ISSN 1392 - 1207. MECHANIKA. 2010. Nr.1(81)

Effect of explosive loading on mechanical properties of concrete and reinforcing steel: towards developing a predictive model

Virm. Juocevičius*, E. R. Vaidogas**
**Vilnius Gediminas Technical University, Saulėtekio al.11, 10223 Vilnius, Lithuania,*
E-mail: virmantas.juocevicius@conserela.lt
***Vilnius Gediminas Technical University, Saulėtekio al.11, 10223 Vilnius, Lithuania, E-mail: erv@st.vgtu.lt*

1. Introduction

The ability of reinforced concrete (RC) to absorb energy under dynamic transient nonlinear conditions has led to its utilisation for several classes of important structures which may be subjected to impact or explosive loading (e.g., nuclear containment vessels, power plant structures, protective barriers, shelter structures) [1-4]. The low probability of occurrence of this loading necessitates a limit state approach to design in which irreversible structural deformation and material damage is acceptable provided that overall structural integrity is maintained. The numerical simulation of such structural responses therefore requires the simultaneous consideration of both dynamic properties of concrete and steel and geometrical nonlinearities (see the papers included in the book [5, 6]).

The stress and strain rate sensitivity of concrete and steel plays a considerable role in its dynamic load-carrying capacity. By increasing either stress or strain loading rates, the strength of these materials is significantly increased. The prevailing approach to accounting for dynamic effects on properties of steel and concrete is an application of enhancement factors (enhancement of static properties) [1, 7, 8]. Recently, several newer approaches were introduced in an attempt to predict dynamic properties of concrete by an in-depth understanding and modelling of the physical processes which occur in concrete under dynamic loading. These processes are described by applying the methods of thermo-fluctuation theory and dynamic fracture mechanics [9, 10]. However, the enhancement factor approach is still not without strong appeal because it is well-founded by experimental data and applicable to highly specific design problems (e.g., specific types and classes of concrete and steel). In addition, the fact that enhancement factors are directly derived from statistical data makes them naturally amenable to an analysis of uncertainties related to the degree of dynamic enhancement.

Despite the random nature of material properties under both static and dynamic loading, the modelling of enhancement of static properties remained predominantly deterministic. Even those few articles which were devoted to a probabilistic analysis of RC structures subjected to explosive loading applied fixed (nonrandom) enhancement factors [11, 12]. It is needless to say that a realistic prediction of a dynamic property should have the form of a probability distribution expressing uncertainties related to this property. Constructing this distribution will require to integrate enhancement factor models into a broader modelling. This process should link deterministic models of the physical phenomena influencing the dynamic properties and include models quantifying uncertainties involved in the problem (Fig. 1).

The prediction of the dynamic properties by means of modelling will require to solve three tasks:

1. to link the deterministic models used for predicting individual physical phenomena related to dynamic enhancement (models I to III in Fig. 1);
2. to quantify uncertainties related to accuracy of the models I to III as well as the uncertainty in the static material properties (to build models IV and V, Fig. 1);
3. to propagate uncertainties mentioned in the previous item to the final output of the problem, namely, a probabilistic model of a dynamic material property.

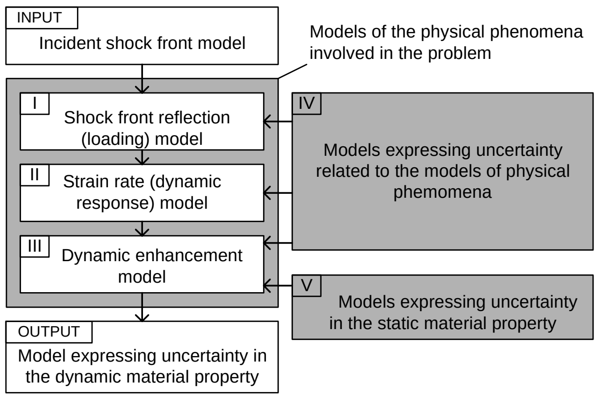

Fig. 1 The scheme of modelling aimed to predict the dynamic properties of concrete and reinforcing steel

The uncertainties targeted in the second task are of two different types. They can be quantified by a separate treatment of aleatory (stochastic) and epistemic (state-of knowledge) uncertainties involved in the problem. Such a treatment of uncertainties is based on engineering applications of the Bayesian statistical theory [13]. The results of this quantification can be further propagated to a model quantifying structural fragility to explosive loading (fragility function) [14, 15]. Models expressing uncertainties in the dynamic material properties can also be used for specifying characteristic values of these properties.

The present paper seeks to solve the first task. Despite a considerable number of publications on individual models denoted by the numbers I to III, the appealingly simple task of their application in one combined set remains to be solved, at least in part. Our finding was that this task involves several tricky facets. This paper identifies them and attempts to address them.

2. High-rate loading generated by an explosion

In the case of an impulsive explosive loading, the response of an RC structure will occur in so short a time that no viscous damping can be invoked [16]. For a structure subjected to such a loading, the first displacement peak will be the most severe. Subsequent cycles will decrease significantly in magnitude and the oscillation will die down rapidly. Moreover, under severe loading, the structure is likely to undergo excessive permanent deformation during its first displacement, and it is very unlikely for the structure to fail during its second displacement peak. Therefore, in most cases, only the first displacement peak is considered in analyzing structural response to explosive loading [11].

A loading of a structure directly exposed to an incident shock front generated by an above ground explosion takes place during the reflection of this front. The typical pressure signal $p(t)$ of incident and reflected shock fronts of a large and distant free field explosion is characterised by the peak overpressure P_{max} and the positive phase duration $t_{rise} + t_{decay}$ (Fig. 2, e.g., [17]). The negative phase is usually ignored in the explosive damage assessment. In order to distinguish between the pressure signals of incident and reflected shock fronts, characteristics of these signals will be denoted by the subscripts "i" and "r", respectively.

