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Worm gears with optimized main geometrical parameters and their efficiency

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1. Introduction

When designing mechanical transmissions one of the important factors is assessing of the efficiency, which must be as high as possible. The paper gives a method of designing worm gears with the main geometrical parameters obtained so that the load is transmitted in appropriate circumstances, with the highest efficiency and the lowest overall size. For the study of the load transmission normal section between the worm spire and a tooth of the worm gear is considered. In this section, the main geometrical parameters and the load are used to find the basic elements for the design of cylindrical worm gear.

2. Determination of the module for the worm gears

The normalized module of worm gears can be determined, with a good approximation, using Hertz relation. This relation, used at gears, in order to find surface tension between the teeth flanks, has the following general form [1 - 4]

$$\sigma_H = \sqrt{\frac{F_n}{L_K}\frac{\rho_1+\rho_2}{\rho_1\rho_2}\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}} \le \sigma_{H\,accepted} \quad (1)$$

where F_n is the force in the direction of common normal; L_K is the length of contact line between the flanks (that can be computed by the formula $L_K \approx 0.55 m_x q$); ρ_1 and ρ_2 are curvatures of the teeth flank rays at the contact point; ν_1 and ν_2 are Poisson coefficients for the material of the teeth in contact; E_1 and E_2 are elastic moduli of the teeth; $\sigma_{H\,accepted}$ is the accepted surface stresses between the teeth. For steel worm we have, $\nu_1 = 0.3$ and $E_1 = 2.1\times10^5$ N/mm^2 and for bronze wheel we have $\nu_2 = 0.34$ and $E_2 = 1\times10^5$ N/mm^2; m_x is the module of the worm gear measured at axial section of the worm; q is the diameter factor.

At normal section, the flank of the worm spire for the teeth in contact is considered as straight-lined, while the tooth of the wheel is considered as curved. From Fig. 1, for worm gears, the elements from Eq. (1) are $F_n = F_{N2}$ is the force in the direction of common normal at C point on a tooth of the worm wheel given by the expression [1]

$$F_{N2} = \frac{2T_2}{m_x(z_2+2x)}\frac{cos(\varphi_1)}{cos(\alpha_n)cos(\gamma_m+\varphi_1)} \quad (2)$$

where $\rho_1 = \rho_1 \rightarrow \infty$ is curvature of the worm profile; $\rho_2 = CN_2$ is curvature of the wheel tooth; T_2 is the torque on the axis of the worm wheel; x is specific addendum modification of the wheel; φ_1 is reduced friction angle.

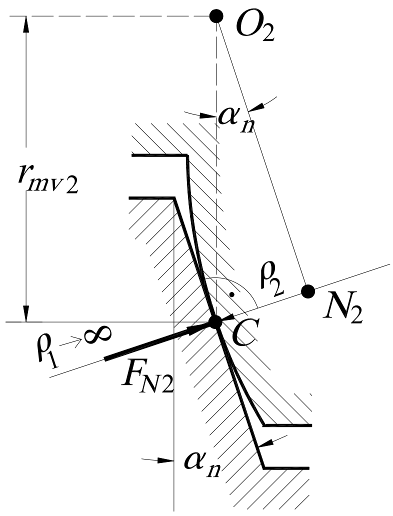

Fig. 1 Contact of the teeth flanks, at normal section

The wheel tooth curvature is given by the expression

$$\rho_2 = \frac{d_{m2}}{2}\frac{sin\,\alpha_n}{cos^2\,\gamma_m} = \frac{m_x(z_2+2x)}{2}\frac{sin\,\alpha_n}{cos^2\,\gamma_m} \quad (3)$$

The reduced friction angle is

$$tan\,\varphi_1 = \frac{\mu}{cos\,\alpha_n} = \mu_1 \quad (4)$$

where μ is friction coefficient between the teeth flanks; z_2 is the number of teeth of the worm wheel; α_n is profile angle of the worm at normal section ($\alpha_n = 20°$); γ_m is the angle of fall of the worm spire on the reference cylinder given by the expression

$$tan\,\gamma_m = \frac{z_1}{q} \quad (5)$$

where z_1 is the number of threads (starts) of the worm.

