Critical rigidity of stiffener for increasing shear stability of rectangular plate

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1. Introduction

Efficiency of application of the plate girder with slender web (GSW) is determining with the relatively small thickness of web compare with usual welded beams that give sufficient reducing expense of metal for web and more rational using of it[1, 2].

Experience of using such beams show, that loss of local stability of web take place near supports with typical view of inclined half waves, indicating that reason of buckling is shear deformation (Fig. 1).

Increase loading capacity of web is possible by installation of vertical or inclined stiffeners located near the supports [3-17]. If width of panel between transversal stiffeners is equal to height of web, than we consider stability of square plate otherwise it is necessary to deal with rectangular plate. Namely these two variants are considering in the article to determine efficient sizes of stiffeners for increasing of critical load of the girder web.
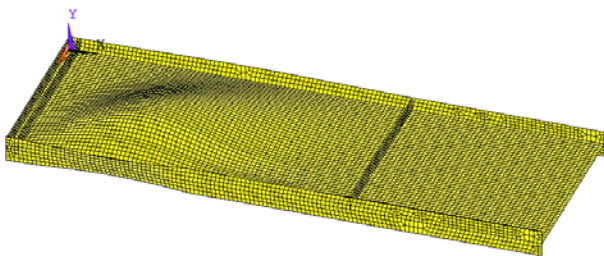

Fig. 1 Form of buckling of GSW near support from the shear deformation

2. Analysis of plate stability without stiffener

For analysis of efficiency of stiffeners it was elaborated the computer program for creation of model of rectangular plate, allowing uniform loading of plate on contour with shear forces. The calculations were performed by the finite element method using complex AN-SYS. Using any program it is necessary to be convenient in correctness of its work, i.e. in obtaining reliable results. With this aim it was initially calculated stress state of the square and rectangular plates and calculations its shear buckling under two kinds of boundary conditions: clamped one (Fig. 2, a) and simply supported (Fig. 2, b).

For prevention of the plate displacement as rigid whole under calculation by FEM it were used boundary conditions of absence of lineal (in directions of axis *OX* and *OY*) and angular (*rotz*) displacements of the plate center. In that case the initial stress state under pure shear of plate is uniform that indicate on correctness of application of boundary conditions and loads.
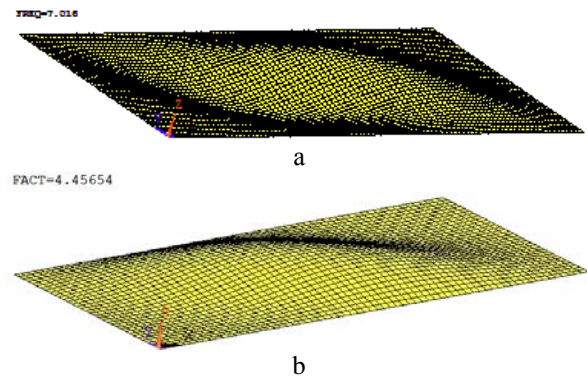

Fig. 2 Forms of buckling of (a) clamped one and (b) simply supported rectangular plate under shear

Estimate reliability of FEM results is possible, using theoretical values for critical shear stresses. In common case dependence for τ_{cr} takes view:

$$\tau_{cr} = K \frac{\pi^2 E t^2}{12(1-\mu^2)b^2},\qquad(1)$$

here K is numerical coefficient, depending on relation of plate sides a/b and boundary conditions on contour.

For simply supported plate it can be written as [18]:

$$K = 5.34 + 4(b/a)^2,\qquad(2)$$

here b is short side of plate. For rectangular plate with dimensions $200\times100\times0.19$ mm $a = 2b$; $K = 6.34$ and in accordance with Eq. (1) and Eq. (2) critical stress is determined as:

$$\tau_{cr} = 6.34 \frac{\pi^2 2.1 \cdot 10^5 \cdot 0.19^2}{12(1-0.3^2)100^2} = 4.34 \, \text{MPa}.\qquad(3)$$

Calculation with FEM gives $\tau_{cr} = 4.45$ MPa (Fig. 2, b), i.e. divergence is about 2.5%. The reason of such big divergence is incorrectness of factor K determined on Eq. (2). As it is marked in handbook [3], accuracy value $K = 6.47$, substitution of which in Eq. (3) gives value $\tau_{cr} = 4.43$ MPa, practically coinciding with FEM calculation.