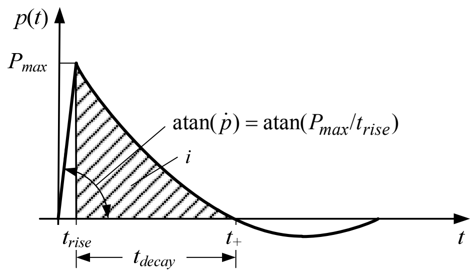

Fig. 2 Pressure signal of the shock fronts, incident or reflected, resulting from a distant explosion

Records of the pressure time-history of distant explosions allow to assume a linear form of the function $p_i(t)$ within the rise time $t_{i,rise}$ (Fig. 2, [18, 19]). This linear part of $p_i(t)$ is expressed as

$$p_i(t) = \frac{P_{i,max}}{t_{i,rise}} t = \dot{p}_i t, \qquad t < t_{i,rise} \tag{1}$$

where $P_{i,max}$ is the peak overpressure of the incident shock front; $\dot{p}_i$ is the rate of pressure increase. The assumption of the linear form of $p_i(t)$ within $t_{i,rise}$ implies that the pressure increases at a constant rate $\dot{p}_i$. During the reflection time $t_{r,rise} + t_{r,decay}$, the structure will be subjected to a varying straining rate. However, the traditional approach is to assume this rate to be constant for within the time $t_{r,rise} + t_{r,decay}$. It is stated that this assumption gives good results [20].

The rise time $t_{i,rise}$ can be roughly assumed as maximum 25% of the decay time $t_{i,decay}$ [21]. Low & Hao used for their calculations the value $t_{i,rise} = 0.1$ ms [12]. However, for TNT (trinitrotoluene) explosions, a more accurate modelling exists and allows to assess $t_{i,rise}$ of the incident shock front from the following empirical relation [20]

$$t_{i,rise} = \kappa_1 \, \Delta^{\kappa_2} \tag{2}$$

with the scaled distance

$$\Delta = R/Q^{1/3} \tag{3}$$

where R is the distance from the charge centre; Q is the TNT charge weight; κ_1 and κ_2 are parameters (regression coefficients) given in Table 1.

Table 1

Parameters used for establishing the predictive model

Explosive loading parameters κ			
Symbol	Value	Symbol	Value
κ_1	0.0019	κ_5	0.051
κ_2	1.3	κ_6	1.008
κ_3	1.059	κ_7	-2.01
κ_4	-2.56		
Parameters related to the enhancement factors γ			
Symbol	Value	Symbol	Value
α_1	$1/(5 + 0.75 f_{cm})$ (1)	α_5	0.026
α_2	1/3	α_6	0.02
α_3	$6\alpha_1 - 2$	α_7	4.3 (2); 6 (3); 12 (4)
α_4	$6.156\,\alpha_1 - 0.492$		
(1) f_{cm} is the mean static cube strength of concrete (2,3,4) values related to cold-worked steel, hot rolled steel and mild reinforcing steel, respectively [7]			

In the course of the reflection, the overpressure on the face of the structure rises to $P_{r,max}$ at the instant $t_{r,rise}$ and then decreases to the ambient pressure after the time $t_{r,decay}$. It is reasonable to suppose that the enhancement of mechanical properties of concrete and steel is affected by loading rate $P_{r,max} / t_{r,rise}$ (hereafter denoted by $\dot{p}_r$) within the rise phase (0, $t_{r,rise}$). The question how the enhancement might be influenced by sudden stop of pressure increase at $t_{r,rise}$ and a decrease in the early part of the decay phase ($t_{r,rise}$, t_{r+}) remains unanswered, to the best of our knowledge.

One can roughly estimate the reflected specific impulse i_r by assuming a similarity between the time-histories of the overpressure in the incident and reflected shock fronts. This assumption yields the following relation [22]

$$\frac{P_{r,max}}{P_{i,max}} \approx \frac{i_r}{i_i} \tag{4}$$

where i_i is the specific impulse of the incident shock front.

The similarity of pressure signals allows to make also an assumption about the ratio of the rising times of the incident and reflected shock fronts

$$\frac{P_{r,max}}{P_{i,max}} = \frac{t_{r,rise}}{t_{i,rise}} \tag{5}$$

The above expression means that the rates of pressure increase in the incident and reflected fronts, $\dot{p}_i$ and $\dot{p}_r$, are equal. To make this assumption less stringent one can assume the linear relationship. The assumption (5) implies that the rate $\dot{p}_r$ can be estimated by the parameters of the incident pressure signal

$$\dot{p}_r = \frac{P_{i,max}}{t_{rise}} \tag{6}$$

The incident overpressure $P_{i,max}$ can be estimated by means of an empirical function of the scaled distance $\varDelta$ [20]

$$P_{i,max} = \begin{cases} \kappa_3 \varDelta^{\kappa_4} - \kappa_5 & \text{if } 0.1 \le \varDelta \le 1 \\ \kappa_6 \varDelta^{\kappa_7} & \text{if } 1 < \varDelta \le 10 \end{cases} \tag{7}$$

where κ_3 to κ_7 are parameters given in Table 1.

As the empirical relations (3) and (7) depend on $\varDelta$, the rate $\dot{p}_r$ can be expressed as a function of the incident overpressure $P_{i,max}$ alone

$$\left.\begin{array}{l} \dot{p}_r = \kappa_1^{-1} P_{i,max} \left(\dfrac{P_{i,max}}{\kappa_6} \right)^{-\kappa_2/\kappa_7} \\ 0.0098 \text{ MPa} < P_{i,max} < 1.008 \text{ MPa} \end{array}\right\} \tag{8}$$

$$\left.\begin{array}{l} \dot{p}_r = \kappa_1^{-1} P_{i,max} \left(\dfrac{P_{i,max} + \kappa_5}{\kappa_3} \right)^{-\kappa_2/\kappa_4} \\ 1.008 \text{ MPa} < P_{i,max} < 384.5 \text{ MPa} \end{array}\right\} \tag{9}$$

Eqs. (6), (8) and (9) imply that the loading rate $\dot{p}_r$ can be predicted with the peak overpressure of the incident (not the reflected) shock front $P_{i,max}$. The loading rate $\dot{p}_r$ depends on $P_{i,max}$ nonlinearly although this nonlinearity is not strong (Fig. 3).

