6.5633
	0.75	1.2848	1.5468	2.0183	2.4606	2.8808	3.2817
4	0	6.4432	9.4152	14.6917	19.2579	23.2504	26.7707
	0.25	4.8324	7.0614	11.0188	14.4434	17.4378	20.0780
	0.5	3.2216	4.7076	7.3459	9.6290	11.6252	13.3853
	0.75	1.6108	2.3538	3.6729	4.8145	5.8126	6.6926
6	0	6.6812	9.9874	15.7990	20.7515	25.0224	28.7425
	0.25	5.0109	7.4905	11.8492	15.5636	18.7668	21.5569
	0.5	3.3406	4.9937	7.8995	10.3757	12.5112	14.3712
	0.75	1.6703	2.4968	3.9497	5.1878	6.2556	7.1855
8	0	6.7642	10.1858	16.1786	21.2585	25.6186	29.4008
	0.25	5.0731	7.6394	12.1340	15.9438	19.2139	22.0506
	0.5	3.3821	5.0929	8.0893	10.6292	12.8092	14.7003
	0.75	1.6910	2.5465	4.0446	5.3146	6.4046	7.3501

Table 5

Effect of layer number and modulus ratio on the natural frequency of plates under various uniform temperature rise (a/b=1; a/h=10; $K_f = 0$; $S = 0$; $T_g/T_o = 0$)

n	T_o/T_{cr}	E_x/E_y					
		5	10	20	30	40	50
2	0	7.1219	7.8142	8.9264	9.8558	10.6643	11.3822
	0.25	6.1678	6.7674	7.7305	8.5354	9.2356	9.8573
	0.5	5.0360	5.5256	6.3119	6.9692	7.5409	8.0485
	0.75	3.5610	3.9073	4.4632	4.9280	5.3323	5.6912
4	0	7.9745	9.6397	12.0417	13.7866	15.1484	16.2548
	0.25	6.9061	8.3483	10.4284	11.9395	13.1189	14.0771
	0.5	5.6388	6.8164	8.5148	9.7486	10.7116	11.4939
	0.75	3.9873	4.8200	6.0209	6.8934	7.5743	8.1274
6	0	8.1204	9.9284	12.4872	14.3112	15.7151	16.8428
	0.25	7.0325	8.5982	10.8143	12.3939	13.6097	14.5863
	0.5	5.7420	7.0204	8.8298	10.1196	11.1123	11.9097
	0.75	4.0602	4.9642	6.2436	7.1556	7.8576	8.4214
8	0	8.1707	10.0265	12.6364	14.4850	15.9012	17.0346
	0.25	7.0760	8.6832	10.9434	12.5444	13.7708	14.7524
	0.5	5.7776	7.0899	8.9353	10.2424	11.2438	12.0453
	0.75	4.0853	5.0133	6.3182	7.2425	7.9506	8.5173

The effect of different temperature gradient on buckling load and natural frequency of plates is presented in Tables 6-7. When the linear gradient temperature increases the buckling load and natural frequency coefficient slightly decrease. The laminated plate with lower modulus ratio and under higher uniform temperature and temperature gradient has a smaller critical buckling and vibration frequency. The influence of temperature gradient on the buckling load and natural frequency for laminate plates is less apparent than that of uniform temperature.

Table 6

Effect of linear temperature rise on the critical buckling coefficient ($a/b = 1$; $a/h = 10$; $n = 8$; $S = 0$)

E_x/E_y	T_o/T_{cr}	T_g/T_o				
		0	5	10	20	40
10	0	10.1858	10.1858	10.1858	10.1858	10.1858
	0.25	7.6394	7.6385	7.6356	7.6243	7.5792
	0.5	5.0929	5.0892	5.0779	5.0327	4.8515
	0.75	2.5465	2.5380	2.5126	2.4109	2.0013
40	0	25.6186	25.6186	25.6186	25.6186	25.6186
	0.25	19.2139	19.2127	19.2092	19.1950	19.1381
	0.5	12.8092	12.8045	12.7903	12.7335	12.5049
	0.75	6.4046	6.3939	6.3620	6.2338	5.7151

Table 7

Effect of linear temperature rise on the natural frequency ($a/b = 1$; $a/h = 10$; $n = 8$; $K_f = 0$; $S = 0$)

E_x/E_y	T_o/T_{cr}	T_g/T_o				
		0	5	10	20	40
10	0	10.0265	10.0265	10.0265	10.0265	10.0265
	0.25	8.6832	8.6827	8.6811	8.6747	8.6490
	0.5	7.0899	7.0872	7.0794	7.0478	6.9198
	0.75	5.0133	5.0050	4.9799	4.8780	4.4444
40	0	15.9012	15.9012	15.9012	15.9012	15.9012
	0.25	13.7708	13.7704	13.7691	13.7640	13.7436
	0.5	11.2438	11.2418	11.2355	11.2105	11.1095
	0.75	7.9506	7.9440	7.9241	7.8439	7.5105

Variations of critical temperature and natural frequency with the linear temperature change for initially stressed laminate plates are shown in Tables 8-9. It is evident that the compressive initial stress ($K_f < 0$) produces a decreasing effect on the critical temperature and natural frequency, and the tensile initial stress has a reverse effect. Likewise, the initially stressed laminate plate with higher modulus ratio has a larger critical temperature than the one with lower modulus ratio.