For clamped plate expression for K can be written as [18]:

$$K = 8.98 + 5.6(b/a)^2 . \qquad (4)$$

For rectangular plate $(a = 2b)$ it gives $K = 10.38$ and in accordance with Eq. (1) and Eq. (4) value of τ_{cr} is:

$$\tau_{cr} = 10.38 \frac{\pi^2 2.1 \cdot 10^5 \cdot 0.19^2}{12(1 - 0.3^2)100^2} = 7.10 \, \text{MPa} . \qquad (5)$$

Divergence of theoretical value of critical stress with calculated by FEM τ_{cr}=7.01 MPa (Fig. 2, a) does not exceed 1.3%.

As it can be seen from Table, obtained results confirm high reliability of program and now it is possible to transfer to determination of influence of stiffener on the plate buckling.

Table
Critical stresses (MPa) of rectangular plates under shear

Type of plate	Rectangular		Square	
Dimensions	200×100×0.19mm		100×100×0.19mm	
Boundary conditions	Simple	Clamped	Simple	Clamped
τ_{cr}^{FEM}	4.45	7.01	6.38	10.02
τ_{cr}^{Theor}	4.43	7.1	6.39	9.98
Divergence, %	0.5	1.3	0.2	0.4

3. Analysis of plate stability with stiffener

Under investigation of effect of stiffener it is necessary to get response at three questions: how to install stiffener – vertically or inclined; what are to be dimensions of stiffener; what is the critical rigidity of stiffener? As a critical rigidity of stiffener it can be understand value EI_{cr} with minimum dimensions h_r and t_r, assuring linearity of stiffener during the plate buckling.

Answer the first question is almost known. Installation of stiffener is most effective when it cross buckled surface normally to its location. In case of simply supported rectangular plate (Fig. 3) location of stiffener must be inclined. Calculations with FEM (Fig. 3, a and b) confirm that inclined stiffener under the same parameters of plate is more effective than transversal one.

For answer the second question put attention to SN&R. Height h_r of one side stiffener in accordance with p.8.5.9 [19], is recommended to adopt no less than:

$$h_r = h_w / 24 + 40 \, \text{mm} , \qquad (6)$$

here h_w is height of web.

Thickness of stiffener must be not less than:

$$t_r = 2h_r \sqrt{R_y / E}. \qquad (7)$$

It is necessary to mark that under choice of stiffener dimensions the Eq. (6) is primary and Eq. (7) – secondary, i.e. the bigger height of stiffener the bigger is it thickness. It is clear, that choice of height of stiffener from Eq. (6) does not correspond to parameters of plate: for

width of plate 100 mm stiffener according to Eq. (6) will be bigger 44 mm. Of course for plate with dimensions 200×100 mm such recommendation is not suitable. It is obvious nonsense, leaking from presence in Eq. (6) the term not depending from parameters of plate. That is why for any plate the height of stiffener according to Eq. (6) will not be less of 40 mm.
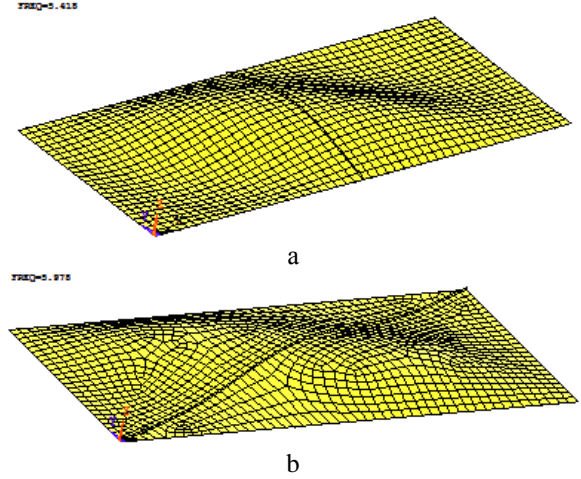

a

b

Fig. 3 Critical stresses of simply supported rectangular plate with vertical (a) and inclined (b) stiffener $h \times t = 1 \times 0.19$ mm

