ISSN 1392–1207. MECHANIKA. 2015 Volume 21(4): 290–295

Finite element analysis of free vibration of beams with composite coats

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crossref http://dx.doi.org/10.5755/j01.mech.21.3.9849

1. Introduction

In the modern civil engineering structures, such as buildings, steel framed structures and bridges, use of coated laminated composite beams has increased rapidly in recent years. Many studies exist on the dynamic behavior of isotropic beams using the analytical, experimental and numerical methods (Biggs, [1]; Clough [2]; Khadri et al., [3, 4]). However, the number of studies related to the free vibration of beams with composite coats is relatively less. Hamada et al. [5] studied the variations in the natural frequencies and damping properties of laminated composite coated beams utilizing a numerical technique to compute the Eigen parameters of coated laminated composite beams (sandwich structures). Kiral et al. [6] studied the dynamic behavior composite beam subjected to vertical moving force using the commercial finite element. Using analytical technique, Zibdeh et al. [7] studied the vibration of a simply-supported laminated composite coated beam traversed by a random moving load. Tekili et al., [8] utilised the analytical analysis of free vibration of simply-supported laminated composite coated beams. Kadivar et al., [9] the one dimensional finite elements based on classical lamination theory, first-order shear deformation theory, and higher-order shear deformation theory are developed to study the dynamic response of an unsymmetric composite laminated orthotropic beam. Mohebpour et al., [10] presented the free vibration and moving oscillator problems analysis of isotropic and composite laminated beams are presented using the finite element method. There are numerous publications on composite structures which employed the experimental method. In this study, the free vibration of strengthened beams by composite coats has been investigated by use of finite element method (FEM). For this purpose, a computer code is developed using MATLAB to perform the finite element vibration analysis. The parametric analysis is conducted to study the effects of the variation of different parameters such as the thickness of faces, core thickness and the fiber orientation, and type of core isotropic material (steel and foam) on natural frequencies of the beam are examined with different boundary conditions imposed on the beam. The beam frequencies extracted in this regard will be compared with those obtained analytically.

2. Theoretical formulation

A laminated composite coated beam with its physical dimensions shown in Fig. 1. The core is made from an isotropic material (steel and foam), where L, b, and $2H$ are the length, the width and thickness of the beam, respectively. The top and bottom lamina are made from composite material (glass/epoxy) with the thickness ($H - h$) as shown in Fig. 1.

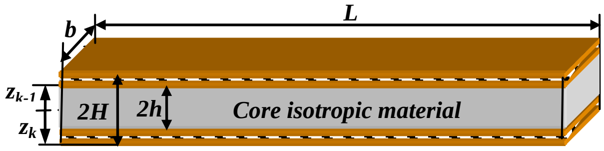


Fig. 1 Geometry of a laminated composite coated beam

In the case of pure bending of a symmetric laminate beam, the constitutive equation to the momentum equation [11]:

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}, \quad (1)$$

where M_x, M_y, and M_{xy} are the bending and twisting moments, and κ_x, κ_y, and κ_{xy} are the curvatures of plate, whichare defined by

$$\kappa_x = \frac{\partial \varphi_x}{\partial x}, \quad \kappa_y = \frac{\partial \varphi_y}{\partial y}, \quad \kappa_{xy} = \frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}, \quad (2)$$

with φ_x and φ_y are rotations and the stiffness parameters is:

$$D_{ij} = \frac{1}{3}\sum_{k=1}^{n}\left(z_k^3 - z_{k-1}^3\right)\left(Q_{ij}^{'}\right)_k, \quad (3)$$

where $Q_{ij}^{'}$ is the reduced stiffness constant of a unidirectional. The beam theory makes the assumption that in the case of bending along x-direction, the bending and twisting moments M_y and M_{xy} are zero. Eq. (1) thus lead to:

$$M_x = D_{11}\kappa_x = -D_{11}\frac{\partial^2 w}{\partial x^2}, \quad (4)$$

where w is displacement and the stiffness parameter is:

$$D_{11} = \frac{1}{3}\sum_{k=1}^{n}\left(z_k^3 - z_{k-1}^3\right)\left(Q_{11}^{'}\right)_k. \quad (5)$$