Based on the Eqs. (2) - (5) and the given values for the worm and wheel, from Eq. (1) the expression of the module can be obtained as

$$m_x \ge \sqrt[3]{\frac{546616.15T_2}{(z_2+2x)^2}\frac{q}{\sqrt{q^2+z_1^2}\left(q-z_1\mu_1\right)}\frac{1}{\sigma^2_{H\,accepted}}} \quad (6)$$

2. Determination of the worm diameter factor

The second important design parameter of worm gears is the diameter factor. Its value must be determined so that no bending of the worm shaft would appear if this is laid on two supports, as the bending would have a bad influence on teeth meshing. The bending deformation f is given mainly by radial force F_{r1} and the tangential force F_{t1} on the worm and can be computed using [5 - 8]

$$f = \frac{l^3}{48 E_1 I_m}\sqrt{F_{t1}^2 + F_{r1}^2} \tag{7}$$

where l is distance between the supports [1] $l = \psi_a a$, where $\psi_a = 1.5 - 2$; $a = \dfrac{m_x(q + z_2 + 2x)}{2}$ is distance between the axes; $I_m = \dfrac{\pi(qm_x)^4}{64}$ is geometric moment of inertia.

Based on the expressions of forces from [6] and the deduced geometrical relations from above, the expression of bending deformation can be obtained as

$$f = \frac{\psi_a^3 (q + z_2 + 2x)^3}{m_x^3 q^5}\frac{T_1}{1979203.4}\times$$
$$\times\sqrt{1 + 0.1324743\frac{z_1^2 + q^2}{(z_1 + \mu_1 q)^2}} \tag{8}$$

where T_1 is torque on the shaft measured in Nm.

The bending deformation f given by Eq. (8) must satisfy the condition

$$f \le f_a \tag{9}$$

where f_a is the accepted bending deformation ($f_a = 0.004 m_x$ at hardened worm and $f_a = 0.01 m_x$ at improved worm [1]).

3. Efficiency of the worm gear with optimized main parameters

The efficiency of cylindrical worm gears is determined by

$$\eta = \frac{tan(\gamma_m)}{tan(\gamma_m + \varphi_1)} = \frac{z_1(q - \mu_1 z_1)}{q(z_1 + \mu_1 q)} \tag{10}$$

The reduced friction coefficient μ_1 is determined from Eq. (4). The value of friction coefficient μ between the teeth flanks can be computed considering the sliding velocity V_a given in [6, 7]

$$\mu = \frac{0.04}{\sqrt[4]{V_a}} = \frac{0.04}{\sqrt[4]{\dfrac{\pi m_x q n_1}{60 \times 1000}\dfrac{\sqrt{q^2 + z_1^2}}{q}}} \tag{11}$$

where n_1 is the rotative speed of the worm (given in $\min^{-1}$).

Knowing efficiency of the gear the torque T_2 on the shaft of the worm wheel is

$$T_2 = \eta\frac{z_2}{z_1}T_1 = \frac{z_2}{q}\left(\frac{q - \mu_1 z_1}{z_1 + \mu_1 q}\right)T_1 \tag{12}$$

where

$$T_1 = 9550000\frac{P_1}{n_1} \tag{13}$$

with P_1 the power on the driving wheel (given in kW).

4. Numerical results

Using the MATLAB [9 - 11] computing environment the expressions from (4) to (13) were used to find the basic geometrical dimensions of the worm gear. The values from Fig. 2 to Fig. 6 are obtained for the following input data: $z_1 = 2$, $z_2 = 35$, $\psi_a = 1.5$, $P_1 = 5$ kW, $n_1 = 950 \min^{-1}$, $\sigma_{Haccepted} = 300$ N/mm^2. The variation of the m_x module is between 1 mm and 10 mm (the step considered in the computations is 1 mm), the q diameter factor variation is between 4 and 20 (the step considered in the computations is 1), while the x addendum modification variation is between -1 and 1 (the step considered in the computations is 0.5).

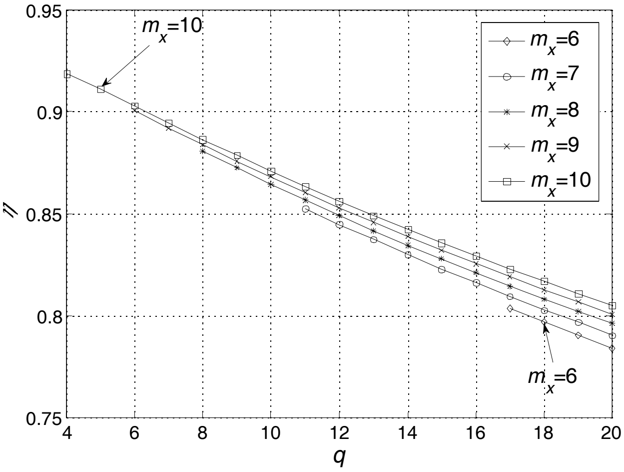

Fig. 2 Variation of η for different q and m_x values ($x = 0$)