For webs of beams of big sizes (for example, h_w= = 1000 mm) such recommendation is quite suitable, only it is necessary to restrict field of application the Eq. (6). It is clear that and second Eq. (7) lead to incorrect result. In order to obtain more a equate relation it will be considered stability of clamped and simply supported plate with different stiffeners using FEM. It allows understanding what real sizes of stiffener are to be.

In Eurocode EN 1993-1-5[20] demands to transverse stiffeners, creating rigid support for web, are one of two conditions:

$$I_{st} \geq 1.5 h_w^3 t_w^3 / a^2 , \text{ if } a / h_w \leq \sqrt{2} ,$$

or:

$$I_{st} \geq 0.75 h_w t_w^3 , \text{ if } a / h_w \geq \sqrt{2} , \qquad (8)$$

here a is width of BFW panel between transverse stiffeners; h_w is height of web; t_w is thickness of web.

Plate with sides $a / b = 2$ is suit to Eq. (8). Substitution in Eq. (8) of moment inertia of stiffener, allow get condition for demanded height of stiffener:

$$h_r \geq 2.08 t_w \sqrt[3]{h_w / t_r} . \qquad (9)$$

If thickness of stiffener is equal to thickness of plate $t_r = t_w$ then from Eq. (9) we obtain:

$$h_r \geq 2.08 t_w \sqrt[3]{\lambda_w} , \qquad (10)$$

here $\lambda_w = h_w / t_w$ is the web slenderness.

If demand Eq. (10) is not satisfied, then stiffener is flexible. Its real rigidity can be determined by the finite element calculation of the plate stability.

4. Critical rigidity of stiffener

Determine optimal sizes of stiffener are possible if take into account results obtained in work [21]. For rectangular plate such sizes can be find from relation of rigidity of stiffener and plate:

$$EI / Dl_r = 4.5 , \qquad (11)$$

here $I = t_r h_r^3 / 12$ is moment of inertia of stiffener; $D = Et_w^3 / 12(1 - \mu^2)$ is cylindrical rigidity of plate; l_r is length of stiffener. After substitution of expressions for I and D in Eq. (11) give:

$$t_r h_r^3 (1 - \mu^2) / l_r t_w^3 \geq 15 . \qquad (12)$$

If thickness of stiffener t_r is adopted distinct from thick-ness of plate t_w, then from Eq. (12) with $\mu = 0.3$ is possible to express critical height of stiffener as:

$$h_r = 1.7 t_w \sqrt[3]{l_r / t_r} . \qquad (13)$$

So far as most rational location of inclined stiffeners 45°, the length of rib need to adopt equal:

$$l_r = h_w \sqrt{2} . \qquad (14)$$

Then substitution Eq. (14) in Eq. (13) give:

$$h_r = 1.91 t_w \sqrt[3]{h_w / t_r} . \qquad (15)$$

In case of equal thicknesses of plate and rib $t_r = t_w$ instead of Eq. (15) we get:

$$h_r = 1.91 t_w \sqrt[3]{\lambda_w} . \qquad (16)$$

Check up Eq. (16) for plate with sizes 200×100×0.19 mm. Substitution of $h_w = 100$ mm and $t_w = 0.19$ mm in Eq. (16) give:

$$h_r = 1.91 \cdot 0.19 \sqrt[3]{100 / 0.19} = 2.93 \, \text{mm} ,$$

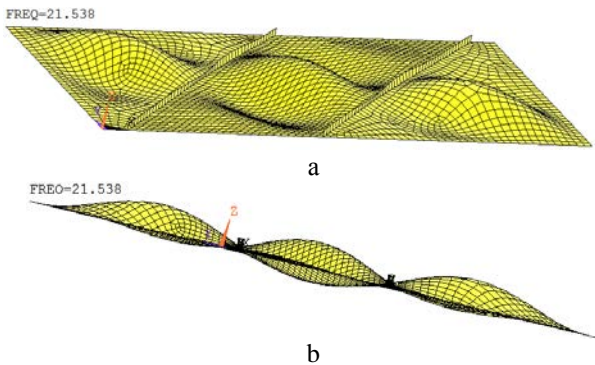that is in good correlation with result represented in Fig.4.