The reduced stiffness constant of a unidirectional layer, off its material directions is obtained by:

$$Q'_{11} = Q_{11}cos^4\theta + Q_{22}sin^4\theta + 2(Q_{12}+Q_{66})\,sin^2\theta\,cos^2\theta\,, \qquad (6)$$

where θ is the angle between the principal laminate's direction and the axis of the beam. The elastic constants Q_{ij} in the principal material coordinate system are expressed as follows:

$$\left.\begin{aligned} &Q_{11} = \frac{E_{11}}{1-\upsilon_{12}^2\dfrac{E_{22}}{E_{11}}}; \quad Q_{22} = \frac{E_{22}}{E_{11}}Q_{11}; \\ &Q_{12} = \upsilon_{12}Q_{11}; \quad Q_{66} = G_{12}, \end{aligned}\right\} \qquad (7)$$

where E_{11}, E_{22}, G_{12} and υ_{12} are the engineering parameters of the kth lamina. The equivalent mass per unit length of the laminated composite beam is expressed as:

$$\rho_s = b\sum_{k=1}^{n}\rho_k\left(z_k - z_{k-1}\right) = 2b\left(\rho_c h + \rho_f\left(H-h\right)\right), \qquad (8)$$

where ρ_c and ρ_f are the densities of the core and faces of the beam, respectively. Lastly, the beam theory makes the additional assumption that the deflection is a function of x only: $w = w(x, t)$. So, the mode shape of beam only depends on the coordinate x. In the framework of the beam theory, in this case the fundamental equations of laminates are simplified as:

$$\frac{\partial^2 M_x}{\partial x^2} + q = \rho_s\frac{\partial^2 w}{\partial t^2}, \qquad (9)$$

where q is pressure load applied to the beam. For free vibration analysis ($q = 0$), the relevant equation can be written as:

$$\rho_s\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(bD_{11}\frac{\partial^2 w}{\partial x^2}\right) = 0. \qquad (10)$$

It may be noted here that for uniform composite beam and for each uniform segment of an externally tapered composite beam, bD_{11} is constant.

3. Finite element formulation

The approach of separation of variables is being applied for $w(x, t)$, and it can be expressed as the product of two functions one in displacement 'x' and the other in time 't' as

$$w\left(x,t\right) = \sum_{j=1}^{n} N_j\left(x\right)u_j\left(t\right), \qquad (11)$$

where $w(x, t)$represents the solution of the governing differential equation in hand. The displacement components of a beam element shown in Fig. 2 can be expressed in the form:

$$w^{(e)}\left(x,t\right) = N_1 v_i + N_2\theta_i + N_3 v_j + N_4\theta_j\,, \qquad (12)$$

where $\{u\} = \{v_i\ \varphi_i\ v_j\ \varphi_j\}^T$ is the nodal displacement vector for the element with v and φ are in transverse displacement and slope at the nodal.

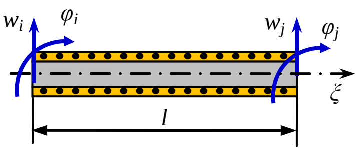


Fig. 2 Beam element

The N_i (i = 1, 4) are shape functions of the beam element which can be obtained as follows [2]:

$$\left.\begin{aligned} &N_1 = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3}; \quad N_2 = x - \frac{2x^2}{l} + \frac{x^3}{l^2}; \\ &N_3 = \frac{3x^2}{l^2} - \frac{2x^3}{l^3}; \quad N_4 = -\frac{x^3}{l} + \frac{x^3}{l^2}, \end{aligned}\right\} \qquad (13)$$

where l is the beam element and x is the local coordinate of the beam element. In the finite element formulation an integral statement is to be established to develop algebraic relations. The Galerkin method leads to the following equation:

$$\int_0^l N > \left(\rho_s\frac{\partial^2 w}{\partial t^2} + \frac{\partial^2}{\partial x^2}\left(bD_{11}\frac{\partial^2 w}{\partial x^2}\right)\right)dx = 0. \qquad (14)$$

Substituting Eq. (11) into Eq. (14), then:

$$\left(\int_0^l N_i\rho_s N_j dx\right)\frac{\partial^2 u_j}{\partial t^2} + \\ + \left(\int_0^l N_i\left(\frac{\partial^2}{\partial x^2}\left(bD_{11}\frac{\partial^2 N_j}{\partial x^2}\right)\right)dx\right)u_j = 0 \quad i,\ j = 1,4. \quad (15)$$