Table 8
Effect of initial stresses on the critical temperature of plates under linear temperature rise
($a / b = 1$; $a / h = 10$; $n = 8$; $S = 0$)

E_x / E_y	K_f	T_g / T_o				
		0	5	10	20	40
10	4	8.8753	8.8571	8.8034	8.5996	7.9259
	0	6.3727	6.3634	6.3355	6.2285	5.8585
	-4	3.8702	3.8667	3.8564	3.8161	3.6697
40	4	14.1523	14.1402	14.1042	13.9628	13.4365
	0	12.2410	12.2320	12.2050	12.0990	11.7016
	-4	10.3297	10.3233	10.3041	10.2284	9.9425

Table 9
Effect of initial stresses on the natural frequency of plates under linear temperature rise
($a / b = 1$, $a / h = 10$, $n = 8$, $S = 0$, $T_o / T_{cr} = 0.5$)

E_x / E_y	K_f	T_g / T_o				
		0	5	10	20	40
10	4	9.4734	9.4714	9.4655	9.4419	9.3468
	0	7.0899	7.0872	7.0794	7.0478	6.9198
	-4	3.2844	3.2788	3.2617	3.1927	2.8991
40	4	12.8803	12.8785	12.8731	12.8512	12.7632
	0	11.2438	11.2418	11.2355	11.2105	11.1095
	-4	9.3244	9.3219	9.3144	9.2842	9.1620

The effect of bending stress ratio on the critical buckling coefficient for initially stressed plates under uniform temperature is given in Table 10. As can be seen, the increasing bending stress ratio decreases the critical buckling load. The influence of bending stress on the natural frequency of initially stressed plates under a fixed uniform temperature is presented in Table 11. The vibration frequency decreases with the increase in bending stress. However, the natural frequency is not affected by the increasing bending stress when the plate is subject to the pure bending stress only. The lowest natural frequency can be observed for the plate with a lower modulus ratio and under a higher bending stress.

Table 10
Effect of bending ratios on critical buckling coefficient of plates under different uniform temperature rise
($a / b = 1$, $a / h = 10$, $n = 8$, $T_g / T_o = 0$)

E_x / E_y	T_o / T_{cr}	S					
		0	10	20	30	40	50
10	0	10.1858	10.1263	9.9553	9.6925	9.3639	8.9952
	0.25	7.6394	7.6058	7.5084	7.3560	7.1608	6.9358
	0.5	5.0929	5.0780	5.0341	4.9641	4.8721	4.7627
	0.75	2.5465	2.5427	2.5316	2.5135	2.4890	2.4587
40	0	25.6186	25.5432	25.3213	24.9643	24.4897	23.9190
	0.25	19.2139	19.1715	19.0461	18.8430	18.5704	18.2383
	0.5	12.8092	12.7904	12.7343	12.6429	12.5186	12.3650
	0.75	6.4046	6.3998	6.3857	6.3625	6.3305	6.2903

Table 11
Effect of initial bending stress on the natural frequency of plates under uniform temperature rise
($a / b = 1$, $a / h = 10$, $n = 8$, $T_o / T_{cr} = 0.5$, $T_g / T_o = 0$)

E_x / E_y	K_f	S					
		0	10	20	30	40	50
10	4	9.4734	9.4685	9.4540	9.4297	9.3956	9.3514
	0	7.0899	7.0899	7.0899	7.0899	7.0899	7.0899
	-4	3.2844	3.2704	3.2281	3.1563	3.0528	2.9140
40	4	12.8803	12.8796	12.8775	12.8739	12.8690	12.8626
	0	11.2438	11.2438	11.2438	11.2438	11.2438	11.2438
	-4	9.3244	9.3235	9.3205	9.3156	9.3088	9.3000

5. Conclusions

The vibration and buckling behaviors of initially stressed and thermally stressed laminate plates have been described and discussed in this paper. The results demonstrate the influence of the modulus ratio, number of layer, initial stress and thermal stress on the vibration and buckling behaviors of laminate plates. Following the above discussions, the preliminary results are summarized as follows:

1. The modulus ratio, number of layer and uniform temperature has an apparent influence on natural frequency and buckling load. They are slightly affected by the temperature gradient rise and bending stress.
2. With the increasing modulus ratio and number of layer, the critical temperature, buckling load and natural frequency increase. The uniform temperature has a reverse effect.
3. The compressive stress significantly reduces the natural frequency and critical temperature but the tensile stress produces an opposite effect.

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THERMALLY INDUCED STABILITY AND VIBRATION OF INITIALLY STRESSED LAMINATED COMPOSITE PLATES

S u m m a r y

In this paper, the thermal effect on the buckling and vibration of laminated composite plates with an arbitrary initial stress is presented. The governing equations including the transverse shear deformation effects are established using the variation method. The initial stress is taken to be a combination of pure bending stress and a uniform normal stress in the example problems. Temperature distribution in the laminate plate is assumed to be combined uniform and linear temperature change in the transverse direction. The effects of various parameters on the thermal induced vibration and stability of laminated composite plates are studied. It is found that the initial stress, rise temperature and elastic modulus cause drastic changes in the thermal vibration and buckling behavior of laminated composite plates.

Keywords: thermal effect, buckling, vibration, laminated composite plate, initial stress.

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