a

b

Fig. 4 Critical stress of clamped rectangular plate 200×100×0.19 mm with two stiffeners 2.93×0.19 mm (a, b – different view)

5. Installation of two stiffeners

Performed calculations by FEM of clamped plate with two inclined ribs show that on critical load big influence have not only sizes of ribs but their location also (Fig. 5). All ribs of the same size 2.5×0.19 mm, have inclination 45°, but different distance from angle of plate: x=5, 20 and 40 mm.
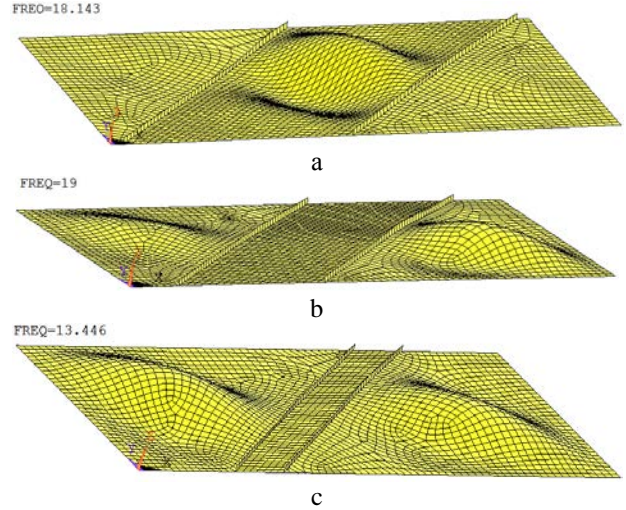

a

b

c

Fig. 5 Critical stresses of clamped plate 200×100×0.19 mm with different location of ribs: a - x = 5 mm; b - x = 20 mm; c - x = 40 mm

Analysis show (Fig. 6), that transfer of ribs from angle to middle of the plate side initially increase critical load and then reduce it. Practically in all cases optimal location of ribs correspond to distance x=0.1b (b is width of plate).
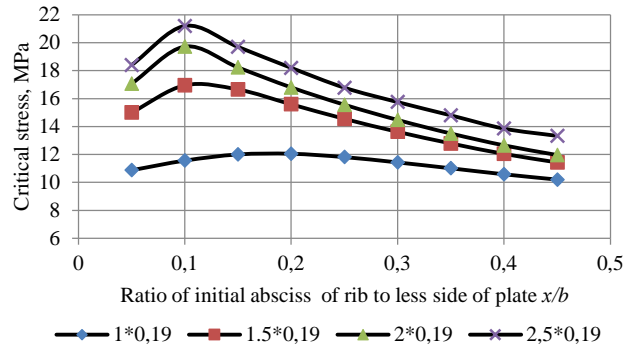

Fig. 6 Dependence of critical stress τ_{cr} on location of two inclined ribs in clamped plate 200×100×0.19 mm

It is possible to appreciate efficiency of stiffeners from Fig. 7, which is constructed in dimensionless coordinates: along axis OX–ratio of height of rib h to the plate thickness t, and along axis OY– ratio of critical stress of stiffened plate to critical stress of plane plate. Such approach allows understand growth of stability due to stiffeners and determine critical height of rib. Fig. 7 show that increasing of height of ribs from $13 t_w$ to $23 t_w$ lead to growth τ_{cr} only at 5% (3,19/3,05=1.05), i.e. $h_{cr} \approx 15.4 t_w$.