Integration by parts the weighted integral statement twice, one can get the following equation:

$$\left(\int_0^l N_i\rho_s N_j dx\right)\frac{\partial^2 u_j}{\partial t^2} + \\ + \left(\int_0^l \frac{\partial^2 N_i}{\partial x^2}bD_{11}\frac{\partial^2 N_j}{\partial x^2}dx\right)u_j = 0 \quad i,\ j = 1,4 \qquad (16)$$

Then, the element stiffness $k^{(e)}$ and element mass matrices $m^{(e)}$ are given by:

$$k^{(e)} = k_{ij} = \int_0^l \frac{d^2 N_i}{dx^2}bD_{11}\frac{d^2 N_j}{dx^2}dx \quad i,\ j = 1,4; \qquad (17)$$

$$m^{(e)} = m_{ij} = \int_0^l N_i\rho_s N_j dx \quad i, j = 1,4. \qquad (18)$$

The overall mass and stiffness matrices are obtained by assembling the element matrices:

$$K = \sum_{e=1}^{n} k^{(e)}; \qquad (19)$$

$$M = \sum_{e=1}^{n} m^{(e)}, \qquad (20)$$

where n is the total number of discretized elements. The equation of the undamped free vibration of beams with composite coats may be expressed as:

$$M\frac{\partial^2 u_j}{\partial t^2}+Ku_j=0. \tag{21}$$

If the system is vibrating in a normal mode, we obtain an eigenvalue problem as:

$$\left(K-\omega_j^2 M\right)\phi=0, \tag{22}$$

where ω_j is the j-th natural frequency and ϕ is the corresponding modal displacements. The Eq. (22) has a solution if and only if its determinant is zero, ie:

$$det\left(K-\omega_j^2 M\right)=0. \tag{23}$$

The roots of the Eq. (23) are the characteristic values, which are equal to the squares of the natural frequencies.

4. Numerical results

A computer code is developed using the MATLAB in order to calculate the natural frequencies and the modes of natural vibrations of an undamped beam with composite coats. The material geometrical properties and physical dimensions of the beam are the same as [7]. The beam has length, L = 500 mm, width b = 25 mm, thickness H = 4 mm. Table 1 shows material properties for the face and core of the beam models used in the study. The material of the core is the steel and the foam (Divinycell H200) for model I and II respectively. The faces are made from glass/epoxy composite material for the two models.

Table 1

Material properties for the face and core of the beam models

Model	Layer	Material	ρ, kg/m^3	E_{11},GPa	E_{22}, GPa	G_{12}, GPa	υ_{12}
I	Core I	Steel	7850	200	200	77	0.3
II	Core II	Foam	200	0.277	0.277	0.11	0.33
	Face	Glass/Epoxy	1759	38.6	8.27	4.14	0.26

For validation tests, the natural frequencies calculated by analytical and numerical method for simply supported sandwich beam.The natural frequencies of the simply supported of a sandwich beam are expressed analytically by [8, 11]:

$$\omega_j=\frac{j^2\pi^2}{L^2}\sqrt{\frac{1}{\rho_s D_{11}^{-1}}}\,. \tag{24}$$

In the first validation test, the beam is viewed with $h/H=0$ and $\theta=0°$ and for ten first modes. From Fig. 3 one observes that the natural frequencies computed by the finite element method agrees generally well with the analytical one (Eq. (24)) with the considerable deviation for number of finite element ne = 10 and with the slight deviation for ne = 50.