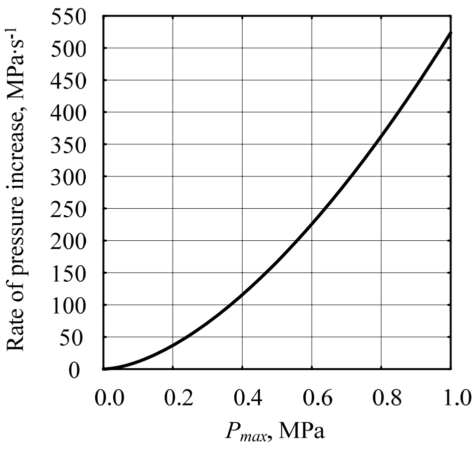

Fig. 3 Illustration of the relation in Eq. (8): rate of pressure increase $\dot{p}$ versus overpressure $P_{i,max}$

The models given by Eqs. (2) - (9) are valid for the case of shock front reflection by front wall of a building subjected to a distant explosion. They can be applied to the design of wall panels on the potential front face and other structures directly facing explosive loading (e.g., protective barriers). The pressure signals on the roof as well as side and rare walls will be different from that on the front wall. Simplified models of the pressure signals on roof, side, and rare surfaces of a rectangular box-shape involve a linear rise phase [1, 3]. These models can be used for an assessment of loading rate $\dot{p}_r$ in the rise phase. However, this task lies outside the scope of this paper.

The loading of structural elements inside building engulfed by a shock front will depend on a complex reflection and refraction of this front by building envelope and penetration of the front through openings in the building (e.g., windows). An assessment of such loading with the sufficiently high accuracy required for the prediction of dynamic material properties can be difficult and tedious process. It can be even more complicated in the case where the behaviour of structural elements influences the loading they receive (the case of "coupled" structures [17]). The question whether the explosive loading of internal elements will have a pronounced rise phase, which might determine the dynamic enhancement, requires special investigation. An answer to this question lies outside the modelling on material level and is not searched for in the present paper.

3. Influence of the loading rate on material properties

3.1. Concrete

The normal rate of static loading for the standard cylinder test is approximately 0.2 - 1 MPa/s [7, 23]. This corresponds to the static strain rate of approximately $30 \cdot 10^{-6}$ s^{-1}. A popular strain rate dependent model of concrete mechanical properties was introduced by CEB [7]. This model consists of enhancement factors γ which can be calculated for a given stress rate $\dot{\sigma}$ or strain rate $\dot{\varepsilon}$ (Table 2).

Table 2

CEB enhancement factors for properties of concrete subjected to uniaxial compression [7]

Property	Enhancement factor
Expressions in terms of the stress rate $\dot{\sigma}$	
Strength: $\dfrac{f_{c,dyn}}{f_{c,st}}$	$\gamma_c = (\dot{\sigma}/\dot{\sigma}_{st})^{\alpha_1}$, $\dot{\sigma} \le 10^6$ MPa/s $\gamma_c = \alpha_3 \dot{\sigma}^{\alpha_2}$, $\dot{\sigma} > 10^6$ MPa/s
Expressions in terms of the strain rate $\dot{\varepsilon}$	
Strength: $\dfrac{f_{c,dyn}}{f_{c,st}}$	$\gamma_c = (\dot{\varepsilon}/\dot{\varepsilon}_{st})^{1.026\alpha_1}$, $\dot{\varepsilon} \le 30$ s^{-1} $\gamma_c = \alpha_4 \dot{\varepsilon}^{\alpha_2}$, $\dot{\varepsilon} > 30$ s^{-1}
Expressions in terms of $\dot{\sigma}$ and $\dot{\varepsilon}$	
Secant modulus: $E_{c,dyn}/E_{c,st}$	$\gamma_E = (\dot{\sigma}/\dot{\sigma}_0)^{\alpha_5}$ or $\gamma_E = (\dot{\varepsilon}/\dot{\varepsilon}_0)^{\alpha_5}$, see Table 1 for α_5
Ultimate strain: $\varepsilon_{u,dyn}/\varepsilon_{u,st}$	$\gamma_\varepsilon = (\dot{\sigma}/\dot{\sigma}_0)^{\alpha_6}$ or $\gamma_\varepsilon = (\dot{\varepsilon}/\dot{\varepsilon}_0)^{\alpha_6}$ See Table 1 for α_6
Explanations: $\alpha_1 = 1/(5 + 0.75 f_{cm})$; $log_{10}\alpha_3 = 6\alpha_1 - 2$; $\dot{\sigma}_{st}$ = 1 MPa/s (the static stress rate); $\dot{\varepsilon}_{st}$ = $30 \cdot 10^{-6}$ s^{-1} (the static strain rate); see Table 1 for α_2 and α_4 to α_6	

The CEB model is purely deterministic. It is unclear what degree of uncertainty is related to the predictions of concrete dynamic properties yielded by this model. Relations presented in Table 2 are suitable to predicting the mean value of $f_{c,dyn}$, as well as mean values of dynamic deformation modulus $E_{c,dyn}$ and dynamic ultimate strain $\varepsilon_{u,dyn}$. It is not clear whether the changes in the strain rate $\dot{\varepsilon}$ influence the scatter of these mechanical properties. The only attempt to quantify the coefficient of variation of $f_{c,dyn}$, $E_{c,dyn}$, and $\varepsilon_{u,dyn}$ known to us is the investigation of Mihashi & Wittmann [24]. They demonstrated theoretically that the coefficient of variation remains unaffected by changes in $\dot{\varepsilon}$. The enhancement factor of the concrete compressive strength, γ_c, raises from the value 1.0 up to the value of 5.97 when $\dot{\varepsilon} \in (3 \cdot 10^{-5};\ 1 \cdot 10^{3})$ (Fig. 4, Table 3). The enhancement factors γ_E and γ_ε used for the calculation of $E_{c,dyn}$ and $\varepsilon_{u,dyn}$ are not particularly sensitive to the changes of $\dot{\varepsilon}$ (Table 3).