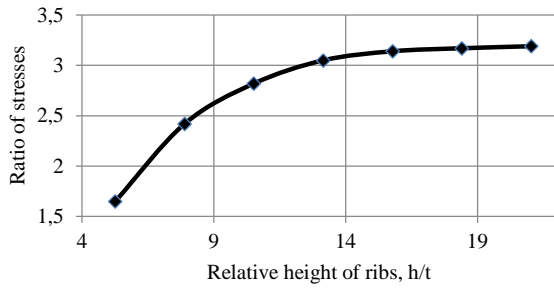To increase height of rib bigger of h_{cr} is not need.

Fig. 7 Ratio of critical stresses of clamped rectangular plate 200×100×0.19 mm with two ribs to 7.01 MPa

After determination of critical rigidity of stiffener and more exactly its height, is important to appreciate value of critical stress of the stiffened plate. Using obtained by FEM results it is possible to note that under different thicknesses and consequently different slenderness of plate in diapason $167 \leq a / t_w \leq 500$ relation between critical stresses of clamped plate without stiffener τ_{cr} and critical stresses of stiffened plate τ_{cr}^{stif} there is very stable tie. It can be determined as:

$$\tau_{cr}^{stif} = (3 \div 3.2) \tau_{cr}. \tag{17}$$

It can be seen from curve shown in Fig. 7.

Beside of clamped plate it was investigated buckling of simply supported rectangular plate with two stiffeners (Fig. 8).
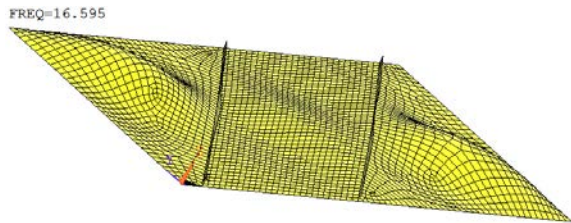

Fig. 8 Buckling of simply supported rectangular plate with two stiffeners 2.93×0.19mm

Compare results of FEM performed for plates with different boundary conditions (Fig. 4, a and Fig. 8), it is possible to note that effect of stiffening for simply supported plate is bigger than for clamped one: for simply supported plate stiffener increase stability almost 3.7 times, but for clamped plate only 3.1 times. But absolute value τ_{cr}^{stif} for plates of the same dimensions for clamped plates is bigger than for simply supported one. Tie between critical stresses of stiffened plates with different boundary conditions can be approximately expressed as:

$$\tau_{cr}^{clamp} = \tau_{cr}^{sipm} \cdot \sqrt{k_{clamp} / k_{sipm}}, \tag{18}$$

here k_{clamp} and k_{sipm} are coefficients in Eqs. (2) and (4) for critical stresses of plates with corresponding boundary conditions, i.e. $k_{clamp} \approx 10.38$, and $k_{simple} \approx 6.34$. For these values $\alpha = \sqrt{10.38 / 6.34} = 1.28$ (compare relation τ_{cr} in Fig. 4 and Fig. 8). For stiffened plates with sizes 200×100×0.19 mm relation of values $\tau_{cr}^{clamp} = 21.54$ MPa

and $\tau_{cr}^{simp} = 16.6$ MPa is equal 1.30. Difference between magnitudes 1.28 and 1.30 is about 1.4%.

Beside of clamped plate with inclined ribs it was analyzed variants with two transversal ribs. Initially these ribs were installed at distance 0.25a and 0.75a from edge of plate, and later distance between them was variable. Results of calculations by FEM show the transverse ribs of the same dimensions increase stability in less degree than inclined ones. Under ribs of sizes 2.93×0.19 mm critical stresses were equal 9.72 MPa (Fig. 9, a).
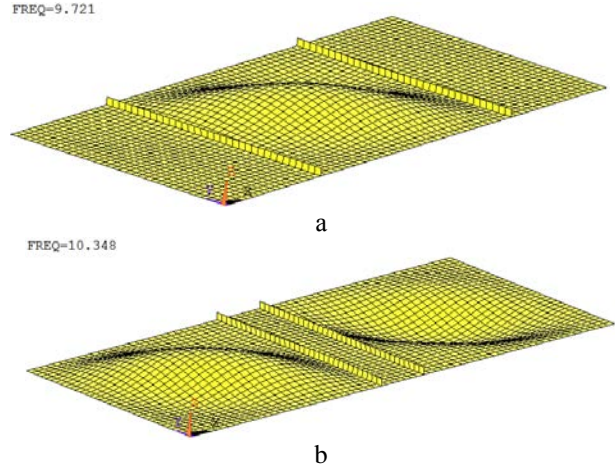

