Effect of liquid physical properties variability on film thickness

S. Šinkūnas*, A. Kiela**

*Kaunas University of Technology, Donelaičio 20, 44239 Kaunas, Lithuania, E-mail: stasys.sinkunas@ktu.lt
**Kaunas University of Applied Sciences, Pramonės 22, 50387 Kaunas, Lithuania, E-mail: algimantas.kiela.@kauko.lt

Nomenclature

a - thermal diffusivity, m²/s; c - specific heat, J/(kg·K); F - frictional force, N; G - liquid mass flow rate, kg/s or gravitational force, N; g - acceleration of gravity, m/s²; Ga_R - Galileo number, gR^3/v^2; Pr - Prandtl number, v/a; Q - heat flux, W; q - heat flux density, W/m²; R - tube external radius, m; r - variable radius in the film; Re - Reynolds number of liquid film, $4\Gamma/(\rho v)$; T - temperature, K; x - longitudinal coordinate, m; w - local velocities of stabilized film, m/s; y - distance from wetted surface, m; Γ - wetting density, $kg/(ms)$; δ - liquid film thickness, m; ε_δ - ratio of film thicknesses; ε_R - relative cross curvature of the film, δ/R; λ - thermal conductivity, W/(m·K); v - kinematic viscosity, m²/s; ρ - liquid density, kg/m³; ς - dimensionless distance from wetted surface, y/δ.

Subscripts: f - film flow; g - gas or vapour; is - isothermal; m - mean; s - film surface; w - wetted surface.

1. Introduction

Gravitational liquid films play an important role in many industrial applications and mathematical modelling thus receives increasing attention. The thickness and velocity of thin liquid films flowing down on vertical surfaces are among the key parameters determining overall performance of gas-liquid contacting apparatuses such as film evaporators, distillation columns, nuclear reactors, boilers, condensers. A significant amount of research work [1-5] carried out over the last few years suggest that rates of momentum and heat transfer are strongly influenced by the liquid film characteristics, fluid physical properties, presence of surfactants. Study [6] analyzed the behaviour of squeeze film between two curved rough circular plates. The results suggested that the use of a film considerably improved the lubrication of bearing system.

In the paper [7], the effect of physical properties of liquids and of surface treatment on wetted area of structured packing was experimentally studied. The liquid film width and thickness were measured for solutions with different surface tension and viscosities. The experimental results showed that the liquid film width, and hence the wetted area, decreased with liquid viscosity, contrary to earlier correlations in the literature. A new statistical correlation for the estimation of the wetted area and for the liquid film thickness is proposed, reflecting the measured variations with viscosity and advancing contact angles.

The effect of liquid properties on flooding in small diameter vertical tubes for various liquids with the aim to contribute to the interpretation of flooding mechanisms in such geometries was studied in [8]. The results confirmed the influence of the liquid properties on the interfacial wave evolution and film characteristics. New correlations based on dimensionless groups for the prediction of flooding in narrow passages are proposed and found to be in good agreement with the available data.

The effect of liquid viscosity on the flow regimes and corresponding pressure gradients along the vertical two-phase flow was investigated [9]. Experiments were carried out in a vertical tube of 0.019 m in diameter and 3 m length and the pressure gradients were measured by a U-tube manometer. It was found that in the annular flow regimes, pressure gradients increased with increasing Reynolds number.

Dewetting of liquid films was experimentally studied in [10]. A dry patch was produced on a liquid film of controlled thickness. The viscosity and the static contact angle were varied using different liquids. The results are compared with a simple model taking into account the balance between viscous and driving forces, finding a very good agreement.

The film flow of water and two aqueous of glycerol on horizontal rotating disk with the aim to obtain the variations of film thickness along the disk radius at different volumetric flow rates and speed of rotation has been investigated in [11]. It has been established that when the centrifugal forces are dominant, the film thickness decreases continuously and can be predicted by the equations which accounts for the Coriolis force. The influence of liquid physical properties and flow rate on the jump position has been correlated by means of Reynolds and Weber numbers.

Study [12] examined the steady state solutions of a laminar falling variable viscosity liquid film along an inclined heated plate. Analytical solutions were constructed for the governing nonlinear boundary value problem using perturbation technique together with a special type of Hermite-Pade approximants. Important properties of the velocity and temperature fields including bifurcations and thermal criticality are discussed.