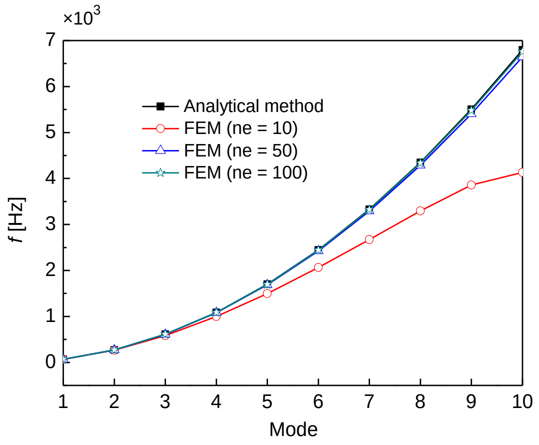


Fig. 3 Comparison between the FE results and the analytical solution for the first ten modes

For second verification (Fig. 4), the model I is considerate with h/H varies from 0 to 1, with $\theta=0°$ and for mode 1. However, the good agreement was carried out for ne = 100. It can be also verified that when the number of elements increases, the numerical results converge to the exact solutions. Thus, the number of the finite elements used in vibration analysis is ne = 100.

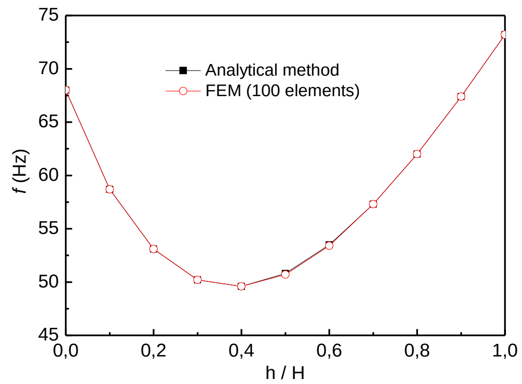
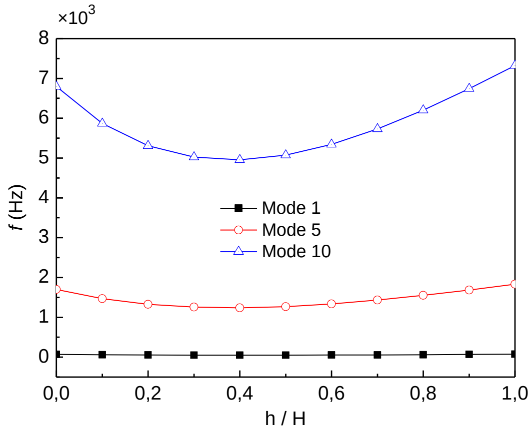


Fig. 4 Comparison between the FE results and the analytical solution for a first mode and with different thickness ratios

The natural frequencies corresponding to mode 1, 5 and 10 are plotted in Figs. 5, a and 5, b, for case of the simply supported laminated composite coated beam and for two models with $\theta=0°$. As, can be seen from of these figures that the natural frequencies are affected by the thickness ratio for high modes. However for mode 1, the curve of the frequency is almost horizontal, it remains nearly independent of the thickness ratio, relatively compared to the higher modes. According to Fig. 5, a, we note that the natural frequencies of the full steel beam are almost identical to those of the full composite beam and this is for frequencies bases, however for the high frequencies, a small difference is found. While, a large difference is found between the natural frequencies of the full steel beam and the full foam beam (Fig. 5, b) and this is due to differences in stiffness of the two materials.



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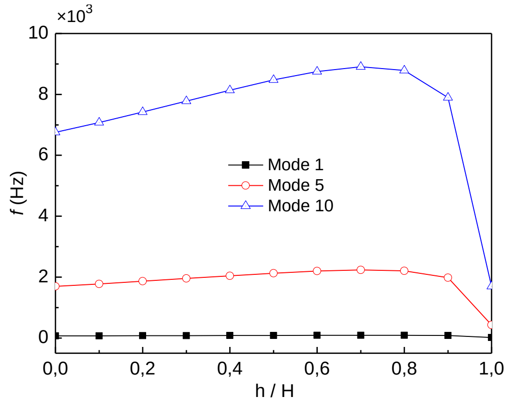
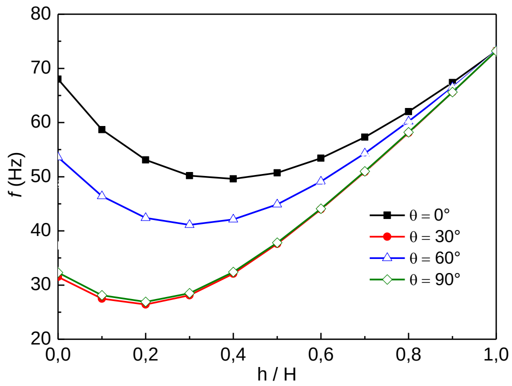