Fig. 9 Critical stress via distance between vertical stiffeners 2.93×0.19 mm: a - x=0.5a; b - x=0.1a

It is twice less then under inclined ribs. Vertical ribs under buckling remain straight, i.e. its critical rigidity is already reached. Reducing distance between ribs up to 0.3a make possible to increase critical stresses almost 1.5 times, but further decreasing of distance between ribs bring down value of critical stress τ_{cr} again (Fig. 9, b).

After numerical investigation we consider now results of experiment performed by authors in laboratory POLEKS of Baltic State Academy for Fishing Fleet.

6. Experimental investigation of critical stresses

Although the calculations with FEM give true information about stability of the plate model, authors performed experimental investigation of the shear buckling of rectangular plate with and without stiffeners, because it is known "the practice is criteria of truth". In experiment it was checked up such factors, which is difficult to take into account in theoretical calculations: for example, dispersion of thicknesses on field of plate, existence of initial deflections, uneven distribution of loads along the edges of plate due to different tightening of bolts and so on.

Experiment was performed on specially designed installation (Fig. 10), formed of vertical pillar 1 fixed on massive basement. On upper end of pillar there is beam 2 with diverse arms, to one of ends of which is fixed tested specimen located in pinned quadrangular 4, joined to dynamometer 5 for measuring of applied load. To other end of beam is joined the tightening screw 3, providing loading of specimen.

It was overall tested three clamped plates with sizes 200×100×0.19 mm without ribs and three stiffened
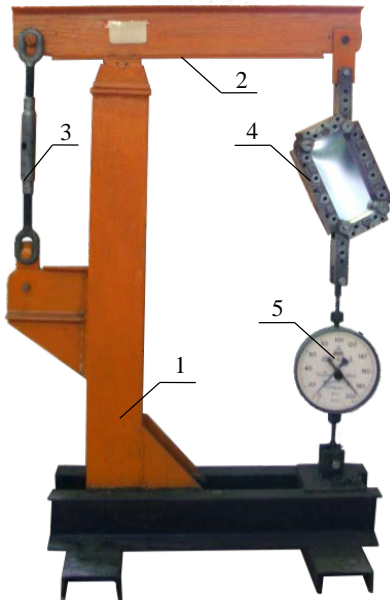
Fig.10 Installation for the shear plate testing

plates with ribs 6×0.19 mm. Material of specimen was steel C350. Binding of plate in quadrangular with help of bolts ensure their clamping in contour, and axial loading of quadrangular along diagonal, as it shown on scheme (Fig. 11, b) lead to shear of plate. As a result of performed tests of clamped plates with two stiffeners it was obtained an average value of critical load:

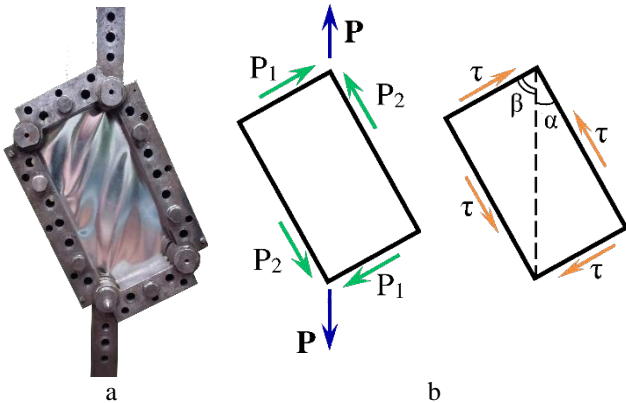$$P_{cr}^{exp} = 1.1\,\mathrm{kN} \,. \tag{19}$$

Fig. 11 Plate-specimen in pinned quadrangular (a), and scheme of distribution of forces along the edges of plate (b)