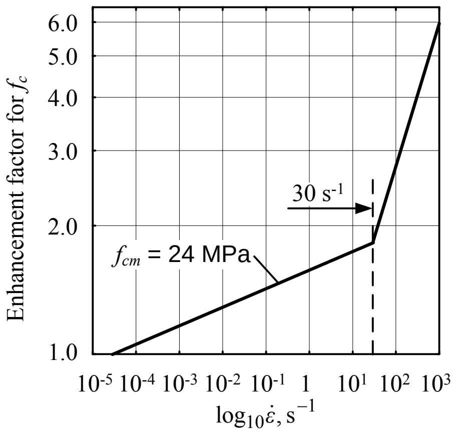

Fig. 4 Influence of the strain rate $\dot{\varepsilon}$ on the enhancement factor γ_c of the compressive strength of concrete, f_c

Table 3

Ranges of values of the enhancement factors γ corresponding to the strain rate interval $\dot{\varepsilon}_{st} \le \dot{\varepsilon} \le 1 \cdot 10^3$ s^{-1}

Factor γ	Range of γ values*	Note
γ_c	(1.0; 5.97)	for f_{cm} = 24 MPa
γ_E	(1.0; 1.57)	—
γ_ε	(1.0; 1.41)	—
γ_s	(1.0; 1.12)	for $f_{y,st}$ = 400 MPa, α_7 = 12
* The upper limits of the intervals correspond to the strain rate $\dot{\varepsilon} = 1 \cdot 10^3$ s^{-1}		

In line with the CEB model, the dynamic strength $f_{c,dyn}$ can be related to the class of concrete through the parameter α_1. This parameter depends on the mean cube strength of concrete, f_{cm}. One can expect that the expressions of the enhancement factor γ_c in terms of the stress rate $\dot{\sigma}$ are more accurate than the ones in terms of the strain rate $\dot{\varepsilon}$. The former were fitted to the raw data and this yielded the parameters α_1 and α_2 (Table 2). The expressions in terms of $\dot{\varepsilon}$ were simply derived from those based on $\dot{\sigma}$ by assuming elastic behaviour of concrete and a fixed elasticity modulus. It is obvious that this assumption is not very firm. The parameters $1.026\alpha_1$ and α_4 present in the models based on $\dot{\varepsilon}$ are results of simple calculations and not a regression analysis.

The CEB enhancement factors are not the only models which can be used for predicting dynamic properties of concrete. For example, Liu & Owen [25] introduced the enhancement factor

$$\gamma_{c1} = \frac{f_{c,dyn}}{f_{c,st}} = \alpha_8 \left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_{st}} \right)^{\alpha_9} + 1 \qquad \text{with } \dot{\varepsilon}_{st} = 10^{-5} \text{ s}^{-1} \quad (10)$$

where α_8 and α_9 are function parameters depending on the strength of concrete. For a concrete with static strength $f_{c,st}$ lower than 44 MPa, $\alpha_8 = 0.02789$; $\alpha_9 = 0.3303$ and for $f_{c,st}$ equal to 60.5 MPa, $\alpha_8 = 0.011768$.

The enhancement factor γ_{c1} is less transparent than the factors γ_c given in Table 2. The former is related to specific class of concrete by two parameters α_8 and α_9 which do not have clear physical meaning. In addition, it is unclear how accurate is the model (10) at predicting the mean value of $f_{c,dyn}$. The publications presenting the factors γ_c and γ_{c1} do not contain any information allowing to compare their accuracy in terms of predicting the mean of $f_{c,dyn}$. Due to the fact that the CEB factor γ_c has a direct relation to the concrete class through the quantity f_{cm}, this factor is more practicable than γ_{c1}.

3.2. Steel

Similarly to the concrete, the yield strength of steel, f_y, increases with increasing the strain rate $\dot{\varepsilon}_s$. Mill tests are generally carried out at much greater strain rates (approximately $1.04 \cdot 10^{-3}$ s^{-1}) than encountered in the structures subjected to static loads [25-27]. In the CEB methodology, the static strain rate of steel, $\dot{\varepsilon}_{s,st}$, is assumed to be $5 \cdot 10^{-5}$ s^{-1} [7].

The mostly known dynamic enhancement factor for the yield strength of steel has the following form

$$\gamma_s = \frac{f_{y,dyn}}{f_{y,st}} = 1 + \alpha \log_{10} \left(\frac{\dot{\varepsilon}_s}{\dot{\varepsilon}_{s,st}} \right) \quad (11)$$

where α is a material dependent parameter (regression coefficient). This model is quoted by Liu and Owen [25] who state that the experimental value of α for austenitic steels is equal to 0.03 within the range ε_s = 2 - 3%. The static strain rate $\dot{\varepsilon}_{s,st}$ is assumed to be 10^{-2} s^{-1}. In case of steels used for the reinforcement of RC structures, the parameter α depends on the static yield strength and the type of steel

$$\alpha = \frac{\alpha_7}{f_{y,st}} \quad (12)$$

where α_7 is a regression coefficient. Values of α_7 are given in Table 1. The model (11) was evaluated for the strain rates up to $\dot{\varepsilon}_s = 10$ s^{-1}. The range of values of γ_s calculated for α_7 = 12 and $f_{y,st}$ = 400 MPa is given in Table 3. One can see that f_y is not particularly strain rate sensitive.

Another model suggested in the literature is the so-called Cowper-Symonds equation expressed in the following form [28]

$$\gamma_{s1} = \frac{f_{y,dyn}}{f_{y,st}} = 1 + \left(\frac{\dot{\varepsilon}_s}{\alpha_{10}} \right)^{\frac{1}{\alpha_{11}}} \quad (13)$$

where α_{10} and α_{11} are material dependent constants.