The effects of variable viscosity, variable thermal conductivity and thermocapillarity on the flow and heat transfer in a laminar liquid film on a horizontal stretching sheet were analyzed in [13]. Using a similarity transformation the governing time dependent boundary layer equations for momentum and thermal energy were reduced to a set of coupled ordinary differential equations. The resulting five parameter problem was solved numerically for some representative value of the parameters. It was shown that the film thickness increases with the increase in viscosity of the fluid.

2. Analysis of liquid film thickness variation

We consider the stabilized heat transfer for laminar liquid film flow. In this case the thicknesses of hydrodynamic and thermal boundary layers are both equal to the film thickness. Let us take elementary film volume of height dx and width dr on the outside surface of a vertical tube (Fig. 1).
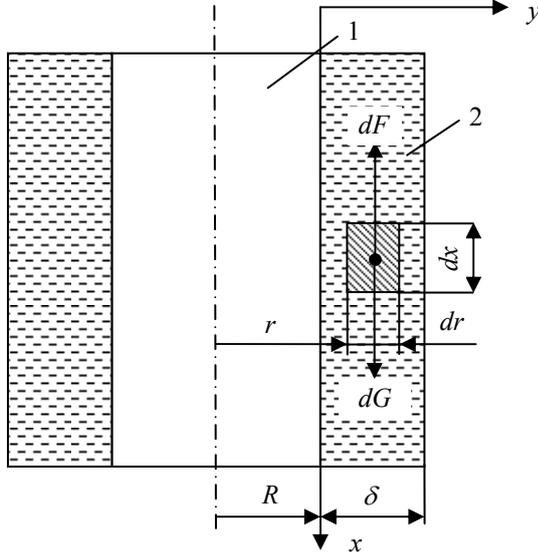

Fig. 1 The geometry of the film flowing down on outside surface of vertical tube: *1* - vertical tube; *2* - liquid film

Then, the elementary gravitational force of the film can be written as follows

$$dG = 2\pi g \left[\int_r^{R+\delta} (\rho - \rho_g) r dr \right] dx \qquad (1)$$

and elementary frictional force of the film, respectively

$$dF = 2\pi r \rho v (dw/dr) dx \qquad (2)$$

In the case of stabilized film flow, the numerical values of these forces are equal. By equating them, one can obtain that

$$dw = (g/r\rho) \left[\int_r^{R+\delta} (\rho - \rho_g) r dr \right] dr \qquad (3)$$

By solving Eq. (3) with the following boundary conditions

$$w = 0 , \text{ for } r = R \qquad (4)$$

we obtain velocity distribution across the film

$$w = g \int_R^r (1/r\rho v) \left[\int_r^{R+\delta} (\rho - \rho_g) r dr \right] dr \qquad (5)$$

When temperature gradient in the film is equal to zero and pressure is negligible, then $\rho = const$, $v = const$ and $\rho_g \ll g$. Integrating of Eq. (5), allows obtaining the expression of velocity distribution in gravitational laminar liquid film

$$w = (g/2v) \left[(R+r)^2 \ln(r/R) - 0.5(r^2 - R^2) \right] \qquad (6)$$

In the case of heat exchange between flowing film and tube surface, liquid density ρ and kinematic viscosity v becomes of variables values and in order to determine them, the film temperature field is to be known. Then, heat transfer problem must be solved together with momentum transfer one.

Heat flux across the elementary volume dx of laminar film can be expressed as follows

$$dQ = -2\lambda \pi r (dT/dr) dx \qquad (7)$$

Heat flux density falling to the unit of tube surface can be written as

$$q = (dQ/2\pi R dx) = -\lambda(r/R)(dT/dr) \qquad (8)$$

For the momentum transfer analysis, it is more reasonable variable r to express through the distance from wetted surface

$$y = r - R \qquad (9)$$

By using the dimensionless quantities $\varepsilon_R = \delta/R$ and $\varsigma = y/\delta$, we obtain that

$$dT = -(q\delta/\lambda)(1 + \varepsilon_R \varsigma)^{-1} d\varsigma \qquad (10)$$

By solving Eq. (10) with the following boundary conditions

$$T = T_w \text{ for } \varsigma = 0 \qquad (11)$$

we obtain the expression of temperature field in the film

$$T = T_w \pm \frac{q_w \delta}{\lambda_f} \int_0^\varsigma \frac{q/q_w}{(\lambda/\lambda_f)(1 + \varepsilon_R \varsigma)} d\varsigma \qquad (12)$$

The negative sign is put in the case of film heating and positive sign, when the film is cooling.