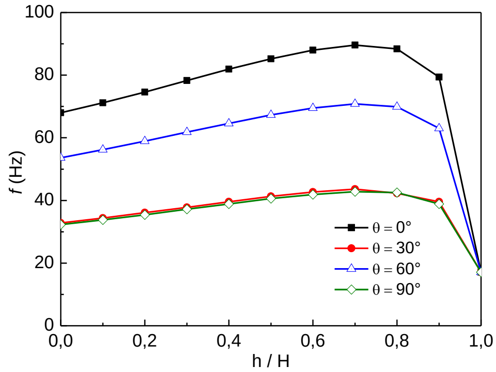
b

Fig. 5 Natural frequency of sandwich beam with different mode: a - Model I; b - Model II

For the following analysis, we assumed that the fiber angle of the composite layer vary of the 0° to 90° with an increment of 30°. The Figs. 6, a and 6, b shows the variation of the first natural frequency of the sandwich beam versus the thickness ratio, with various fiber orientations. A linear relation is observed between frequency and thickness ratio for Model I (from $h/H = 0.5$ to 1.0) and for Model II (from $h/H = 0.5$ to 0.7). On the other hand, the frequency increases with the increase in the thickness ratio, in these linear domains. The maximum frequency is reached for a sandwich beam model I with $h/H = 0.2$ and $\theta = 0°$ (Fig. 6, a), while the maximum value of the frequency is carried out for the sandwich beam Model II, with $h/H = 0.7$ and $\theta = 0°$ (Fig. 6, b). The first natural frequency of the sandwich beam for two thickness ratio ($h/H = 0.2$ and 0.7) is shows versus fiber orientation angle for Model I and II in the Figs. 7, a and 7, b, respectively. For the Model I (Fig. 7, a) with a strengthening of the order of 30% ($h/H = 0.7$), the effect of fiber orientation on the natural frequency is low, this is due to the domination of the heavy nucleus (steel), which is not the case in the Model II (Fig. 7, b), the composite layer is dominant over the central layer (foam). For the sandwich beam with thickness ratio $h/H = 0.2$, the effect of fiber orientation on the natural frequency is larger and this for the two Models I and II. To investigate the influence of the boundary conditions of the sandwich beam on the first natural frequencies, four different boundary conditions were imposed on the beam (Fig. 8): simply-supported–simply-supported (S–S), clamped–clamped (C–C), clamped–free (C–F), and clamped– simply-supported (C–S).

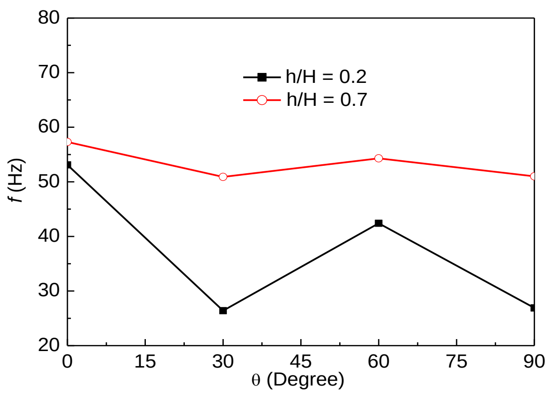


a



b

Fig. 6 First frequency of sandwich beam versus thickness ratio: a - Model I; b - Model II



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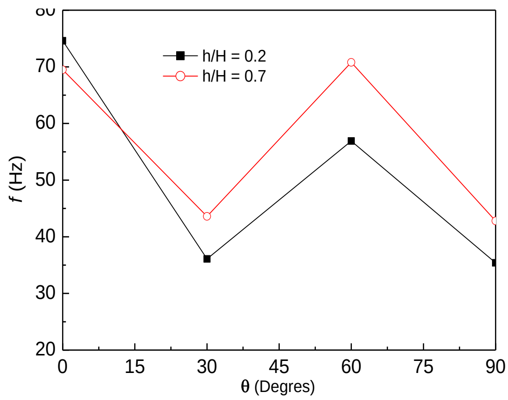


b

Fig. 7 Natural frequency of the sandwich beam versus fiber orientation angle: a - Model I; b - Model II

The first natural frequencies of the beam with coating composite ($h/H = 0.25$ and 0.75) are calculated and presented in Table 2 for both Models I and II with different fiber orientations.