From Eq. (10) we can observe that efficiency of the worm gear is the highest if diameter factor q is low and the module m_x is high (Fig. 2). The diminution of q is limited by bending strength of the worm shaft. Bending deformation given by Eq. (9) that is acting in the gear can't be higher than the values given at [2] because it would have a bad influence on the teeth meshing. Module of the gear is determined using Eq. (6) considering the load to be transmitted and the surface tension between the teeth. The program is using two for cycles, one with the m_x counter variable and the second, inside the first for, with the q counter variable. The code from the second for is testing the Eqs. (6) and (9) expressions eliminating the m_x and q values that are not satisfying the test expressions. For a fixed value of m_x, if the test expressions are satisfied, the η and the q values are stored in vectors - as the interior for cycle, controlled by q, varies faster that the exterior one, controlled by m_x - and plotted as parte of a single curve. For each execution of the exterior cycle we obtain a curve that is plotted, with the help of MATLAB function hold, on the same graph with the previous plots (Figs. 3 - 6). The pseudocode of the program is given below. Lines beginning with two consecutive slashed are comments. Where it was possible the MATLAB programming language syntax and function names were kept.

```
//initialize constants like:
//the Poisson coefficient,
//the accepted surface tension, ...
read z1, z2, x
//the step for mx is 1
for mx=6:10
 //c counts the valid points
 c=0;
 //qp and ep vectors are defined to store the
 //valid results
 //initialize vectors
 qp=zeros(1,2);
 ep=zeros(1,2);
 //the step for q is 1
 for q=4:20
  //the results of the test are stored in
  //two variables: ftest and mxtest
  ftest =  test expression (9)
  mxtest = test expression (6)
  compute eta (10)
  if (ftest is True) and (mxtest is True)
   print q, mx, eta, ftest, mxtest
   //store the valid point
   qp(c) = q
   ep(c) = eta
   //prepare to store the next valid point
   //by incrementing the c counter
   c=c+1
   end //end of if
  end //end of q for
  //prepare the labels for the axes
  xlabel('q')
  ylabel(texlabel('eta'))
  //plot the rezults
  plot(qp',ep')
  //hold from Matlab is used as all the data
  //should be drawn on a single plot, as distinct
  //curves in order to compare the results
  hold on
end //end of mx for
```

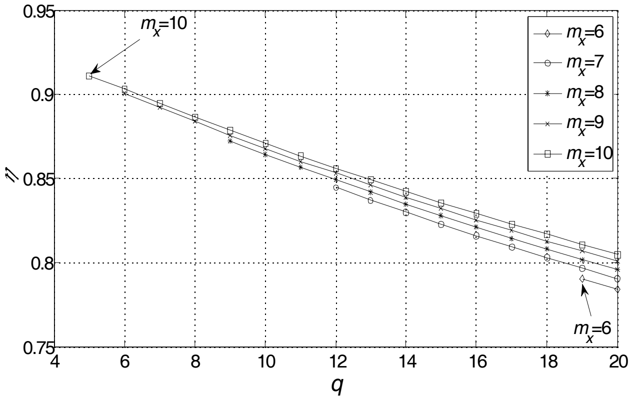

Fig. 3 Variation of η for different q and m_x values (x = -1)

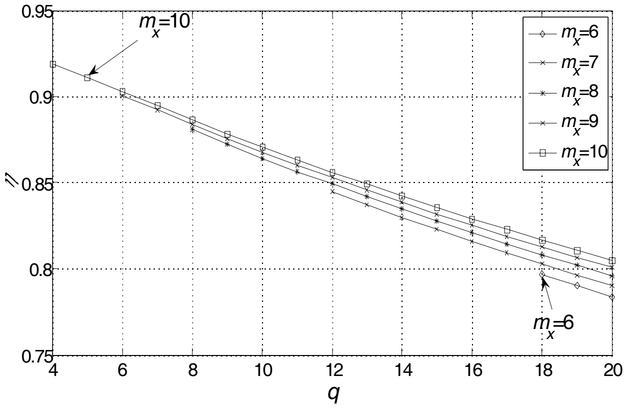

Fig. 4 Variation of η for different q and m_x values (x = -0.5)