Heat flux density in the film first of all is a function of ς. It can be determined by solving the following differential energy equation

$$(1 + \varepsilon_R \varsigma) c \rho w \delta \frac{\partial T}{\partial x} + \frac{\partial q}{\partial \varsigma} = 0 \qquad (13)$$

By integrating Eq. (13) within the limits from 0 to ς and using the boundary condition $q = q_w$ for $\varsigma = 0$, we obtain that

$$q/q_w = 1 - (\delta/q_w) \int\limits_0^\varsigma c\rho w(1+\varepsilon_R\varsigma)\frac{\partial T}{\partial x}d\varsigma \qquad (14)$$

The longitudinal temperature gradient $\partial T/\partial x$ depends upon the boundary conditions on a surface of the tube. Usually it is expressed at the boundary condition $q_w = const$. Then, we have

$$\frac{\partial T}{\partial x} = \frac{dT_f}{dx} \qquad (15)$$

The derivative dT_f/dx one can determine from the heat balance equation written for the elementary volume of the film

$$2\pi R(q_w - q_s)dx = Gc_m dT_f \qquad (16)$$

The mean specific capacity can be defined as follows

$$c_m = \frac{2\pi}{G}\int\limits_R^{R+\delta} c\rho wrdr = \frac{2\pi\delta}{G}\int\limits_0^1 c\rho w(1+\varepsilon_R\varsigma)d\varsigma \qquad (17)$$

By taking into account Eq. (17), we obtain

$$\frac{dT_f}{dx} = \frac{q_w - q_s}{\delta\int\limits_0^1 c\rho w(1+\varepsilon_R\varsigma)d\varsigma} \qquad (18)$$

Let us denote that

$$(wv_f)/(\delta^2 g) = u \qquad (19)$$

By substituting expression $\partial T/\partial x = dT_f/dx$ into Eq. (14) in accordance with Eq. (18), we obtain that

$$\frac{q}{q_w} = 1 - \left(1-\frac{q_s}{q_w}\right)\frac{\int\limits_0^\varsigma (1+\varepsilon_R\varsigma)(c\rho/c_f\rho_f)ud\varsigma}{\int\limits_0^1 (1+\varepsilon_R\varsigma)(\rho v/\rho_f v_f)ud\varsigma} \qquad (20)$$

By rearranging Eq. (19) and substituting of variable r for variables y and ς, we obtain the following expression

$$u = \int\limits_0^\varsigma \frac{\int\limits_\varsigma^1 [(\rho-\rho_g)/\rho_f](1+\varepsilon_R\varsigma)d\varsigma}{(1+\varepsilon_R\varsigma)(\rho v/\rho_f v_f)}d\varsigma \qquad (21)$$

In the case of liquid density variation, mass flow rate of the film can be defined as

$$G = 2\pi R\int\limits_R^{R+\delta}\rho wrdr = 2\pi\delta R\int\limits_0^1 \rho w(1+\varepsilon_R\varsigma)d\varsigma \qquad (22)$$

The wetting density can be expressed as follows

$$\Gamma = (G/2\pi R) = \delta\int\limits_0^1 \rho w(1+\varepsilon_R\varsigma)d\varsigma \qquad (23)$$

By taking into account Eq. (19), we can define the Reynolds number for film flow through the mean film temperature T_f by the following expression

$$Re_f = 4\Gamma/\rho_f v_f = 4Ga_f\int\limits_0^1 \frac{\rho}{\rho_f}(1+\varepsilon_R\varsigma)d\varsigma \qquad (24)$$

or

$$Ga_f = Re_f \bigg/ 4\int\limits_0^1 (\rho/\rho_f)(1+\varepsilon_R\varsigma)ud\varsigma \qquad (25)$$