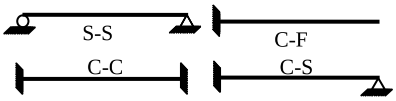


Fig. 8 Different boundary conditions were imposed on the beam with composite coats

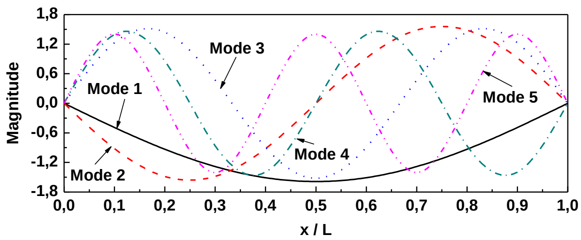
For both Models I and II, it can be indicated that the frequency of the sandwich beam with fiber orientation $\theta = 0°$ is relatively high compared with the others cases. This is explained by the fact that the fiber orientation to 0° (direction x) provides a maximum rigidity of the sandwich beam. Moreover, the maximum natural frequency value is reached to boundary conditions clamped–clamped (C–C) and this is evident. By comparing the natural frequencies of the beam with a composite coating $h/H = 0.25$ and 0.75, one can conclude that the maximum frequency is 201.4 Hz**,** that of the sandwich beam with thickness ratio $h/H = 0.75$.

Table 2

The first natural frequencies (Hz) of the beam with and without coating composite

Model	Boundary conditions	without coating	with coating							
			$h/H = 0.25$				$h/H = 0.75$			
			θ = 0°	30°	60°	90°	θ = 0°	30°	60°	90°
I	S-S	72.8	51.4	26.9	41.5	27.4	59.2	54.2	56.9	54.2
	C-C	164.8	**116.4**	61.	94.	62.1	**134.1**	122.6	128.7	122.8
	C-F	26.1	18.3	9.6	14.8	9.8	21.3	19.4	20.4	19.5
	C-S	113.6	80.2	42.	64.8	42.8	92.4	84.5	88.7	84.6
II	S-S	17.	76.4	35.4	60.3	36.3	89.	41.6	70.3	42.6
	C-C	38.4	**173.3**	80.3	136.8	82.3	**201.4**	94.2	159.2	96.5
	C-F	6.1	27.2	12.6	21.5	12.9	31.9	14.9	25.2	15.3
	C-S	26.5	119.4	55.4	94.3	56.7	138.8	65.	109.8	66.5

The first five mode shapes of the beam without coating are presented in Figs. 9 and 10 for both Models I and II. These two Figs. 9 and 10 show the mode shapes for boundary conditions simply-supported–simply-supported (S–S) and clamped–clamped (C–C), respectively. Since the Model II is more flexible than the Model I, it produces the larger amplitude of the modes.



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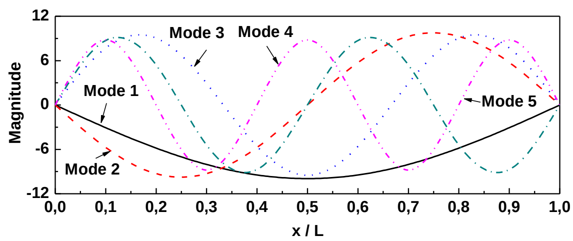
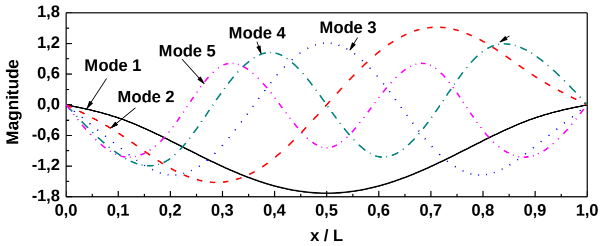


b

Fig. 9 First five modes shape of the beam for BC: S-S: a - Model I; b - Model II



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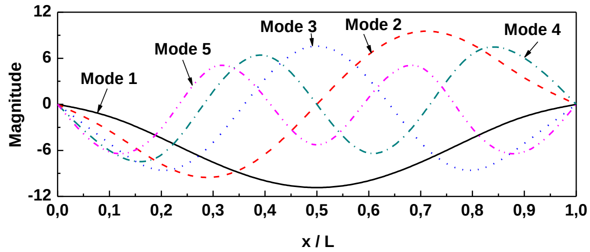

b

Fig. 10 First five modes shape of the beam for BC; C-C: a - Model I; b - Model II