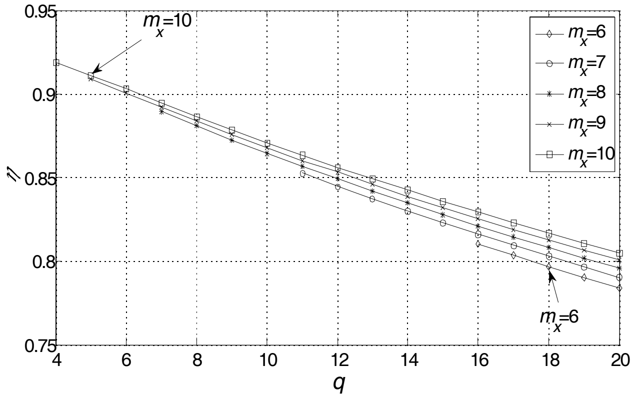

Fig. 5 Variation of η for different q and m_x values (x = 0.5)

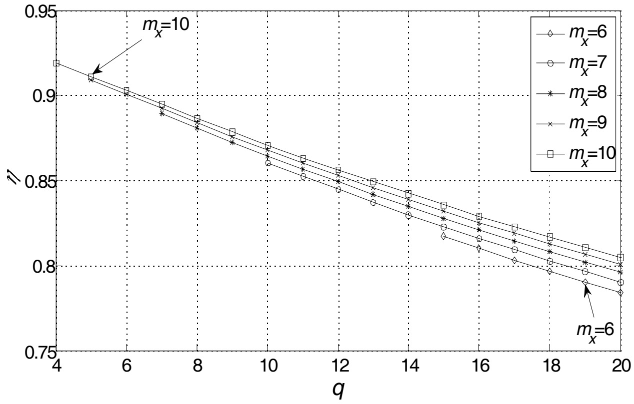

Fig. 6 Variation of η for different q and m_x values (x = 1)

4. Conclusions

The design method given in the paper concerning the main parameters of worm gears gives a method of choosing the module m_x and the diameter factor q. The technical literature doesn't give a method for choosing these parameters and in most cases arbitrary values are used. The value of m_x and q are determined so that efficiency of the worm gear is the highest, the overall size is the lowest, while strength conditions are satisfied and the worm gear work conditions are appropriate. The specific addendum modification must have positive values in order to obtain a better efficiency of the gear.

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OPTIMIZUOTUS PAGRINDINIUS PARAMETRUS TURINČIŲ SLIEKINIŲ PAVARŲ PROJEKTAVIMAS IR JŲ EFEKTYVUMAS

R e z i u m ė

Straipsnyje aprašomas sliekinių pavarų, kurių pagrindiniai parametrai (modulis m_x bei slieko skersmens koeficientas q) užtikrina mažus jų matmenis ir didelį naudingumo koeficientą, projektavimo metodas, kad naudojant gautas lygtis būtų galima suprojektuoti minimalių dydžių sliekines pavaras, turinčias didžiausią naudingumo koeficientą. MATLAB sistema nustatoma pavaros perduodama jėga ir sukuriami galimų krumplio dydžio modifikacijų grafikai. Grafikuose matyti, kad, norint gauti didelį pavaros naudingumo koeficientą, sliekinė pavara turi būti projektuojama tokia, kad turėtų didelį modulį, mažą skersmens koeficientą ir teigiamą krumplio korekcijos koeficientą.

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WORM GEARS WITH OPTIMIZED MAIN GEOMETRICAL PARAMETERS AND THEIR EFFICIENCY

S u m m a r y

The paper gives a method of designing worm gears with the main geometrical parameters (module m_x and diameter factor q) obtained so that the overall size is low and the efficiency is high. Using the obtained expressions, a given load to be transmitted and the possible variations for addendum modification plots were generated in MATLAB to help the designer to obtain maximum efficiency with minimum overall size. As shown in the plots, in order to have a high efficiency, the worm gear must be designed with high module, low diameter factor and positive addendum modification.

Т. А. Антал

ПРОЕКТИРОВАНИЕ ЧЕРВЯЧНЫХ ПЕРЕДАЧ С ОПТИМИЗИРОВАННЫМИ ОСНОВНЫМИ ПАРАМЕТРАМИ И ИХ ЭФФЕКТИВНОСТЬ

Р е з ю м е

В статье рассматривается метод проектирования червячных передач, основные параметры которых (модуль упругости m_x и коэффициент диаметра червяка q) определены таким образом, чтобы получить высокий коэффициент полезного действия и малые габариты. Для обеспечения минимальных размеров червячной передачи с максимальным коэффициентом полезного действия, используя полученные уравнения, с помощью системы МАТЛАБ определена ее передаваемая сила и получены графики для определения возможных модификаций размеров зуба. Из графиков очевидно, что для получения высокого коэффициента полезного действия червячной передачи необходимо подобрать большой модуль, малый коэффициент диаметра червяка и положительный коэффициент коррекции зубца.

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