The mean temperature of the film can be expressed as follows

$$T_f = \frac{\int\limits_0^1 (1+\varepsilon_R\varsigma)c\rho uTd\varsigma}{\int\limits_0^1 (1+\varepsilon_R\varsigma)c\rho ud\varsigma} \qquad (26)$$

By denoting that

$$\psi = \int\limits_0^\varsigma \frac{q/q_w}{(\lambda/\lambda_f)(1+\varepsilon_R\varsigma)}d\varsigma \qquad (27)$$

in accordance with Eqs. (12) and (26), we obtain that

$$T_w = T_f \pm \frac{q_w\delta}{\lambda_f}\frac{\int\limits_0^1 (1+\varepsilon_R\varsigma)c\rho u\psi d\varsigma}{\int\limits_0^1 (1+\varepsilon_R\varsigma)c\rho ud\varsigma} \qquad (28)$$

By employing Eqs. (12), (27) and (28), the liquid film temperature field can be defined by the following expression

$$T = T_f \pm \frac{q_w \delta}{\lambda_f} \left[\frac{\int\limits_0^1 \psi (1 + \varepsilon_R \varsigma) c \rho u d\varsigma}{\int\limits_0^1 (1 + \varepsilon_R \varsigma) c \rho u d\varsigma} - \psi \right] \qquad (29)$$

For the case of isothermal flow, by denoting the film thickness as δ_{is}, the temperature T_{is}, the Reynolds number as Re_{is} and the Galileo number as $Ga_{fis} = g \delta_{is}^3 / v_f^2$ from Eqs. (21) and (24), we obtain that

$$u_{is} = \int\limits_0^1 \frac{\int\limits_\varsigma^1 (1 - \rho_g / \rho_f)(1 + \varepsilon_R \varsigma) d\varsigma}{1 + \varepsilon_R \varsigma} d\varsigma \qquad (30)$$

$$Re_{fis} = 4 Ga_{fis} \int\limits_0^1 (1 + \varepsilon_R \varsigma) u_{is} d\varsigma \qquad (31)$$

The influence of liquid physical properties variation on film thickness can be evaluated using the ratio $\varepsilon_\delta = \delta / \delta_{is}$. For $Re_f = Re_{fis}$ and $T_f = T_{fis}$, this ratio in accordance with Eqs. (24) and (30) is as follows

$$\varepsilon_\delta = \frac{\delta}{\delta_{is}} = \frac{\left[\int\limits_0^1 (1 + \varepsilon_R \varsigma) u_{is} d\varsigma \right]^{1/3}}{\left[\int\limits_0^1 (\rho / \rho_f)(1 + \varepsilon_R \varsigma) u d\varsigma \right]^{1/3}} \qquad (32)$$

Liquid viscosity variation has a significant influence on the film thickness. However, viscosity variation depends on the film temperature field, which is determined by the liquid thermal properties as well. Therefore, the influence of liquid physical properties variation on film thickness it is purposely to evaluate using the ratio Pr_f / Pr_w. The calculation results were obtained by evaluating a function $\varepsilon_\delta = f(Pr_f / Pr_w)$. This function is presented in Fig. 2. As can be seen from Fig. 2, despite the analyzed of very different liquid physical properties dependences on temperature, the calculation data unambiguously can be defined by the following expression

$$\varepsilon_\delta = A(Pr_f / Pr_w)^{-n} \qquad (33)$$

where

$A = 1.2$, $n = 0.088$, for $0.01 \le (Pr_f / Pr_w) \le 0.1$

$A = 1$, $n = 0.17$, for $0.1 \le (Pr_f / Pr_w) \le 1$

$A = 1$, $n = 0.22$, for $1 \le (Pr_f / Pr_w) \le 10$

$A = 1.2$, $n = 0.3$, for $10 \le (Pr_f / Pr_w) \le 100$.