Recalculate value of critical load P_{cr} into shear stresses is rather easy. Taking in a view scheme of loading (Fig. 11, b) it can be written:

$$P_{cr} = P_2 \cos\alpha + P_1 \sin\alpha \,. \tag{20}$$

Projection of forces at axis OX gives:

$$P_2 \sin\alpha - P_1 \cos\alpha = 0 \,. \tag{21}$$

For rectangular plate with $a = 2b$ value $tg\,\alpha = 0.5$ and $P_1 = \tau_{cr} at_w$ then from Eq. (21) we get $0.5 P_2 = P_1$.

Substitution in Eq. (20) obtained values of forces

P_1 and P_2 allow to express τ_{cr} via P_{cr} in form:

$$\tau_{cr} = \frac{P_{cr}}{(\cos\alpha + 0.5\sin\alpha) at_w} \,. \tag{22}$$

In correspondence with Eq. (19) and Eq. (22) it is possible for plate 200×100×0.19 mm without ribs get:

$$\tau_{cr}^{exp} = \frac{1100}{(0.8962 + 0.5\cdot 0.4481)\,200\cdot 0.19} = 25.8\,\mathrm{MPa} \,.$$

Comparison of $\tau_{cr}^{exp} = 25.8\,\mathrm{MPa}$ obtained from test and $\tau_{cr}^{FEM} = 21.9\,\mathrm{MPa}$ received from calculations by FEM (Fig. 12) indicates that divergence is above 15%. Such big difference can be explained with form and sizes of stiffeners, which have ∟-form and joined to plate with pointed welding. Small side of the corner rib was 4 mm and its height was 6 mm. Thickness of rib was equal to plate thickness t_w. All that increase value of P_{cr} significantly.

Beside of stiffened plates with two inclined ribs there were also tested plates without any ribs (Fig. 11, a). In this case critical loads measured in experiment $P_{cr}^{exp} = 0.302\,\mathrm{kN}$ and calculated theoretically via Eq. (20) $P_{cr}^{theor} = 0.298\,\mathrm{kN}$ practically coincided. As it can be seen the divergence is near 1.3%.
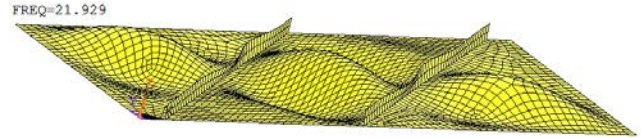

Fig. 12 Critical stress of clamped rectangular plate 200×100×0.19 mm with two stiffeners 6×0.19 mm

Completed investigations show effectiveness of the plate stiffening with inclined ribs which significantly increases stability of plate under shear. Results of these investigations can be used in estimation of load capacity of beams with slender web.

7. Conclusions

1. The inclined stiffeners are more efficient in arising of stability of rectangular plate than transverse one.

2. Existing recommendations of SN&R on reinforcement of plates are needed in correction.

3. Optimal sizes of rib for stiffened plate were obtained by authors in form (16) that is near to demands of Eurocode 3(9). All this is confirmed with calculations by FEM.

4. Experiments on finding of critical load performed on models of clamped rectangular plates 200×100×0.19 mm without ribs show satisfactory correspondence to theoretical values.

5. Installation of two inclined stiffeners can to arise the clamped plate stability almost 3.1 times.

6. The increasing of the stiffener rigidity above critical is not lead to significant growth of τ_{cr}, so as it remains almost straight during of plate buckling.

7. Field of application of Eq. (11) corresponds to values of the plate slenderness $\lambda_w > 120$.

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CRITICAL RIGIDITY OF STIFFENER FOR INCREASING SHEAR STABILITY OF RECTANGULAR PLATE

S u m m a r y

In plate girder with slender web the big transverse forces lead to web buckling produced with shear. One of the ways to increase stability of web near supports is to install stiffeners. In work the task of finding of critical stresses of clamped rectangular plate with installed inclined stiffener is considering. Investigations were performed with the finite element method and were checked with the experiment. Recommendations were given on choice of optimal size of stiffener.