The first mode of the Model I and II of the simply supported the beam with and without coating composite (fiber angle, $\theta = 0°$) with different thickness of the glass/epoxy composite layer are shown in Fig. 11. For the Model I (Fig. 11, a) when the thickness ratio increases (the thickness of the layer composite decreases), i.e., the stiffness the sandwich increases, and thus the amplitude decreases. That is due to the fact that in the Model I, the stiffness of the core layer is higher than that of the face layer. However, for Model II (Fig. 11, b), it be found that the opposite effect is produced, because the rigidity of the sandwich decreases with the increase thickness ratio.

As seen from the results, it is clear that the natural frequency and mode shapes of the beam with coating layer composite can be controlled by choosing the proper fiber orientation, the laminate thickness and the boundary conditions imposed on the sandwich beam.

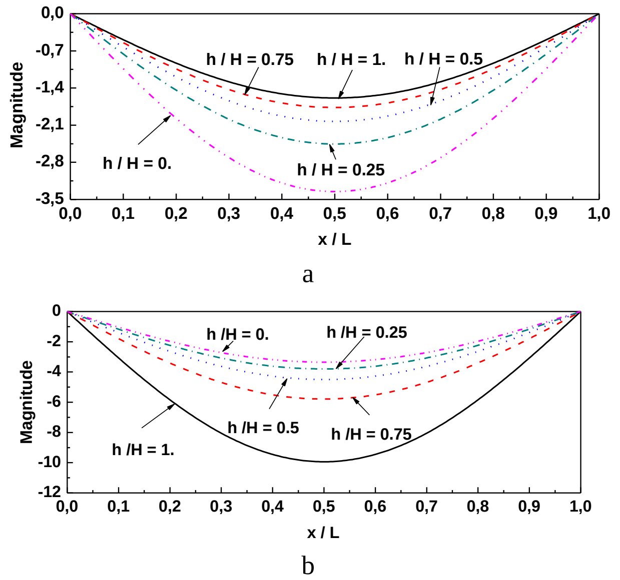

Fig. 11 Mode shapes plot of the beam with coating composite and for S-S: a - Model I; b - Model II

5. Conclusions

In the present study, the analysis of free vibration of beams with composite coats has been investigated numerically by finite element method and verified analytically. The following conclusions can be drawn from the present study: (1) the natural frequencies obtained by the FEM are in good agreement with those of the method analytical, (2) the natural frequency increases generally with the increase in the thickness ratio, (3) when the stiffness of the face layers is higher than that of the core layer, a linear relation is observed between frequency and thickness ratio and the amplitude of the mode increases when the thickness ratio increases, (4) the frequencies are larger with a fiber orientation of 0° and this for any thickness of the reinforcing layer, This is explained by the fact that the fibers are the direction of the sandwich beam, (5)the boundary conditions clamped–clamped (C–C) give the maximum natural frequency value.

Acknowledgements

This work is one of the research projects of the Ministry for High Education and Scientific Research of Algeria. Code of project: J0301120130097. The National Commission for the Evaluation of the Research Projects gave a favorable opinion for three years (2013-2016).

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FINITE ELEMENT ANALYSIS OF FREE VIBRATION OF BEAMS WITH COMPOSITE COATS

S u m m a r y

This paper presents a finite element model to investigate the analysis of free vibration of beams with composite coats. We used two sandwich beam models; the core is made from an isotropic material, the steel as heavy material, for first model, and the foam as light material, employed in the second model. The faces are made from glass/epoxy composite material for the two models. The natural frequency and mode shapes of the sandwich beam are controlled by choosing the proper fiber orientation, the laminate thickness and the boundary conditions. The effects of these parameters are examined for the two models with different boundary conditions imposed on the beam. Good agreements were achieved between the finite element method and analytical solutions.

Keywords: free vibration, composites coats, finite element method, dynamic beams.

Received February 15, 2015
Accepted April 14, 2015