For the case of isothermal laminar film flow, the film thickness can be calculated by the following equations: when the film flows down a vertical plane surface

$$\delta_{is} = \left(\frac{3}{4} \frac{v_f^2}{g} Re \right)^{1/3} \qquad (34)$$

and when the film flows down an outside surface of vertical tube

$$\delta_{is} = 1.67 \left\{ \left[1 + 1.09 \left(Re_f / Ga_{Rf} \right)^{1/3} \right]^{1/2} - 1 \right\} \qquad (35)$$

In the case of flowing film heating or cooling, its thickness can be determined by the formula

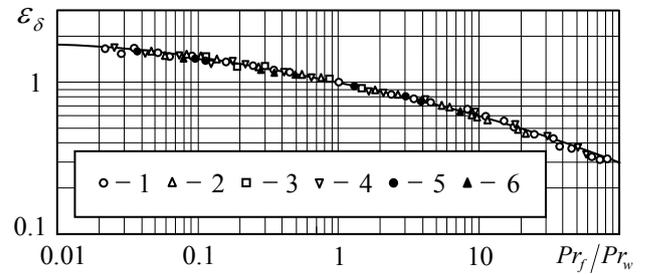$$\delta = \varepsilon_\delta \delta_{is} \qquad (36)$$

Fig. 2 Dependence of the film thickness alteration on the size of nonisothermality: *1, 2, 3* - fuel oil, compressor oil and water films respectively, when $\varepsilon_R = 0.5$; *4, 5, 6* - fuel oil, compressor oil and water films respectively, when $\varepsilon_R = 1$

4. Conclusions

For the most part, transformation of the film thickness is related to variation of liquid viscosity. However, viscosity variation depends on the film temperature field, which is determined by liquid thermal properties. Therefore, the effect of liquid physical properties on film thickness was evaluated using the ratio Pr_f / Pr_w.

The calculation data analysis showed that film cross curvature and external heat exchange between the film surface and surrounding medium of gas or vapour influence on liquid film thickness variation practically is negligible.

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S. Šinkūnas, A. Kiela

SKYSČIO FIZIKINIŲ SAVYBIŲ POKYČIO POVEIKIS JO PLĖVELĖS STORIUI

R e z i u m ė

Straipsnyje pateikta metodika stabilizuotai laminarinei skysčio plėvelei, tekančiai vertikalaus vamzdžio išoriniu paviršiumi tirti, taikant jėgų pusiausvyros lygtis. Atlikti tekančių vandens, kompresorinės alyvos ir mazuto plėvelių analitiniai tyrimai, esant įvairioms plėvelių santykinio kreivumo ir šilumos srautų tankių santykių vertėms. Gauta funkcija, leidžianti įvertinti plėvelės storio pokyčio priklausomybę nuo neizotermiškumo dydžio. Nustatyta, kad plėvelės storio kitimas priklauso nuo skysčio fizikinių savybių pokyčio. Teoriškai išnagrinėta skysčio plėvelės kreivumo ir išorinių šilumos mainų įtaka jos storio pokyčiui.

S. Šinkūnas, A. Kiela

EFFECT OF LIQUID PHYSICAL PROPERTIES VARIABILITY ON FILM THICKNESS

S u m m a r y

A model for research of stabilized laminar liquid film flow on an outside surface of vertical tube based on force equilibrium equations is presented in the paper. The calculations for water, compressor oil and fuel oil films at various values of relative cross curvature of the films and ratios of heat fluxes densities were carried out. A function allowing estimating nonisothermality effect on film thickness was established. It has been determined that film thickness variation depends on the variability of liquid physical properties. Theoretical analysis of film cross curvature and external heat exchange influence on liquid film thickness variation was performed as well.

С. Шинкунас, А. Кела

ВЛИЯНИЕ ФИЗИЧЕСКИХ СВОЙСТВ ЖИДКОСТИ НА ТОЛЩИНУ ПЛЕНКИ

Р е з ю м е

В статье представлена методика для исследования течения стабилизированной ламинарной жидкостной пленки по наружной поверхности вертикальной трубы, основанная на применении выражений сил равновесия. Проведены расчеты течения пленок воды, компрессорного масла и мазута при различных значениях относительной кривизны пленок и отношениях плотностей тепловых потоков. Определена зависимость, позволяющая оценить влияние неизотермичности на толщину пленки. Установлено, что трансформация толщины пленки связана в первую очередь с изменением физических свойств жидкости. Также проведен теоретический анализ по влиянию поперечной кривизны пленки и наружной теплоотдачи на изменение толщины жидкостной пленки.