The choice between the enhancement factors γ_s and γ_{s1} is relatively unproblematic, because discrepancy between the two models is small [7]. However, formal investigation of the accuracy of these two models and the goodness of their fit to the same set of raw experimental data seems not to be available. The factor γ_s is simpler in that sense that it contains only one experimental parameter α_7 which depends on the type of reinforcing steel. The factor γ_{s1} includes two material-dependent parameters α_{10} and α_{11} and this may contribute to additional uncertainty related to the predictions made by this model. Contrary to γ_s, the factor γ_{s1} does not contain any explicit relation to the static strength of steel and this may be considered to be a clear deficiency of the latter model. It is known that the influence of strain rate on the strength of steel decreases with the increasing strength and becomes practically negligible in case of highly strong pre-stressing wires. For the aforementioned reasons, the factor γ_s is preferable to γ_{s1} in terms of practical application.

4. Relating loading rate to strain rate

The main input information necessary to apply the enhancement factors γ is the strain rates $\dot{\varepsilon}$ and $\dot{\varepsilon}_s$. These factors can be used only for the case where $\dot{\varepsilon}$ and $\dot{\varepsilon}_s$ are time-independent or can be assumed to be approximately time-independent ones.

While the expressions of γ are valid for any critical section of an RC structure analysed for an explosive loading, $\dot{\varepsilon}$ and $\dot{\varepsilon}_s$ may be different in each individual section. Consequently, the strain rates and thus the dynamic properties of concrete and steel will depend on the location of the section. In each individual section, the rates $\dot{\varepsilon}$ and $\dot{\varepsilon}_s$ will have to be determined in a coupled analysis.

If a section is subjected to an action effect expressed by a scalar function $e(t)$ (e.g., a bending moment), the strains $\varepsilon(t)$ and $\varepsilon_s(t)$ are calculated using the two composite functions

$$\varepsilon(t) = \varepsilon(e(p_r(t))) \tag{14}$$

$$\varepsilon_s(t) = \varepsilon_s(e(p_r(t))) \tag{15}$$

where $p_r(t)$ is the pressure signal of reflected front. With the functions $\varepsilon(t)$ and $\varepsilon_s(t)$, the strain rates $\dot{\varepsilon}(t)$ and $\dot{\varepsilon}_s(t)$ can be determined by applying the standard chain rule of differentiation. Due to the assumption of the constant increase of the reflected pressure in the period $(0, t_{rise})$, the strain rates $\dot{\varepsilon}(t)$ and $\dot{\varepsilon}_s(t)$ become time-independent quantities $\dot{\varepsilon}$ and $\dot{\varepsilon}_s$ which can be expressed in the standard way

$$\dot{\varepsilon} = \dot{\varepsilon}(e)\dot{e}(p)\dot{p} = \frac{d\varepsilon(e)}{de}\frac{de(p)}{dp}\dot{p}_r \tag{16}$$

$$\dot{\varepsilon}_s = \dot{\varepsilon}_s(e)\dot{e}(p)\dot{p} = \frac{d\varepsilon_s(e)}{de}\frac{de(p)}{dp}\dot{p}_r \tag{17}$$

The functions $\varepsilon(e)$ and $\varepsilon_s(e)$ have a fairly complex form and are not amenable to an analytical differentiation even in a simple case of a cracked fragment of an RC element subjected to bending (e.g. [29]). In principle, the derivative $\dot{e}(p)$ can also be difficult to assess analytically. Therefore, a numerical differentiation of $\varepsilon(t)$ and $\varepsilon_s(t)$ seems to be the only practical approach allowing an approximate assessment of the strain rates $\dot{\varepsilon}$ and $\dot{\varepsilon}_s$. Although the numerical differentiation must be applied with caution, its formulas can be used by a simple calculation of the pairs $(\varepsilon(t_i), t_i)$ and $(\varepsilon_s(t_i), t_i)$ for a small value of the differences $t_{i+1} - t_i$ $(i = 1, 2, \ldots; t_1 = 0)$ (e.g. [30]).

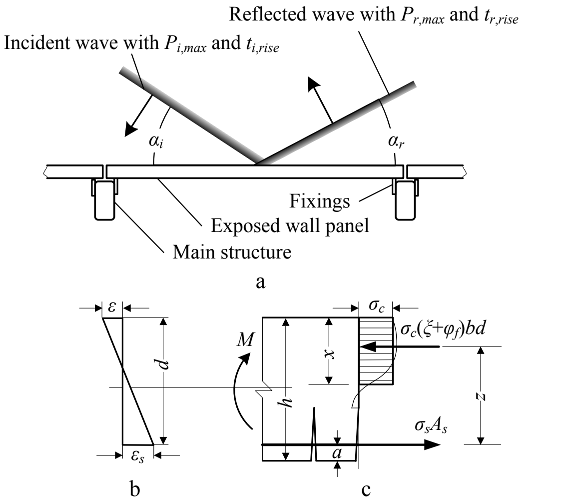

Fig. 5 RC wall panel subjected to explosive loading: (a) idealisation as a simple beam; (b) strain distribution; (c) equivalent stress distribution [29]

The Eq. (12) can be generalised to the case where the action effect in an RC section under analysis is expressed by a vector function $\boldsymbol{e}(\boldsymbol{p}_r(t))$, the components of which, $e_1(t)$, $e_2(t)$, …, are, for instance, compressive force and its eccentricity. The arguments of such a function are expressed by the vector $\boldsymbol{p}_r(t)$ which models the loading on all surfaces involved in the reflection (refraction) of the shock front. The functions $\varepsilon(\boldsymbol{e}(\boldsymbol{p}_r(t)))$ and $\varepsilon_s(\boldsymbol{e}(\boldsymbol{p}_r(t)))$ will be still amenable to a numerical differentiation as long as it is possible to calculate the pairs $(\varepsilon(t_i), t_i)$ and $(\varepsilon_s(t_i), t_i)$.

In principle, one can apply a dynamic finite element analysis for the calculation of the strain values in the pairs $(\varepsilon(t_i), t_i)$ and $(\varepsilon_s(t_i), t_i)$ [2, 3, 19, 20]. It is not clear how accurate such analysis can be, especially in the case of complex structures. Studies allowing to assess this accuracy and its influence of the prediction of dynamic enhancement are not known to the authors of this paper.

An application of Eqs. (14) and (15) to a simple RC beam subjected to a uniform explosive loading is given in the next section. The simple RC beam can be used as an idealisation of exterior wall panel which first meets the shock front generated by a distant explosion (Fig. 5).

5. Example: wall panel subjected to blast

Consider a singly-reinforced RC beam subjected to a uniformly distributed explosive load $p_r(t)$ with the rising part characteristics $P_{i,max}$ = 14 kPa and $t_{i,rise}$ = 0.1 ms (Fig. 2). For the mid-span section of this beam, the action effect $e(t)$ is given by the bending moment $M = \mathrm{C}\,p_r(t)$ and strains $\varepsilon(t)$ and $\varepsilon_s(t)$ are expressed by the functions

$$\varepsilon(t) = \varepsilon(M(t) \mid \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3) \quad (18)$$

$$\varepsilon_s(t) = \varepsilon_s(M(t) \mid \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3) \quad (19)$$

with $\boldsymbol{\theta}_1 = (h, d, b, l, A_s, a)$, $\boldsymbol{\theta}_2 = (E_s, E_c, \nu, f_{c,st}, f_{ct,st})$ and $\boldsymbol{\theta}_3 = (\varphi_{c1}, \psi_c, \psi_s, \beta, \gamma)$, where $\boldsymbol{\theta}_1$ is the vector of geometry parameters; $\boldsymbol{\theta}_2$ is the vector representing material properties of concrete and steel; and $\boldsymbol{\theta}_3$ is the vector of parameters (coefficients) included in the deterministic model used for the calculation of $\varepsilon(t)$ and $\varepsilon_s(t)$. Components of $\boldsymbol{\theta}_1$, $\boldsymbol{\theta}_2$, and $\boldsymbol{\theta}_3$ are explained in part by Fig. 5; a detailed explanation can be found in the design code [29]. This code explains also the functions given in Eqs. (18) and (19).

The strains $\varepsilon(t)$ and $\varepsilon_s(t)$ and the strain rates $\dot{\varepsilon}(t)$ and $\dot{\varepsilon}_s(t)$ were calculated until the instant $t_y = 5.2 \cdot 10^{-5}$s, at which the dynamic yielding strain of steel was reached (see Figs. 6 and 7). This strain was determined using a pilot run of the calculation. A simple two-point formula was used to evaluate the derivatives $\dot{\varepsilon}(t)$ and $\dot{\varepsilon}_s(t)$ [30].

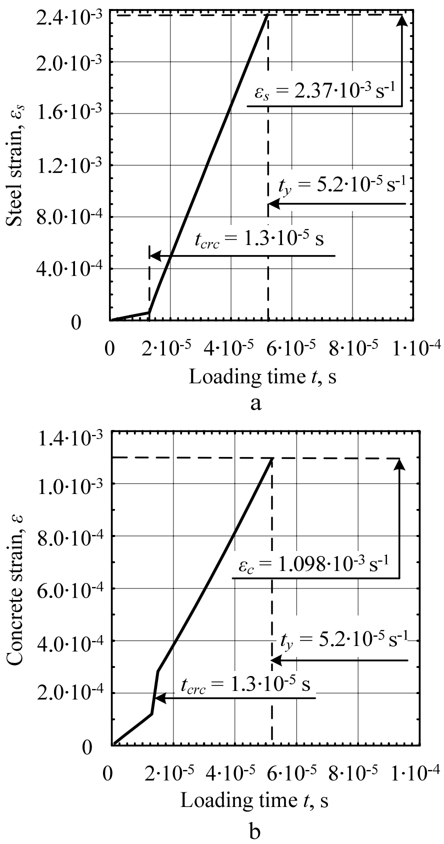

Fig. 6 Time-histories of strain values terminated at reaching the dynamic yielding strain of steel $2.37 \cdot 10^{-3}$ (t_{crc} = cracking time)

One can see from Fig. 6, a that the steel strain $\varepsilon_s(t)$ remains virtually linear after the section cracks at the instant $t_{crc} = 1.3 \cdot 10^{-5}$s. Therefore, the steel strain rate, $\dot{\varepsilon}_s(t)$, is almost constant after t_{crc} (Fig. 7, a). This result allows a direct application of the enhancement factor γ_s defined by Eq. (11). Unfortunately, the model (11) was evaluated only for the strain rates not exceeding $10\ \mathrm{s}^{-1}$. Thus the value of γ_s calculated with the time-averaged strain rate $\bar{\dot{\varepsilon}}_s = 58.7\ \mathrm{s}^{-1}$ shown in Fig. 7, a should be considered an extrapolation.

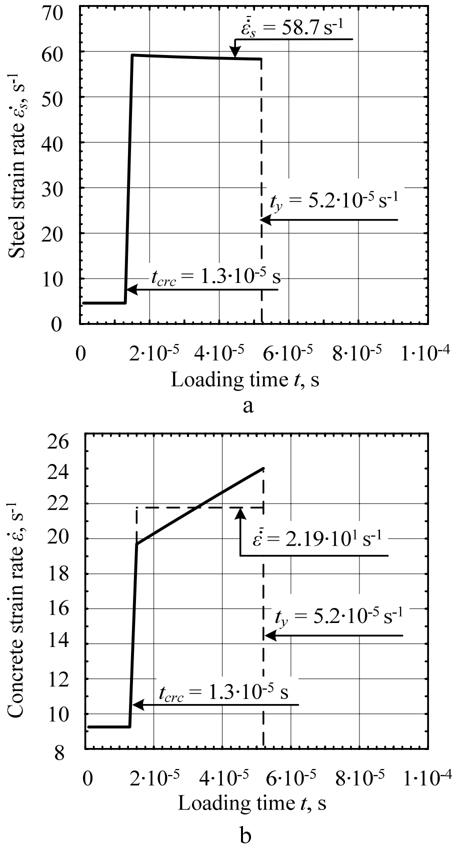

Fig. 7 The time-history of strain rates of concrete and steel (t_{crc} = cracking time)

The concrete strain $\varepsilon(t)$ showed a slight nonlinearity after t_{crc} (Fig. 6, b). This resulted in an inconstant strain rate $\dot{\varepsilon}(t)$ (Fig. 7, b). The values of $\dot{\varepsilon}(t)$ at t_{crc} and t_y differ by 22%. The strain value $\bar{\dot{\varepsilon}}$ obtained by a time-averaging of $\dot{\varepsilon}(t)$ over $[t_{crc}, t_y]$ was $21.9\ \mathrm{s}^{-1}$. As the difference between the strain rates $\dot{\varepsilon}(t_{crc})$ and $\dot{\varepsilon}(t_y)$ is relatively small, the time-averaged value $\bar{\dot{\varepsilon}}$ can be used for the calculation of the enhancement factor γ_c with the formula given in Table 2 for the case $\dot{\varepsilon} \leq 30\ \mathrm{s}^{-1}$.

6. Conclusions

A prediction of dynamic properties of concrete and reinforcing steel cast in RC structures subjected to explosive loading has been considered. Such a prediction requires a combined application of three mathematical models: model I of loading which takes place when a shock front generated by an explosion is reflected and refracted by an exposed structure; model II used to relate the explosive loading to the stress (strain) rates of the concrete and steel in the section under analysis; model III used for an assessment of the dynamic mechanical properties depending on the stress (strain) rates. The third model is a key component of the model set just listed. The prevailing types of this model are the so-called enhancement factors used to relate dynamic properties to corresponding static properties of concrete and steel.

A combined application of the models I to III raises several problems of different nature:

Firstly, the models I and II can be applied only by making several strong assumptions about the physical phenomena of explosive loading and response of structure to this loading. Experimental evidence underpinning these assumptions is either unavailable or difficult to find in the literature.

Secondly, an application of the enhancement factors representing the model III is confined to constant strain rates in concrete and steel. The constant strain rates may not be the case for complex, real world structures subjected to explosive loading. In addition, several candidate enhancement factors are proposed in the literature for both concrete and steel without comparing statistical accuracy of these factors. The choice among them requires a certain degree of guessing which can be based on regarding practical factors (e.g., applicability of the factors to a specific type (class) of concrete or steel).

Thirdly, expressions of the enhancement factors proposed in the literature are purely deterministic and this contradicts the random nature of mechanical properties, static and dynamic. Although it is stated that an application of the enhancement factors yields an average value of the mechanical property in question for a given input produced by the models I and II, a rigorous statistical proof of this statement is not known to us.

The first two of the aforementioned problems will not be decisive for relatively simple structural elements which directly reflect shock front and respond to explosive loading in an easily predictable way (e.g., for wall panels and protective barriers). However, the prediction of dynamic material properties of such elements will remain deterministic unless the models I to III are supplemented by the components (further models) which allow to assess model accuracy and so the uncertainties related to the dynamic properties. A special study is necessary for a statistical extension of the models I to III.

References

1. **Krauthammer, T.** Modern Protective Structures. -Boca Raton etc: CRC Press, 2008.-509p.
2. **Bangash, M.Y.H.** Shock, Impact and Explosion. Structural Analysis and Design. -Berlin etc: Springer, 2009. -1365p.
3. **Bangash, M.Y.H., Bangash, T.** Explosion Resistant Buildings. Design, Analysis, and Case Studies. -Berlin etc: Springer, 2006.-450p.
4. **Federavičius, A., Šaulys, P., Griškevičius, P.** Research of mine imitator interaction with deformable surface. -Mechanika. -Kaunas: Technologija, 2009, Nr.2(76), p.24-27.
5. **Virdi K.S., Matthews, R.S, Clarke, J.L., Garas, F.K. (eds.).** Abnormal Loading on Structures. Experimental and Numerical Modelling. -London & New York: E&F Spon, 2000.-360p.
6. **Raue, E., Timmler, H.-G., Garke, R.** On the physically nonlinear analysis of cyclic loaded reinforced concrete cross-sections with mathematical optimisation.-Journal of Civil Engineering and Management, 2009, 15(2), p.189-195.
7. **CEB.** Concrete Structures under Impact and Impulsive Loading. Synthesis Reprort. Lausanne: Comité Euro-International du Béton. 1988.-184p.
8. **Zhou, X.Q., Kuznetsov, V.A., Hao, H., Waschl, J.** Numerical prediction of concrete slab response to blast loading.-International Journal of Impact Engineering, 2008, 35(10), p.1186-1200.
9. **Shkolnik, E.** Influence of high strain rates on stress-strain relationship, strength and elastic modulus of concrete.-Cement and Concrete Composites, 2008, 30(1), p.1000-1012.
10. **Zheng, D., Li, Q., Wang, L.** A microscopic approach to rate effect on compressive strength of concrete. -Engineering Fracture Mechanics, 2005, 72(15), p.2316-2327.
11. **Low, H.Y., Hao, H.** Reliability analysis of reinforced concrete slabs under explosive loading. -Structural Safety, 2001, 23(2), p.157-178.
12. **Low, H.Y., Hao, H.** Reliability analysis of direct shear and flexural failure modes of RC slabs under explosive loading. -Engineering Structures, 2002, 24(2), p.189-198.
13. **Vaidogas, E.R.** Prediction of Accidental Actions Likely to Occur on Building Structures. An Approach bases on Stochastic Simulation. -Vilnius: Technika, 2007.-247p.
14. **Vaidogas, E.R., Juocevičius, V.** Assessment of structures subjected to accidental actions using crisp and uncertain fragility functions.-Journal of Civil Engineering and Management, 2009, 15(1), p.95-104.
15. **Vaidogas, E.R., Juocevičius, V.** Reliability of a timber structure exposed to fire: estimation using fragility function. -Mechanika. -Kaunas: Technologija, 2008, Nr.5(73), p.35-42.
16. **Tedesco, J.W., McDougal, W.G., Ross, C.A.** Structural Dynamics. Theory and Applications. -Menlo Park, California: Addison Wesley, Longman, Inc., 1999. -870p.
17. **Bulson, P.S.** Explosive Loading on Engineering Structures. London: E&F Spon, 1997.-236p.
18. **Wu, C., Hao, H.** Modeling of simultaneous ground shock and airblast pressure on nearby structures from surface explosions.-International Journal of Impact Engineering, 2005, 31(6), p.699-717.
19. **Wu, Ch., Hao, H.** Numerical simulation of structural response and damage to simultaneous shock and airblast loads.-International Journal of Impact Engineering, 2007, 34(3), p.556-572.
20. **Krauthammer, T., Shanaa, H.M., Assadi, A.** Response of structural concrete elements to severe impulsive loads.-Computer & Structures, 1994, 53(1), p.119-130.
21. **Zhou, X.Q., Hao, H.** Prediction of air blast loads on structures behind a protective barrier.-International Journal of Impact Engineering, 2008, 35(5), p.363-375.
22. **Baker, W.E.** et al. Explosion Hazards and Evaluation. - Amsterdam etc: Elsevier, 1983.-807p.
23. **Mirza, Sh.A., Hatzinikolas, M., MacGregor, G.** Statistical descriptions of strength of concrete.-Journal of Structural Division, 1979, 105, p.1021-1037.
24. **Mihashi, H., Wittmann, F.H.** Stochastic approach to study the influence of rate of loading on strength of concrete. Heron (The Netherlands), 1980.-25p.
25. **Liu, G.Q., Owen, D.R.J.** Ultimate load behaviour of reinforced concrete plates and shells under dynamic transient loading.-International Journal for Numerical Methods in Engineering, 1986, 22(1), p.189-208.
26. **Mirza, Sh.A., MacGregor, G.** Variability of mechanical properties of reinforcing bars. -Journal of Structural

Division, 1979, 105(4), p.921-937.
27. **Janutėnienė, J., Didžiokas, R., Gintalas, M.** Analysis of the variation of metals mechanical properties depending on operation time. -Mechanika. -Kaunas: Technologija, 2009, Nr.1(75), p.26-30.
28. **Kala, Z., Melcher, J., Puklicky, L.** Material and geometrical characteristics of structural steels based on statistical analysis of metallurgical products. -Journal of Civil Engineering and Management, 2009, 15(3), p.299-307.
29. **Hashmi, M.S.J., Hamouda, A.M.S.** Development of a one-dimensional constitutive equation for metals subjected to high strain rate and large strain. -Journal of Strain Analysis, 1994, 29(2), p.117-127.
30. **STR 2.05.05:2005**. Design of concrete and reinforced concrete structures. -Vilnius, Lithuania, 2005 (in Lithuanian).
31. **Faires, J.D., Burden, R.L.** Numerical Methods. 3rd ed. -Boston: Brooks Cole, 2002.-640p.

V. Juocevičius, E. R. Vaidogas

MECHANINIŲ BETONO IR ARMATŪROS SAVYBIŲ PROGNOZAVIMAS SPROGIMŲ SUKELIAMO APKROVIMO ATVEJU

R e z i u m ė

Nagrinėjamas dinaminių betono ir armatūros savybių prognozavimas. Dėmesys telkiamas į dinamines gelžbetoninių konstrukcijų apkrovas, sukeliamas didelių nuotolinių sprogimų. Prognozuojama taikant matematinius modelius, atspindinčius sprogimo apkrovą, betono ir armatūros deformavimą veikiant šiai apkrovai ir dinaminį statinių šių medžiagų savybių didėjimą. Aptariamos problemos, kylančios derinant šiuos modelius vieną su kitu. Daroma išvada, kad dinaminių savybių prognozė daugiausia yra konstrukcijos atsparumo sprogimo apkrovai skaičiavimo rezultatas. Prognozavimas medžiagų lygmenyje suvedamas į didinančiųjų daugiklių, siejančių statines ir dinamines medžiagų savybes, taikymą. Juos taikyti sunku dėl to, kad informacija, reikalinga daugikliams skaičiuoti, turi būti gauta skaičiuojant konstrukcijos atsparumą sprogimo apkrovai, o tai gali būti sudėtinga, kai konstrukcinės sistemos yra kompleksinės.

V. Juocevičius, E. R. Vaidogas

EFFECT OF EXPLOSIVE LOADING ON MECHANICAL PROPERTIES OF CONCRETE AND REINFORCING STEEL: TOWARDS DEVELOPING A PREDICTIVE MODEL

S u m m a r y

Prediction of dynamic mechanical properties of concrete and reinforcing steel is considered. The attention is focussed on dynamic loading imposed on reinforced concrete structures by large distant explosions. The prediction is made by a combined application of mathematical models describing explosive loading, straining of concrete and steel under this loading and dynamic enhancement of static mechanical properties of these materials. Problems arising at coupling these models in a combined set are discussed. The main finding is that prediction of dynamic properties is to a large extent a problem of structural analysis for explosive loading. The prediction on material level consists mainly in an application of enhancement factors relating static and dynamic properties. The intricacy of these factors lies in the input information required for their calculation. This information is obtained by a structural analysis for explosive loading. Such an analysis can be a complicated task for complex, real world structural systems.

В. Юоцявичюс, Э. Р. Вайдогас

ПРОГНОЗИРОВАНИЕ МЕХАНИЧЕСКИХ ХАРАКТЕРИСТИК БЕТОНА И АРМАТУРНОЙ СТАЛИ ДЛЯ СЛУЧАЯ ВЗРЫВНОЙ НАГРУЗКИ

Р е з ю м е

Рассматривается прогнозирование динамических свойств бетона и арматурной стали. Внимание сосредоточено на динамических нагрузках на железобетонные конструкции, вызываемых большими удаленными взрывами. Прогнозирование осуществляется применяя математические модели, описывающие взрывную нагрузку, деформирование бетона и арматуры при действии этой нагрузки и динамическое увеличение свойств этих материалов. Обсуждаются проблемы, возникающие при комбинировании этих моделей. Получено, что прогноз динамических свойств по большей части является результатом расчета конструкции на динамическую нагрузку взрыва. Прогнозирование на уровне материалов сводится к применению увеличивающих факторов, связывающих статические и динамические свойства материалов. Проблема применения этих факторов состоит в том, что для их расчета необходимо осуществлять расчет конструкции на взрывную нагрузку. Такой расчет может быть затруднительным для сложных конструкционных систем.

Received November 16, 2009
Accepted January 21, 2010